Cellular Systems with Non-Regenerative Relaying and Cooperative Base Stations

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***Abstract*—In this paper, the performance of cellular networks with joint multicell processing and dedicated relay terminals is investigated. It is assumed that each relay terminal is capable of full-duplex operation and receives the transmission of relay terminals in adjacent cells. Focusing on intra-cell time division multiple access and non-fading channels, a simplified relay-aided uplink cellular model is considered. Addressing the achievable per-cell sum-rate, two non-regenerative relaying schemes are considered. Interpreting the received signal at the base stations as the outcome of a two-dimensional linear time invariant system, the multicell processing rate of an amplify-and-forward scheme is derived and shown to decrease with the inter-relay interference level. A novel form of distributed compress-and-forward scheme with decoder side information is then proposed. The corresponding multicell processing rate, which is given as a solution of a simple fixed-point equation, reveals that the compress-and-forward scheme is able to completely eliminate the inter-relay interference, and it approaches a "cut-set-like" upper bound for strong relay terminal transmission power. The benefits of base-station cooperation via multicell processing over the conventional single site processing approach is also demonstrated for both protocols.**

***Index Terms*—Non-regenerative relaying, amplify and forward, compress and forward, multicell processing.**

I. Introduction

THE ever growing demand for mobile data rate services and better coverage of cellular networks perpetuates massive research efforts aimed at developing new communication techniques. In this paper, we study the combination of two cooperation-based technologies that are promising candidates for achieving such goals, extending previous work in [1]-[3]. The first technology is relaying, whereby the signal transmitted by a mobile terminal (MT) is forwarded by a dedicated relay terminal (RT) to the intended base station (BS) [4]. (see also [5] for a more recent account). The throughput of such hybrid networks has recently been studied in the limit of asymptotically many nodes [6][7]. Moreover, information theoretic characterization of related single-cell scenarios has been reported in [8]. The second technology of interest here is multicell processing (MCP), which allows the BSs to jointly decode the received signals, equivalently creating a distributed receiving antenna array [9]. The performance gain provided by this technology within a simplified cellular model was first studied in [10][11], and then extended to include fading channels by [12] , under the assumption that BSs are connected to a central receiver by an ideal backbone (see [13] for a survey on MCP).

Recently, the interplay between these two technologies has been investigated for amplify-and-forward (AF) and decode-and-forward (DF) protocols in [1] and [2][3], respectively. The basic framework employed in these works is the Wyner uplink cellular model introduced in [11]. According to the linear variant of this model, cells are arranged in a linear geometry and only adjacent cells interfere with each other. Moreover, inter-cell interference is described by a single parameter $\alpha \in [0, 1]$, which defines the gain experienced by signals travelling to interfered cells. Notwithstanding its simplicity, this model captures the essential structure of a cellular system and it provides insight into the system performance. The RTs added to the basic Wyner model are assumed in [1][2] to operate in half-duplex (HD) mode.

In this work, unlike [1][2], we consider dedicated full-duplex (FD) RTs in a linear Wyner uplink channel model and include the signal path between adjacent RTs (i.e., inter-relay interference). With coverage extension in mind, we focus on distant users having no direct connection to the BSs. Focusing on an intra-cell time-division multiple-access (TDMA) operation and non-fading channels, we study the per-cell throughput of the network applying non-regenerative relay protocols. For each protocol we assess the gain provided by the joint MCP approach over a corresponding naive conventional single-cell processing (SCP) scheme.

In particular, we consider AF scheme and present an extension to a relaying scenario of the analytical framework introduced in [11], whereby the signal received by the BSs is

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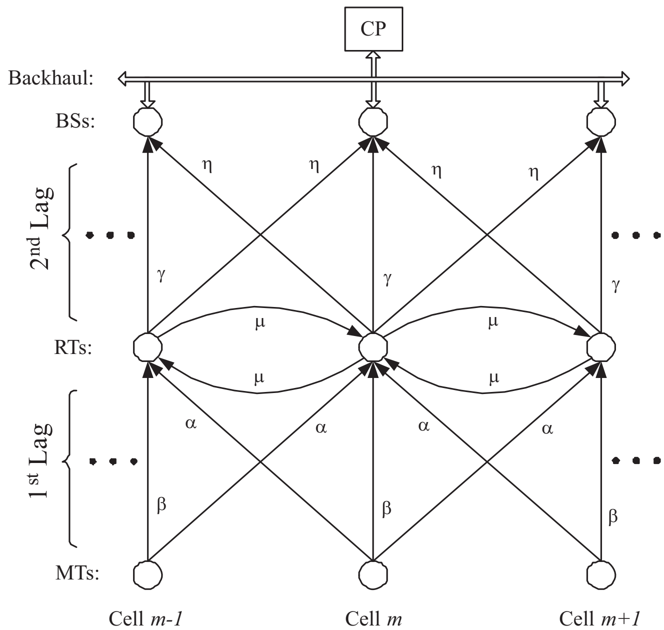

Fig. 1. Schematic diagram of the relay-aided linear Wyner uplink model.

interpreted as the outcome of a two-dimensional (2D) linear time-invariant system. This approach is a further contribution of this paper with respect to [1][2]. Next, a form of distributed compress-and-forward (CF) scheme with decoder side information, similar to that of [14], is analyzed. It is noted that this scheme resembles the single-user multiple-relay CF scheme considered in [8, Thm. 3]. Focusing on a setup with infinitely large number of cells, the achievable per-cell sum-rate of the CF scheme is derived using the methods applied in [15]. The achievable rate, which is given as a solution of a simple fixed point equation, shows that the MCP CF scheme completely eliminates the inter-relay interferences. Moreover the rate is shown to approach a "cut-set-like" upper bound for strong RTs transmission power. Finally, the performance of the CF schemes are compared numerically with the those of the AF schemes revealing the superiority of the MCP CF scheme for a wide range of the system parameters. Specifically, MCP CF outperforms MCP AF for high RTs' power especially when the inter-relay interference levels are also high. The significant benefits of MCP (or BS cooperation) over SCP for both AF and CF protocols are demonstrated as well.

