

Cellular Systems with Full-Duplex Amplify-and-Forward Relaying and Cooperative Base-Stations

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Abstract—In this paper the benefits provided by multi-cell processing of signals transmitted by mobile terminals which are received via dedicated relay terminals (RTs) are assessed. Unlike previous works, each RT is assumed here to be capable of full-duplex operation and receives the transmission of adjacent relay terminals. Focusing on intra-cell TDMA and non-fading channels, a simplified uplink cellular model introduced by Wyner is considered. This framework facilitates analytical derivation of the per-cell sum-rate of multi-cell and conventional single-cell receivers. In particular, the analysis is based on the observation that the signal received at the base stations can be interpreted as the outcome of a two-dimensional linear time invariant system. Numerical results are provided as well in order to provide further insight into the performance benefits of multi-cell processing with relaying.

I. INTRODUCTION

Techniques for provision of better service and coverage in cellular mobile communications are currently being investigated by industry and academia. In this paper, we study the combination of two cooperation-based technologies that are promising candidates for such a goal, extending previous work in [1] [2]. The first is relaying, whereby the signal transmitted by a mobile terminal (MT) is forwarded by a dedicated relay terminal (RT) to the intended base station (BS) [3] (see also [4] for a more recent account). The throughput of such hybrid networks has recently been studied in the limit of asymptotically many nodes [5][6]. Moreover, information theoretic characterization of related single-cell scenarios has been reported in [7]. The second technology of interest here is multi-cell processing (MCP), which allows the BSs to jointly decode the received signals, equivalently creating a distributed receiving antenna array [8]. The performance gain provided by this technology within a simplified cellular model was first studied in [9][10], and then extended to include fading channels by [11], under the assumption that BSs are connected by an ideal backbone (see [12][13] for surveys on MCP).

Recently, the interplay between these two technologies has been investigated for amplify-and-forward (AF) and decode-and-forward (DF) protocols in [1] and [2], respectively. The basic framework employed in these works is the Wyner uplink cellular model introduced in [9]. Following the linear variant of this model, cells are arranged in a linear geometry and only adjacent cells interfere with each other. Moreover, inter-cell interference is described by a single parameter $\alpha \in [0, 1]$,

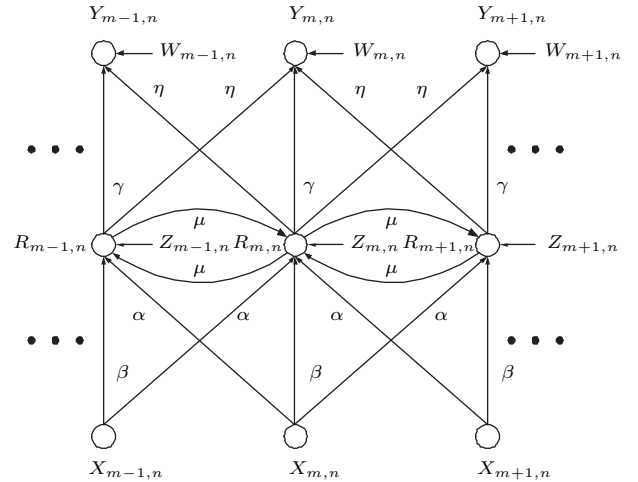


Fig. 1. Network model

which defines the gain experienced by signals travelling to interfered cells. Notwithstanding its simplicity, this model captures the essential structure of a cellular system and it provides insight into the system performance. The RTs added to the basic Wyner model in [1][2] are assumed to operate in a half-duplex mode and to receive signals from the MTs only (and not from adjacent RTs).

In this work we relax the latter restrictions by allowing full-duplex operation at the RTs and considering the signal path between adjacent RTs. Focusing on an intra-cell time-division multiple-access (TDMA) operation and non-fading channels, we assess the gain provided by the joint MCP approach over the conventional single-cell processing (SCP) scheme by deriving the per-cell sum-rate in the two scenarios. We finally remark that a further contribution of this paper with respect to [1][2] is the extension to a relaying scenario of the analytical framework introduced in [9], whereby the signal received by the BSs is interpreted as the outcome of a linear time-invariant system.