For other recent work dealing with the interaction of relaying and interference networks see [16]-[18]. It is noted that unlike the aforementioned works, here we focus on non-regenerative relaying techniques applied to large system setting (although [16] considers AF scheme as well), and we also explicitly account for inter-relay interferences.

II. System Model

We consider the uplink of a cellular system with a dedicated RT for each transmitting MT. We focus on a scenario with *no fading* and adopt the linear cellular uplink channel presented by Wyner [11], where dedicated RTs are added to the basic Wyner model (see Fig. 1 for a schematic diagram of a single cell within the setup and its inter-cell interaction).

Specifically, the system includes M identical cells arranged on a line, with a single MT active in each cell at a given time (intra-cell TDMA protocol), and a dedicated single RT to relay the signals from the MT to the BS (there is no direct connection between MTs and BSs). Accordingly, each RT receives the signals of the local MT, the two adjacent MTs, and the two adjacent RTs, with channel power gains β^2, α^2, and μ^2 respectively. Likewise, each BS receives the signals of the local RT, and the two adjacent RTs, with channel power gains η^2 and γ^2 respectively. The received signals at the RTs and BSs are affected by statistically independent and identically distributed (i.i.d.) zero-mean complex Gaussian additive noise processes with powers σ_1^2 and σ_2^2, respectively. It is assumed that the MTs use independent randomly generated complex Gaussian codebooks with zero-mean and power P, whereas the RTs are subjected to an average transmit power constraint Q. The RTs operate in a non-regenerative way and thus need not know the codebooks used by the MTs. In addition, the RTs are assumed to be capable of receiving and transmitting simultaneously (i.e., perfect echo-cancellation), and are not allowed to cooperate. It is noted that the propagation delays between the different nodes of the system are negligible with respect to the symbol duration. Finally, it is assumed that the BSs are connected to a central processor (CP) via an ideal backhaul network, and that the channel path gains and noise powers are known to the BSs, MTs, and CP. Hence, both full channel state information at the transmitters (CSIT) and full channel state information at the receiver (CSIR) are assumed.

III. Preliminaries

In this section we first define the per-cell sum-rates supported by the linear Wyner uplink with and without MTs cooperation, and with cooperative BSs. These rates are then used to present an upper bound on the rates of any transmission scheme in the relay aided linear Wyner uplink.

A. Wyner's Model - Sum-Rate Capacities

Putting aside the inter-relay interference paths and the lack of joint MCP among the RTs, the relay aided cellular network of Fig. 1 is composed of two Wyner models (or two "Wyner lags") [11]. The close relations of the current setup and the Wyner model renders the following definitions useful in the sequel.

The per-cell sum-rate capacity of the linear (or circular) Wyner uplink channel with infinitely large number of cells ($M \to \infty$), no MT cooperation, optimal MCP, signal-to-noise ratio (SNR) ρ, inter-cell interference path gain a (e.g. α or η in Fig. 1), and local path gain b (e.g. β or γ in Fig. 1), is given by [11]

$$R_{\mathrm{w}}(a,b,\rho) = \int_0^1 \log_2\left(1+\rho H(f)^2\right) df \ , \tag{1}$$

where $H(f) = b + 2a\cos 2\pi f$. If one instead assumes that the MTs of a Wyner uplink model can perfectly cooperate (i.e., all MTs know the messages of all other MTs), the per-cell sum-rate capacity is given by the "waterfilling" solution and is given by

$$R_{\mathrm{w}}^{\mathrm{wf}}(a,b,\rho) = \int_0^1 \log_2\left(1 + \left(\nu - \frac{1}{H(f)^2}\right)^+ H(f)^2\right) df$$
$$\text{s.t.} \int_0^1 \left(\nu - \frac{1}{H(f)^2}\right)^+ = \rho \ , \qquad (2)$$

where $(x)^+ = \max\{x,0\}$. It is noted that (2) can be expressed in a closed-form expression for a certain range of the channel parameters (see [19, App. A]).

B. Upper Bound

Denoting $\rho_1 = P/\sigma_1^2$ and $\rho_2 = Q/\sigma_2^2$ as the SNRs over the first "MT-RT" and second "RT-BS" lags, respectively, we have the following bound.

Proposition 1 *The per-cell sum-rate of any scheme employed in the relay-aided linear Wyner uplink channel with infinite number of cells $M \to \infty$ and no MT cooperation, is upper bounded by*

$$R_{\mathrm{ub}} = \min\left\{R_{\mathrm{w}}(\alpha,\beta,\rho_1), R_{\mathrm{w}}^{\mathrm{wf}}(\eta,\gamma,\rho_2)\right\} \ . \qquad (3)$$

Proof: (outline) The rate expression is easily derived by considering two cut-sets, one separating the MTs from the RTs and the other separating the RTs from the BSs (or CP). It is noted that the rate of the second lag is achieved by allowing the RTs to fully cooperate. We refer to this bound as "cut-set-like" bound since we also account for the assumption of no MTs cooperation in the first lag. ■

It is noted that the upper bound continues to hold even if we allow multiple MTs to be simultaneously active in each cell (assuming a total-cell transmit power of P). Since both arguments of (3) increase with SNR it is easily verified that $R_{\mathrm{ub}} \underset{\rho_1\to\infty}{\to} R_{\mathrm{w}}^{\mathrm{wf}}(\eta,\gamma,\rho_2)$ and that $R_{\mathrm{ub}} \underset{\rho_2\to\infty}{\to} R_{\mathrm{w}}(\alpha,\beta,\rho_1)$. This means that if the SNR in the first lag, ρ_1, is large, the performance is limited by the upper bound on the per-cell sum-rate of the second lag, and vice-versa when $\rho_2 \to \infty$.

IV. Amplify-And-Forward Scheme

In this section we assume that the RTs amplify and forward the received signal with an integer delay of $\lambda \geq 1$ symbols. With $(\cdot)^{(1)}$, $(\cdot)^{(2)}$ denoting the association to the first "MT-RT" and second "RT-BS" lags, respectively, a baseband representation of the signal transmitted by the mth RT for an arbitrary time index n is given by

$$X_{m,n}^{(2)} = g\cdot\Big(\beta X_{m,n-\lambda}^{(1)} + \alpha X_{m-1,n-\lambda}^{(1)} + \alpha X_{m+1,n-\lambda}^{(1)} + \mu X_{m-1,n-\lambda}^{(2)} + \mu X_{m+1,n-\lambda}^{(2)} + Z_{m,n-\lambda}^{(1)}\Big) \ , \qquad (4)$$

where $X_{m,n}^{(1)}$ are the signals transmitted by the MTs, g is the RTs gain (to be defined in the sequel), and $Z_{m,n}^{(1)}$ denotes the additive noise at the RT. The received signal at the mth BS antenna is given by

$$Y_{m,n}^{(2)} = \gamma X_{m,n}^{(2)} + \eta X_{m-1,n}^{(2)} + \eta X_{m+1,n}^{(2)} + Z_{m,n}^{(2)} \ , \qquad (5)$$

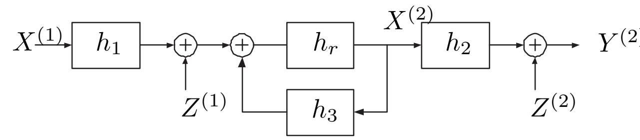

Fig. 2. Equivalent 2D LTI channel.

where $Z_{m,n}^{(2)}$ denotes the additive noise at the BS. In addition, the RTs' gain g is selected to satisfy the average power limitation

$$\sigma_r^2(g) \triangleq \mathbb{E}\left\{\left|X_{m,n}^{(2)}\right|^2\right\} \leq Q \ .$$

A. Joint Multicell Processing

In this section we assume that the signals received at all BSs are jointly decoded by the CP. The CP is connected to the BSs via an ideal backbone and is assumed to be aware of the Gaussian codebooks of all the MTs. It is noted that using similar arguments as in [11], it can be shown that in this setup an intra-cell TDMA protocol is optimal.

Extending the one-dimensional (1D) model introduced in [11], the linear equations (4) and (5) describing the network of Fig. 1 can be interpreted as a 2D linear time invariant (LTI) system. The block diagram of the equivalent 2D LTI system is depicted in Fig. 2 where the 2D filters read

$$\begin{aligned} h_{1m,n} &= \delta_n(\alpha\delta_{m-1} + \beta\delta_m + \alpha\delta_{m+1}) \\ h_{2m,n} &= \delta_n(\eta\delta_{m-1} + \gamma\delta_m + \eta\delta_{m+1}) \\ h_{rm,n} &= g\delta_{n-\lambda}\delta_m \\ h_{3m,n} &= \mu\delta_n(\delta_{m-1} + \delta_{m+1}) \ , \end{aligned} \qquad (6)$$

with δ_n denoting the *Kronecker* delta function. The corresponding 2D *Fourier* transforms of the signals in (6), are given by[1]

$$\begin{aligned} H_1(\theta,\varphi) &= \beta + 2\alpha\cos 2\pi\theta \\ H_2(\theta,\varphi) &= \gamma + 2\eta\cos 2\pi\theta \\ H_r(\theta,\varphi) &= g e^{-j2\pi\lambda\varphi} \\ H_3(\theta,\varphi) &= 2\mu\cos 2\pi\theta \ . \end{aligned} \qquad (7)$$

where θ and φ are the spatial frequency and frequency corresponding to the cells' and time indices m and n, respectively. Since the noise processes $Z^{(1)}$ and $Z^{(2)}$ are zero mean i.i.d. complex Gaussian and statistically independent of each other and of the input signal $X^{(1)}$, the output signal at the BSs can be expressed as

$$Y_{m,n}^{(2)} = S_{m,n} \ + \ N_{m,n} \ , \qquad (8)$$

where $S_{m,n}$ and $N_{m,n}$ are zero mean wide sense stationary (WSS) statistically independent processes representing the useful part of the signal and the noise respectively. Now, using the 2D extension of Szegö's theorem [11], the achievable rate in the channel (8) (without spectral shaping), which is equal