II. SYSTEM MODEL

We consider the uplink of a cellular system with a dedicated RT for each transmitting MT. We focus on a scenario with no fading and employ the framework of a linear cellular uplink

channel presented by Wyner [9]. RTs are added to the basic Wyner model following the analysis in [1][2] (see Fig. 1 for a schematic diagram of the setup). Throughout this paper we make the following underlying assumptions:

- The system includes infinitely many identical cells arranged on a line.
- A single MT is active in each cell at a given time (intra-cell TDMA protocol).
- A dedicated single RT is available in each cell to relay the signal from the MT.
- The signals from the MTs are received by the BSs via the relays (and not directly from the MTs).
- Each RT receives the signals of the MTs from its own cell and the two adjacent cells only.
- Each BS receives the signals of the RTs from its own cell and the two adjacent cells only.
- The channel power gain from the MT to its local RT, and its two adjacent RTs are denoted by β^2 and α^2 respectively.
- The channel power gain from the RT to its local BS, and its two adjacent BSs are denoted by η^2 and γ^2 respectively.
- The channel power gain from the RT to its two adjacent RTs is μ^2 .
- The MTs use independent randomly generated complex Gaussian codebooks with zero mean and power P .
- The average transmit power of each RT is Q .
- The RTs are assumed to be oblivious and to use an AF relaying scheme.
- The RTs are assumed to be capable of receiving and transmitting simultaneously (i.e., we assume full-duplex operation, which amounts to assuming perfect echo-cancellation between transmit and receive paths).
- The RTs amplify and forward the received signal with a delay of $\lambda \geq 1$ symbols (an integer).
- The propagation delays between the different nodes of the system are negligible with respect to the symbol duration.
- No cooperation is assumed among MTs.
- No cooperation is assumed among RTs.
- All the attenuation parameters are known to the BSs.

The main differences between the current model and the model presented in [1] [2], are: (a) full-duplex operation at the relays (which introduces the relaying delay λ); (b) no direct connection between the MTs and the BSs; and (c) the RTs receives also the signals of the two adjacent MTs.

Accounting for the underlying assumptions listed above, a baseband representation of the signal transmitted by the m 'th RT for an arbitrary time index n is given by

$$R_{m,n} = g(\beta X_{m,n} + \alpha X_{m-1,n} + \alpha X_{m+1,n} + \mu R_{m-1,n-\lambda} + \mu R_{m+1,n-\lambda} + Z_{m,n}) , \quad (1)$$

where Z represents the additive complex Gaussian noise process $Z_{m,n} \sim \mathcal{CN}(0, \sigma_Z^2)$, which is assumed to be independent and identically distributed (i.i.d.) with respect to both the time and cell indices. The received signal at the m 'th BS antenna

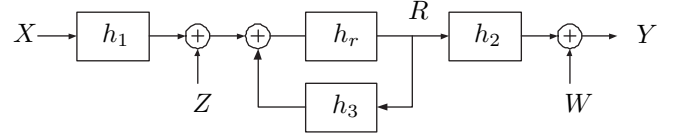


Fig. 2. Equivalent 2D LTI channel.

is given by

$$Y_{m,n} = \gamma R_{m,n-\lambda} + \eta R_{m-1,n-\lambda} + \eta R_{m+1,n-\lambda} + W_{m,n} , \quad (2)$$

where W represents the additive complex Gaussian noise process $W_{m,n} \sim \mathcal{CN}(0, \sigma_W^2)$, which is assumed to be i.i.d. with respect to both the time and cell indices and to be statistically independent of Z . In addition, the RTs' gain g is selected to satisfy the average power limitation

$$\sigma_r^2(g) \triangleq E\{|R_{m,n}|^2\} \leq Q .$$

III. SUM-RATE ANALYSIS

In this section, we derive the per-cell sum-rate of the cellular system at hand with MCP at the BSs and in the reference case with SCP.

A. Joint Multi-Cell Processing

In this section we assume that the signals received at all BSs are jointly decoded by an optimal central receiver. The receiver is connected to the BSs via an ideal backbone and is assumed to be aware of the Gaussian codebooks of all the MTs. It is noted that using similar arguments as in [9], it can be shown that in this setup an intra-cell TDMA protocol is optimal.

Extending the one dimensional (1D) model introduced in [9], the linear equations (1) and (2) describing the network of Fig. 1 can be interpreted as a two dimensional (2D) linear time invariant (LTI) system. The block diagram of the equivalent 2D LTI system is depicted in Fig. 2 where the 2D filters read

$$\begin{aligned} h_{1m,n} &= \delta_n(\alpha\delta_{m-1} + \beta\delta_m + \alpha\delta_{m+1}) \\ h_{2m,n} &= \delta_n(\eta\delta_{m-1} + \gamma\delta_m + \eta\delta_{m+1}) \\ h_{rm,n} &= g\delta_{n-\lambda}\delta_m \\ h_{3m,n} &= \mu\delta_n(\delta_{m-1} + \delta_{m+1}) , \end{aligned} \quad (3)$$

with δ_n denoting the *Kronecker* delta function. The corresponding 2D *Fourier* transforms of the signals in (3) are given by

$$\begin{aligned} H_1(\theta, \varphi) &= \beta + 2\alpha \cos \theta \\ H_2(\theta, \varphi) &= \gamma + 2\eta \cos \theta \\ H_r(\theta, \varphi) &= g e^{-j\lambda\varphi} \\ H_3(\theta, \varphi) &= 2\mu \cos \theta . \end{aligned} \quad (4)$$

Since the noise processes Z and W are zero mean i.i.d. complex Gaussian and statistically independent of each other and of the input signal X , the output signal at the BSs can be expressed as

$$Y_{m,n} = S_{m,n} + N_{m,n} , \quad (5)$$

where $S_{m,n}$ and $N_{m,n}$ are zero mean wide sense stationary (WSS) statistically independent processes representing the useful part of the signal and the noise respectively. Now, using the 2D extension of Szegő's theorem [9], the achievable rate in the channel (5) (without spectral shaping), which is equal to the achievable per-cell sum-rate of the network, is given for arbitrary g by

$$R_{\text{mcp}} = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \log \left(1 + \frac{\mathcal{S}_S(\theta, \varphi)}{\mathcal{S}_N(\theta, \varphi)} \right) d\varphi d\theta, \quad (6)$$

where $\mathcal{S}_S(\theta, \varphi)$ and $\mathcal{S}_N(\theta, \varphi)$ are the 2D power spectral density (PSD) functions of S and N respectively.

On examining Fig. 2, we see that the PSD of the useful signal is given by

$$\mathcal{S}_S(\theta, \varphi) = P |H_S(\theta, \varphi)|^2 = P \left| \frac{H_1 H_r H_2}{1 - H_r H_3} \right|^2, \quad (7)$$

while the PSD of the noise is given by

$$\mathcal{S}_N(\theta, \varphi) = \sigma_Z^2 |H_N(\theta, \varphi)|^2 + \sigma_W^2 = \sigma_Z^2 \left| \frac{H_r H_2}{1 - H_r H_3} \right|^2 + \sigma_W^2, \quad (8)$$

where the transfer functions H_1 , H_2 , H_r , and H_3 are defined in (4).