[1] The 2D *Fourier* transform is done over the the cells' index $m = 1,\ldots,M$ and over the codeword symbol index $n = 1,\ldots,N$, assuming both $M,\ N \to \infty$.

to the achievable per-cell sum-rate of the network, is given for arbitrary g by

$$R_{\rm mcp} = \int_0^1 \int_0^1 \log_2\left(1+\frac{\mathcal{S}_S(\theta,\varphi)}{\mathcal{S}_N(\theta,\varphi)}\right) d\varphi\, d\theta\ , \quad (9)$$

where $\mathcal{S}_S(\theta,\varphi)$ and $\mathcal{S}_N(\theta,\varphi)$ are the 2D power spectral density (PSD) functions of S and N respectively.

On examining Fig. 2, we see that the PSD of the useful signal is given by

$$\mathcal{S}_S(\theta,\varphi) = P\left|H_S(\theta,\varphi)\right|^2 = P\left|\frac{H_1 H_r H_2}{1-H_r H_3}\right|^2\ , \quad (10)$$

while the PSD of the noise is given by

$$\mathcal{S}_N(\theta,\varphi) = \sigma_1^2\left|H_N(\theta,\varphi)\right|^2 + \sigma_2^2 = \sigma_1^2\left|\frac{H_r H_2}{1-H_r H_3}\right|^2 + \sigma_2^2\ , \quad (11)$$

where the transfer functions $H_1,\ H_2,\ H_r$, and H_3 are defined in (7).

Proposition 2 *The per-cell sum-rate of MCP with AF relaying is given by*

$$R_{\rm af-mcp} = \int_0^1 \log_2\left(\frac{A+B+\sqrt{(A+B)^2-C^2}}{B+\sqrt{B^2-C^2}}\right) d\theta\ , \quad (12)$$

where

$$A \triangleq Pg^2(\beta+2\alpha\cos 2\pi\theta)^2(\gamma+2\eta\cos 2\pi\theta)^2$$
$$B \triangleq \sigma_1^2 g^2(\gamma+2\eta\cos 2\pi\theta)^2 + \sigma_2^2(1+4g^2\mu^2\cos^2 2\pi\theta)$$
$$C \triangleq 4\sigma_2^2 g\mu\cos 2\pi\theta\ .$$

Furthermore, the optimal relay gain $g_{\rm o}$ is the unique solution to the equation $\sigma_r^2(g) = Q$ where

$$\sigma_r^2(g) = \frac{(P\beta^2+\sigma_1^2)g^2}{\sqrt{1-(2\mu g)^4}} + \frac{4P\alpha^2 g^2}{\sqrt{1-(2\mu g)^2}+1-(2\mu g)^2} \quad (13)$$

is the relay output power.

Proof: See Appendix A. ■

It is concluded that the optimal gain is achieved when the relays use their full power Q, and that $g_{\rm o} \underset{Q\to\infty}{\longrightarrow} 1/(2\mu)$. Other observations are that the sum-rate $R_{\rm mcp}$ is not interference limited and that it is independent of the actual RT delay value λ. Also noted is that an improvement on the rate of (12) may be achieved by performing spectral shaping ("waterfilling") in the frequency domain (not to be confused with the spatial frequency domain) at the MTs and/ or RTs. In the following, we consider some relevant special cases.

1) No adjacent RTs reception ($\mu = 0$): This scenario refers to the case in which the RTs are employing directional antennas pointed toward their local BSs (see also discussions in [1] [2]). In this case, the general expression (12) reduces to

$$R_{\rm af-mcp-da} = \int_0^1 \log_2\Bigg(1+ \frac{Pg^2(\beta+2\alpha\cos 2\pi\theta)^2(\gamma+2\eta\cos 2\pi\theta)^2}{\sigma_1^2 g^2(\gamma+2\eta\cos 2\pi\theta)^2+\sigma_2^2}\Bigg) d\theta\ . \quad (14)$$

In addition, by setting $\mu = 0$ in (13) we obtain that

$$g_{\rm o}^2 = \frac{Q}{P(\beta^2+2\alpha^2)+\sigma_1^2}\ . \quad (15)$$

2) Half-duplex operation: In this case, the RTs are not capable of simultaneous receive-transmit operation. Accordingly, the time is divided into equal slots: during odd numbered slots the MTs are transmitting with power $2P$ and the RTs only receive, while during even numbered slots the MTs are silent and the RTs transmit. It is easily verified that the per-cell sum-rate in this case is given by multiplying (14) by $1/2$ while replacing P and Q respectively with $2P$ and $2Q$, in both (14) and (15).

B. Single Cell Processing

In this section we consider a conventional SCP scheme in which no cooperation between cells is allowed. According to this scheme, each BS is aware of the codebooks of its own MTs only, and it treats all other cells' signals as interference. Notice that since the RTs are oblivious, their AF operation is not influenced by the fact that the BSs are not cooperating. In addition, since the input signals and noise statistics remain the same, expression (13) is also valid for the current setup.