Proposition 1 *The per-cell sum-rate of MCP with AF relaying is given by*

$$R_{\text{mcp}} = \frac{1}{2\pi} \int_0^{2\pi} \log \left(\frac{A + B + \sqrt{(A+B)^2 - C^2}}{B + \sqrt{B^2 - C^2}} \right) d\theta, \quad (9)$$

where

$$\begin{aligned} A &\triangleq P g^2 (\beta + 2\alpha \cos \theta)^2 (\gamma + 2\eta \cos \theta)^2 \\ B &\triangleq \sigma_Z^2 g^2 (\gamma + 2\eta \cos \theta)^2 + \sigma_W^2 (1 + 4g^2 \mu^2 \cos^2 \theta) \\ C &\triangleq 4\sigma_W^2 g \mu \cos \theta. \end{aligned}$$

Furthermore, the optimal relay gain g_o is the unique solution to the equation $\sigma_r^2(g) = Q$ where

$$\sigma_r^2(g) = \frac{(P\beta^2 + \sigma_Z^2)g^2}{\sqrt{1 - (2\mu g)^4}} + \frac{4P\alpha^2 g^2}{\sqrt{1 - (2\mu g)^2} + 1 - (2\mu g)^2} \quad (10)$$

is the relay output power.

Proof: See Appendix A. ■

It can be seen that the optimal gain is achieved when the relays use their full power Q , and that $g_o \xrightarrow{Q \rightarrow \infty} 1/(2\mu)$. Other observations are that the sum-rate R_{mcp} is not interference limited and that it is independent of the actual RT delay value λ . In the following, we consider some relevant special cases.

1) *No adjacent RTs reception* ($\mu = 0$): This scenario refers to the case in which the RTs are employing directional antennas pointed toward their local BSs (see also discussion in [1] [2]). In this case, the general expression (9) reduces to

$$R_{\text{mcp-da}} = \frac{1}{2\pi} \int_0^{2\pi} \log \left(1 + \frac{P g^2 (\beta + 2\alpha \cos \theta)^2 (\gamma + 2\eta \cos \theta)^2}{\sigma_Z^2 g^2 (\gamma + 2\eta \cos \theta)^2 + \sigma_W^2} \right) d\theta. \quad (11)$$

In addition, by setting $\mu = 0$ in (10) we obtain that

$$g_o^2 = \frac{Q}{P(\beta^2 + 2\alpha^2) + \sigma_Z^2}. \quad (12)$$

2) *Half-duplex operation:* In this case, the RTs are not capable of simultaneous receive-transmit operation. Accordingly, the time is divided into equal slots: during odd numbered slots the MTs are transmitting with power $2P$ and the RTs only receive, while during even numbered slots the MTs are silent and the RTs transmit. It is easily verified that the per-cell sum-rate in this case is given by multiplying (11) by 1/2 while replacing P and Q respectively with $2P$ and $2Q$, in both (11) and (12).

B. Single Cell-Site Processing

In this section we consider a conventional SCP scheme in which no cooperation between cells is allowed. According to this scheme, each cell-site receiver is aware of the codebooks of its own users only, and it treats all other cell-site signals as interference. Notice that since the RTs are oblivious, their AF operation is not influenced by the fact that the BSs are not cooperating. In addition, since the input signals and noise statistics remain the same, expression (10) is also valid for the current setup.

The output signal can be expressed as

$$Y_{m,n} = S_{U_{m,n}} + S_{I_{m,n}} + N_{m,n},$$

where the useful part of the output signal S_U is defined as

$$S_{U_{m,n}} = \sum_{l=-\infty}^{\infty} h_{S0,n-l} X_{m,l},$$

and h_S and h_N are the signal and noise space-time impulse response functions whose Fourier transforms are given in (7) and (8) respectively. The interference part of the output signal S_I is defined as

$$S_{I_{m,n}} = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} h_{S_{m-l_1,n-l_2}} X_{l_1,l_2},$$

and the noise part of the signal is defined as

$$N_{m,n} = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} h_{N_{m-l_1,n-l_2}} Z_{l_1,l_2} + W_{m,n}.$$

Since X , Z , and W are independent of each other, zero-mean complex Gaussian and i.i.d. in space and time, it is easily verified that S_U , S_I , and N are independent and zero-mean complex Gaussian as well. It is also evident that for each m the processes are WSS along the time axis n . Accordingly, the output process at the m 'th cell can be seen as a Gaussian inter-symbol interference (ISI) channel with additive colored independent interference and noise.