The output signal (5) can be expressed as

$$Y^{(2)}_{m,n} = S_{U\,m,n} + S_{I\,m,n} + N_{m,n}\ ,$$

where the useful part of the output signal S_U is defined as

$$S_{U\,m,n} = \sum_{l=-\infty}^{\infty} h_{S\,0,n-l} X^{(1)}_{m,l}\ ,$$

and h_S and h_N are the signal and noise space-time impulse response functions whose Fourier transforms are given in (10) and (11) respectively. The interference part of the output signal S_I is defined as

$$S_{I\,m,n} = \sum_{\substack{l_1=-\infty\\ l_1\neq m}}^{\infty}\ \sum_{l_2=-\infty}^{\infty} h_{S\,m-l_1,n-l_2} X^{(1)}_{l_1,l_2}\ ,$$

and the noise part of the signal is defined as

$$N_{m,n} = \sum_{l_1=-\infty}^{\infty}\ \sum_{l_2=-\infty}^{\infty} h_{N\,m-l_1,n-l_2} Z^{(1)}_{l_1,l_2} + Z^{(2)}_{m,n}\ .$$

Since $X^{(1)}$, $Z^{(1)}$, and $Z^{(2)}$ are independent of each other, zero-mean complex Gaussian and i.i.d. in space and time, it is easily verified that S_U, S_I, and N are independent and zero-mean complex Gaussian as well. It is also evident that for each m the processes are WSS along the time axis n. Accordingly, the output process at the mth BS antenna can be seen as a Gaussian inter-symbol interference (ISI) channel with additive colored independent interference and noise.

Proposition 3 *The per-cell sum-rate of SCP with AF relaying is given for an arbitrary relay gain $0 < g < g_{\rm o}$, by*

$$R_{\rm af-scp} = \int_0^1 \log_2\left(1+\frac{\mathcal{S}_U(\varphi)}{\mathcal{S}_I(\varphi)+\mathcal{S}_N(\varphi)}\right) d\varphi\ ,$$

where $\mathcal{S}_U(\varphi)$, $\mathcal{S}_I(\varphi)$, and $\mathcal{S}_N(\varphi)$ are the 1D PSDs of the useful signal, interference, and noise respectively:

$$\mathcal{S}_U(\varphi) = P\left|\int_0^1 H_S(\theta,\varphi)d\theta\right|^2$$
$$\mathcal{S}_I(\varphi) = P\int_0^1 |H_S(\theta,\varphi)|^2 d\theta - P\left|\int_0^1 H_S(\theta,\varphi)d\theta\right|^2$$
$$\mathcal{S}_N(\varphi) = \sigma_1^2\int_0^1 |H_N(\theta,\varphi)|^2 d\theta + \sigma_2^2 \ .$$

Proof: See Appendix B. ■

It is noted that in contrast to the MCP scheme, $R_{\mathrm{af-scp}}$ is interference limited. It is also easy to verify that $R_{\mathrm{af-scp}}$ is independent of the actual RT delay value λ. As with the MCP AF scheme, additional rate improvement can be achieved by spectral shaping via "waterfiling".

V. Distributed Compress-and-Forward Scheme

Here we describe the proposed CF-based transmission schemes, which organize transmission into successive blocks (or codewords) of N symbols, as sketched in Fig. 3. It should be remarked that transmission in the AF scheme presented in the previous section spans only one block (with some $o(N)$ symbols margin due to the delay λ and the filter effective response time). For this reason, while in the AF scheme the RTs need to maintain only symbol synchronization, for the CF schemes to be discussed below, block synchronization is also necessary.

To simplify the analysis here we adopt a circular version of the linear Wyner model which provides a homogenous setup with all cells being identical and symmetrical. Nevertheless, in the large system limit where the number of cells M is large it can be verified that the achievable per-cell sum-rates supported by both the linear and circular model versions are identical.

With $(\cdot)^{(1)}$, $(\cdot)^{(2)}$ denoting the association to the first "MT-RT" and second "RT-BS" lags, respectively, the received signal at the mth RT in an arbitrary symbol of the nth block is[2]

$$Y_{m,n}^{(1)} = \beta X_{m,n}^{(1)} + \alpha(X_{[m-1],n}^{(1)} + X_{[m+1],n}^{(1)}) + T_{m,n} + Z_{m,n}^{(1)} \ , \quad (16)$$

where $[k] \triangleq k \bmod M$, $X_{m,n}^{(1)}$ are the signals transmitted by the MTs (to be defined in the sequel), $Z_{m,n}^{(1)}$ denotes the additive noise at the RT, and the inter-relay interference is

$$T_{m,n} = \mu(X_{[m-1],n}^{(2)} + X_{[m+1],n}^{(2)}) \ . \quad (17)$$

The received signal at the mth BS is

$$Y_{m,n}^{(2)} = \gamma X_{m,n}^{(2)} + \eta(X_{[m-1],n}^{(2)} + X_{[m+1],n}^{(2)}) + Z_{m,n}^{(2)} \ , \quad (18)$$

where $X_{m,n}^{(2)}$ are the signals transmitted by the RTs, and $Z_{m,n}^{(2)}$ denotes the additive noise at the BS.

[2]It is noted that in contrast to Section IV where the second sub index n denotes the time index, here it denotes the block index.

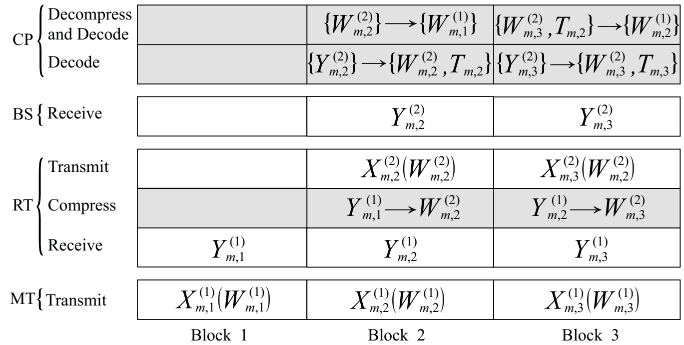

Fig. 3. Illustration of the proposed MCP CF scheme.

A. Joint Multicell Processing

The proposed MCP CF scheme works as follows (see Fig. 3 where shadowed boxes indicate zero time processing). The basic idea is to have the RTs compress the signal $Y_{m,n}^{(1)}$ received in any nth block (say $n = 2$ in Fig. 3) and forward it in the $(n+1)$th block (e.g., $n+1 = 3$) via a channel codeword $X_{m,n+1}^{(2)}$, by exploiting the side information available at the CP about the compressed signals $Y_{m,n}^{(1)}$. In fact, with the proposed scheme, in the nth block, the CP decodes the channel codewords $X_{m,n}^{(2)}$ transmitted by the RTs, and these are correlated with the signal $Y_{m,n}^{(1)}$ (16) via $T_{m,n}$ (17). Based on this side information, distributed CF is implemented at the RTs according to [14] via standard vector quantization and binning. A more formal description of the CF scheme is presented below.

Code Construction:

- *At the MTs*: Each MT generates a rate-$R_{\mathrm{cf-mcp}}$ Gaussian random channel codebook $\mathcal{X}_m^{(1)}$ according to $\mathcal{CN}(0,\rho_1)$;
- *At the RTs*: a) Each RT generates a rate-$R_{\mathrm{w}}(\eta,\gamma,\rho_2)$ Gaussian random channel codebook $\mathcal{X}_m^{(2)}$ according to $\mathcal{CN}(0,\rho_2)$; b) Each RT generates a rate-$\hat{R} = I(Y_m^{(1)};U_m)$ Gaussian quantization codebook $\mathcal{U}_m$ according to the marginal distribution induced by

$$U_m = Y_m^{(1)} + V_m, \quad (19)$$

where the quantization noises V_m are i.i.d. zero-mean complex Gaussian independent of all other random variables, and the block index n is omitted for simplicity matters. Each quantization codebook is randomly partitioned into $2^{NR_{\mathrm{w}}(\eta,\gamma,\rho_2)}$ bins, each of size $2^{N(\hat{R}-R_{\mathrm{w}}(\eta,\gamma,\rho_2))}$;

Encoding at the MTs:

- Each MT sends its message $W_{m,n}^{(1)} \in \mathcal{W}^{(1)} = \{1,\ldots,2^{NR_{\mathrm{cf-mcp}}}\}$ by transmitting the N symbol vector $\boldsymbol{X}_{m,n}^{(1)} = \mathcal{X}_m^{(1)}(W_{m,n}^{(1)})$ over the first "MT-RT" lag;

Processing at the RTs:

- *Compressing*: Each RT employs vector quantization using standard joint typicality arguments via the quantization codebok $\mathcal{U}_m$, to compress the *previously* received vector $\boldsymbol{Y}_{m,n-1}^{(1)}$ into $\boldsymbol{U}_{m,n}$ with the corresponding bin index $W_{m,n}^{(2)}$;

- *Encoding*: Each RT sends its bin index $W^{(2)}_{m,n} \in \mathcal{W}^{(2)} = \{1,\ldots,2^{NR_{\rm w}(\eta,\gamma,\rho_2)}\}$ by transmitting $\boldsymbol{X}^{(2)}_{m,n} = \mathcal{X}^{(2)}_m(W^{(2)}_{m,n})$ over the second "RT-BS" lag;

Decoding at the CP:

- *Decoding the bin indices*: The CP collects the received signal vectors $\boldsymbol{Y}^{(2)}_{\mathcal{M},n}$ (where $\mathcal{M}=\{0,1,\ldots,M-1\}$) from all the BSs through the backhaul links. Then it decodes the resulting multiple-access channel (MAC) using standard methods (e.g., [20]) to recover an estimate $\hat{W}^{(2)}_{\mathcal{M},n}$;
- *Composing the side information*: The CP uses the decoded bin indices $\hat{W}^{(2)}_{\mathcal{M},n}$ to compose the side information vectors $\hat{\boldsymbol{T}}_{\mathcal{M},n}$, where $\hat{\boldsymbol{T}}_{m,n} = \mu(\hat{\boldsymbol{X}}^{(2)}_{[m-1],n} + \hat{\boldsymbol{X}}^{(2)}_{[m+1],n})$, to be used in the *next* block;
- *Decoding the MTs messages*: The CP uses the *previous* side information $\hat{\boldsymbol{T}}_{\mathcal{M},n-1}$ and looks for a unique joint typical triplet $\{\boldsymbol{X}^{(1)}_{\mathcal{M},n-1}, \boldsymbol{U}_{\mathcal{M},n}, \hat{\boldsymbol{T}}_{\mathcal{M},n-1}\}$ within the bins indicated by $\hat{W}^{(2)}_{\mathcal{M},n}$, according to the joint distribution induced by (16), to recover $\hat{W}^{(1)}_{\mathcal{M},n-1}$.