Proposition 2 *The per-cell sum-rate of SCP with AF relaying is given for an arbitrary relay gain $0 < g < g_o$, by*

$$R_{\text{scp}} = \frac{1}{2\pi} \int_0^{2\pi} \log \left(1 + \frac{\mathcal{S}_U(\varphi)}{\mathcal{S}_I(\varphi) + \mathcal{S}_N(\varphi)} \right) d\varphi,$$

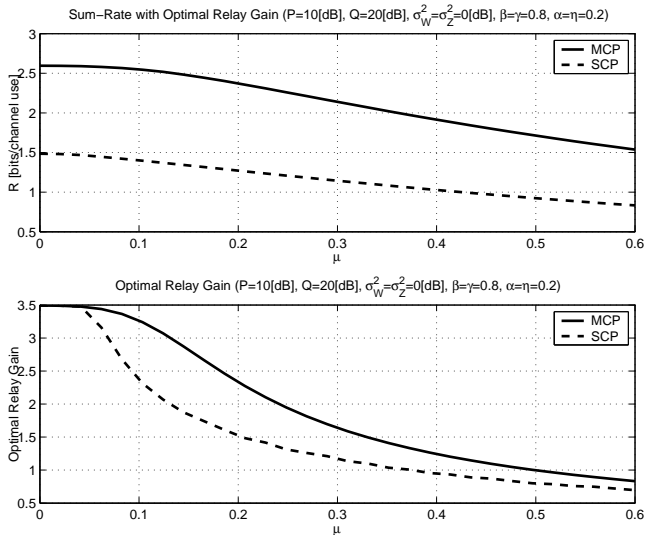


Fig. 3. Sum-rates per-cell

where $S_U(\varphi)$, $S_I(\varphi)$, and $S_N(\varphi)$ are the 1D PSDs of the useful signal, interference, and noise respectively:

$$S_U(\varphi) = \frac{P}{(2\pi)^2} \left| \int_0^{2\pi} H_S(\theta, \varphi) d\theta \right|^2$$

$$S_I(\varphi) = \frac{P}{2\pi} \int_0^{2\pi} |H_S(\theta, \varphi)|^2 d\theta - \frac{P}{(2\pi)^2} \left| \int_0^{2\pi} H_S(\theta, \varphi) d\theta \right|^2$$

$$S_N(\varphi) = \frac{\sigma_Z^2}{2\pi} \int_0^{2\pi} |H_N(\theta, \varphi)|^2 d\theta + \sigma_W^2.$$

Proof: See Appendix B. \blacksquare

It is noted that in contrast to the MCP scheme, R_{SCP} is interference limited. It is also easy to verify that R_{SCP} is independent of the actual RT delay value λ .

IV. NUMERICAL RESULTS

In Fig. 3-a the sum-rates per-cell of the MCP and the SCP schemes are plotted as functions of the inter-relay interference factor μ for $P/\sigma^2 = 10$ [dB], $Q/\sigma^2 \leq 20$ [dB], $\sigma_Z^2 = \sigma_W^2 = \sigma^2 = 1$, $\alpha = \eta = 0.2$, and $\beta = \gamma = 0.8$. The curves are plotted for an optimal selection of the relay gain g , which is shown for both schemes in Fig. 3-b. Examining the figures, it is observed that for this setting the MCP scheme demonstrates a meaningful improvement on performance over the SCP scheme. The deleterious effect of increasing inter-relay interference μ is also demonstrated for both schemes. Moreover, the optimal relay gain for both schemes also decreases with μ . Another observation is that the optimal gain of the SCP scheme is lower than that of the MCP scheme for μ larger than some threshold. Hence, using the full power of the RTs is sub-optimal for the SCP scheme under certain conditions.