It is noted that choosing the MTs' codebooks $\{\mathcal{X}^{(1)}_m\}_{m=0}^{M-1}$ and the RTs' quantization codebooks $\{\mathcal{U}_m\}_{m=0}^{M-1}$ according to a zero mean complex Gaussian distribution is arbitrary and no claim of optimality is made. Next we derive the per-cell sum-rate (or symmetric rate) achievable via the proposed CF scheme.

Proposition 4 *An achievable per-cell sum-rate of the MCP CF scheme employed in the relay-aided Wyner circular uplink channel with infinite number of cells $M \to \infty$, is given by*

$$R_{\rm cf-mcp} = R_{\rm w}\left(\alpha,\beta,\rho_1(1-2^{-r^*})\right) \ , \tag{20}$$

where $r^* \geq 0$ is the unique solution to the following fixed point equation

$$R_{\rm w}\left(\alpha,\beta,\rho_1(1-2^{-r^*})\right) = R_{\rm w}(\eta,\gamma,\rho_2) - r^* \ . \tag{21}$$

Proof: (outline) See Appendix C. ∎

It is concluded that the MCP CF scheme performs as if there are no inter-relay interferences (i.e. $\mu=0$) and its rate coincides with the results of [15] interpreting the second "RT-BS" lag as the backhaul network with limited capacity $C = R_{\rm w}(\eta,\gamma,\rho_2)$. Also note, that the result holds even if we relax the RT perfect echo-cancellation assumption as long as the CP is aware of the residual echo power gain.

Since $R_{\rm w}$ is given in an implicit integral form (1), we can not solve the fixed point equation (21) analytically. Nevertheless, since $R_{\rm w}\left(\alpha,\beta,\rho_1(1-2^{-r})\right)$ is monotonic in r, (21) is easily solved numerically. It is also evident that the CF rate increases with the relay power Q. Hence, as with the AF scheme full relay power usage is optimal.

It is easily verified that when $\rho_1 \to \infty$ then $r^* \to 0$, and $R_{\rm cf-mcp}$ does not achieve the upper bound (3). This is since $R_{\rm cf-mcp} \underset{\rho_1\to\infty}{\to} R_{\rm w}(\eta,\gamma,\rho_2) \leq R^{\rm wf}_{\rm w}(\eta,\gamma,\rho_2)$. On the other extreme when $\rho_2 \to \infty$ then $r^* \to \infty$, and $R_{\rm cf-mcp}$ achieves the upper bound $R_{\rm cf-mcp} \underset{\rho_2\to\infty}{\to} R_{\rm w}(\alpha,\beta,\rho_1)$.

B. Single Cell Processing

In this section it is assumed that each BS is aware of its local MT and RT codebooks only, and decodes its signals treating other BSs signals as noise. Here, each RT quantizes its received signal and transmits it to its local BS. Before describing the scheme in details let us define the signal to interference-plus-noise ratio (SINR) at each RT and each BS $\tilde{\rho}_1$ and $\tilde{\rho}_2$ respectively, by

$$\tilde{\rho}_1 \triangleq \frac{P\beta^2}{\sigma_1^2 + 2\alpha^2 P + 2\mu^2 Q} \quad \text{and} \quad \tilde{\rho}_2 \triangleq \frac{Q\gamma^2}{\sigma_2^2 + 2\eta^2 Q} \ . \tag{22}$$

In addition we denote the capacity of the single user Gaussian channel with SNR ρ to be $R_g(\rho) = \log_2(1+\rho)$.

The straightforward CF scheme where no line-of-sight is available between the transmitter and receiver is described briefly in the sequel, while a detailed description is omitted for the sake brevity.

According to the SCP-CF scheme, each MT sends its message $W^{(1)}_{m,n} = \{1,\ldots,2^{NR_{\rm cf-scp}}\}$ by transmitting the N symbol vector $\boldsymbol{X}^{(1)}_{m,n} = \mathcal{X}^{(1)}_m(W^{(1)}_{m,n})$ over the first "MT-RT" lag. Each RT employs vector quantization using standard joint typicality arguments via the quantization codebook, to compress the *previously* received vector $\boldsymbol{Y}^{(1)}_{m,n-1}$ into $\boldsymbol{U}_{m,n}$ with the corresponding bin index $W^{(2)}_{m,n}$. Then, it sends its bin index $W^{(2)}_{m,n} = \{1,\ldots,2^{NR_{\rm g}(\tilde{\rho}_2)}\}$ by transmitting $\boldsymbol{X}^{(2)}_{m,n} = \mathcal{X}^{(2)}_m(W^{(2)}_{m,n})$ over the second "RT-BS" lag. Each BS receives its signal vector $\boldsymbol{Y}^{(2)}_{m,n}$ and decodes the resulting Gaussian channel to recover an estimate $\hat{W}^{(2)}_{m,n}$. Finally, each BS looks for a unique joint typical pair $\{\boldsymbol{X}^{(1)}_{m,n-1}, \boldsymbol{U}_{m,n}\}$ within the bin indicated by $\hat{W}^{(2)}_{m,n}$, according to the joint distribution induced by (16), to recover $\hat{W}^{(1)}_{m,n-1}$

It is easily verified that applying the SCP CF scheme, each cell (MT, RT, and BS) is equivalent to a single-user single-agent Gaussian channel setup of [21], where the reliable link between the agent and the receiver in the latter is replaced here by the "RT-BS" wireless link (as long as it is being used below its capacity $R_{\rm g}(\tilde{\rho}_2)$).