V. CONCLUDING REMARKS

In this paper, joint MCP of MTs that are received only via dedicated RTs applying full-duplex AF relaying, has been

considered. The received signal at the BSs can be seen as the output of a 2D LTI channel. Using the 2D version of Szegő's Theorem, a closed form expression for the achievable per-cell sum-rate of intra-cell TDMA protocol has been derived. As a reference the rate of a conventional SCP scheme, which treats other cell MTs' signals as interference, has also been derived. Comparing the rates of the two schemes, the benefits of the MCP scheme has been demonstrated. Moreover, we have observed that the rates of both schemes are decreasing with the intra-relay interference factor, μ . The latter can be explained for the MCP scheme, by the fact that the equivalent 2D LTI channel becomes more distorted with increasing μ . Since no MTs cooperation is allowed and no rate splitting is used, this distortion can not be mitigated by power allocation over time or space, and the resulting rate decreases with μ . We also have shown that using the full power of the RTs is unconditionally optimal only for the MCP scheme. Numerical results have revealed that under certain conditions, the SCP setting produces an equivalent noisy ISI channel, the rate of which is not necessarily maximized by using the full RTs power. Other more sophisticated relaying schemes, are currently under further investigation.

APPENDIX

A. Proof of Proposition 1

It is easily verified that the RT output signal $R_{m,n}$ (1) is a WSS complex Gaussian 2D process with zero mean. Hence, its power can be expressed by

$$\begin{aligned} \sigma_r^2(g) &= E\{|R_{m,n}|^2\} \\ &= \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \frac{(P|H_1|^2 + \sigma_Z^2)|H_r|^2}{|1 - H_r H_3|^2} d\varphi d\theta \\ &= \int_0^{2\pi} \int_0^{2\pi} \frac{(2\pi)^{-2}(P(\beta + 2\alpha \cos \theta)^2 + \sigma_Z^2)g^2}{1 - 4g\mu \cos \theta \cos(\lambda\varphi) + 4g^2\mu^2 \cos^2 \theta} d\varphi d\theta, \end{aligned} \quad (13)$$

where the third equality is achieved by substituting (4). Examining (13), it is clear that in order for the relay to transmit finite power (or for the whole system to be stable) the poles of the integrand must lie inside the unit circle. Assuming that g is real this condition implies that

$$g < \frac{1}{2\mu}.$$

It is also verified by differentiating the integrand of (13) with respect to g that $\sigma_r^2(g)$ is an increasing function of g with $\sigma_r^2(0) = 0$. By making a change of variable $\varphi' = \lambda\varphi$, and integrating (13) over φ' we get

$$\sigma_r^2(g) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(P(\beta + 2\alpha \cos \theta)^2 + \sigma_Z^2)g^2}{1 - 4g^2\mu^2 \cos^2 \theta} d\theta, \quad (14)$$

where the last equality is achieved by using formula 3.616.2 of [14] and some algebra. It is noted that (14) implies that the power of the relay signal is *independent* of the actual relay delay duration. Expression (14) can be further simplified into its final closed form of (10), by applying formulas 3.653.2 and 3.682.2 of [14] and some additional algebra.

To derive the per-cell sum-rate expression for an arbitrary RT gain g , we substitute (7) and (8) into (6) to obtain

$$R_{\text{mcp}} = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \log \left(1 + \frac{P |H_1 H_r H_2|^2}{\sigma_Z^2 |H_r H_2|^2 + \sigma_W^2 |1 - H_r H_3|^2} \right) d\varphi d\theta. \quad (15)$$

It is easily verified by differentiating the integrand of (15) with respect to g , that the rate is an increasing function of the RT gain g for $0 \leq g < 1/(2\mu)$. We can conclude that, since $\sigma_r^2(g)$ is also an increasing function of g , the rate is maximized when the RTs use their full power by setting their gain to g_o which is the unique solution to $\sigma_r^2(g) = Q$. Finally, by substituting (4), applying formula 4.224.9 of [14] twice to (15), and using some algebra we obtain (9).

B. Proof of Proposition 2

First, we express the three PSDs of interest in terms of the system signal and noise 2D transfer functions $H_S(\theta, \varphi)$ and $H_N(\theta, \varphi)$. Starting with the noise component, it is easily verified that its PSD is given by

$$\begin{aligned} \mathcal{S}_N(\varphi) &= \frac{1}{2\pi} \int_0^{2\pi} \mathcal{S}_N(\theta, \varphi) d\theta \\ &= \sigma_Z^2 \frac{1}{2\pi} \int_0^{2\pi} |H_N(\theta, \varphi)|^2 d\theta + \sigma_W^2, \end{aligned}$$

where the 2D filter $H_N(\theta, \varphi)$ is defined in (8).

To calculate the useful signal PSD, let us define the following 2D filter $\hat{h}_{U,m,n} \triangleq \delta_m h_{S,m,n}$. It is easily verified that

$$S_{U,m,n} = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \hat{h}_{U,l_1-m,l_2-n} X_{l_1,l_2},$$

and that the 2D Fourier transform of $\hat{h}_{U,m,n}$ is given by

$$\begin{aligned} \hat{H}_U(\theta, \varphi) &= \mathcal{F}\{h_{S,m,n}\} ** \mathcal{F}\{\delta_m\} = H_S(\theta, \varphi) ** 2\pi\delta(\varphi) \\ &= \frac{1}{2\pi} \int_0^{2\pi} H_S(\theta, \varphi) d\theta, \end{aligned}$$

where $**$ denotes a 2D cyclic convolution operation, and $\delta(\varphi)$ denotes the Dirac delta function. Hence, the useful signal PSD becomes

$$\begin{aligned} \mathcal{S}_U(\varphi) &= P \frac{1}{2\pi} \int_0^{2\pi} \left| \hat{H}_U(\theta, \varphi) \right|^2 d\theta \\ &= P \frac{1}{2\pi} \int_0^{2\pi} \left| \frac{1}{2\pi} \int_0^{2\pi} H_S(\theta', \varphi) d\theta' \right|^2 d\theta \\ &= P \frac{1}{(2\pi)^2} \left| \int_0^{2\pi} H_S(\theta, \varphi) d\theta \right|^2. \end{aligned}$$

To calculate the interference PSD, let us define the following 2D filter $\hat{h}_{I,m,n} \triangleq (1 - \delta_m) h_{S,m,n}$. Then we have that

$$S_{I,m,n} = \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-\infty}^{\infty} \hat{h}_{I,l_1-m,l_2-n} X_{l_1,l_2},$$

and that the 2D Fourier transform of $\hat{h}_{S,m,n}$ is given by

$$\begin{aligned} \hat{H}_I(\theta, \varphi) &= \mathcal{F}\{h_{S,m,n}\} ** \mathcal{F}\{1 - \delta_m\} \\ &= H_S(\theta, \varphi) ** ((2\pi)^2 \delta(\theta) \delta(\varphi) - 2\pi \delta(\varphi)) \\ &= H_S(\theta, \varphi) - \frac{1}{2\pi} \int_0^{2\pi} H_S(\theta, \varphi) d\theta. \end{aligned}$$

Hence, the interference PSD is given by

$$\begin{aligned} \mathcal{S}_I(\varphi) &= P \frac{1}{2\pi} \int_0^{2\pi} \left| \hat{H}_I(\theta, \varphi) \right|^2 d\theta \\ &= P \frac{1}{2\pi} \int_0^{2\pi} \left| H_S(\theta, \varphi) - \frac{1}{2\pi} \int_0^{2\pi} H_S(\theta', \varphi) d\theta' \right|^2 d\theta \\ &= P \frac{1}{2\pi} \int_0^{2\pi} |H_S(\theta, \varphi)|^2 d\theta - P \frac{1}{(2\pi)^2} \left| \int_0^{2\pi} H_S(\theta, \varphi) d\theta \right|^2. \end{aligned}$$

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