Proposition 5 *An achievable per-cell sum-rate of the SCP CF scheme employed in the relay-aided Wyner circular uplink channel with infinite number of cells $M \to \infty$ is given by*

$$R_{\rm cf-scp} = R_{\rm g}\left(\tilde{\rho}_1(1-2^{-r^*})\right) \ , \tag{23}$$

where $r^* \geq 0$ is the unique solution to the following fixed point equation

$$R_{\rm g}\left(\tilde{\rho}_1(1-2^{-r^*})\right) = R_{\rm g}(\tilde{\rho}_2) - r^* \ . \tag{24}$$

where $\tilde{\rho}_1$ and $\tilde{\rho}_2$ are the SINRs at each RT and each BS respectively.

In contrast to the MCP scheme analysis, here we can analytically solve (24).

Corollary 6 *The achievable rate of* (23) *is explicitly given by*

$$R_{\rm cf-scp} = \log_2\left(\frac{(1+\tilde{\rho}_1)(1+\tilde{\rho}_2)}{1+\tilde{\rho}_1+\tilde{\rho}_2}\right) \ , \tag{25}$$

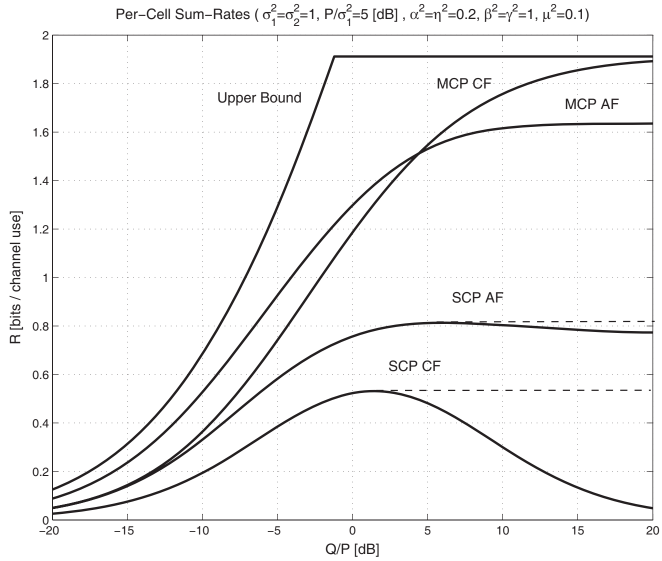

Fig. 4. Rates vs. the RTs' transmission power Q for $P/\sigma_1^2 = 5$ [dB].

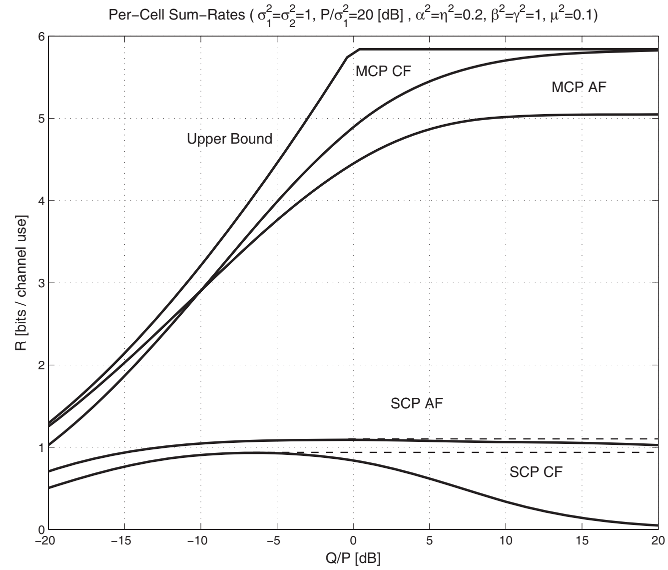

Fig. 5. Rates vs. the RTs' transmission power Q for $P/\sigma_1^2 = 20$ [dB].

It is noted that the rate (25) can also be achieved without binning at the RTs, so that the BSs can recover $U_{m,n}$ directly from $W_{m,n}^{(2)}$ and decoding of $X_{m,n}^{(1)}$ can take place based on $U_{m,n}$ (in a successive fashion, rather than joint, as above), see, e.g., [22]. Examining the rate (25) it is easily verified that it increases with the local path gains ($\beta^2,\ \gamma^2$) while it decreases with the inter-cell path gains ($\alpha^2,\ \eta^2,\ \mu^2$). Hence, in contrast to the MCP scheme, the inter-relay interference is deleterious for the SCP scheme. In addition, the rate increases with the MTs' power and using the full power P is beneficial. On the other hand increasing the RTs' power unboundedly reduces $\tilde{\rho}_1$ to zero which drives $R_{\mathrm{cf-scp}}$ to zero as well. Moreover, fixing the MTs' power P, the optimal RTs' power that maximizes the rate is given by

$$Q_{\mathrm{o}} = \min\left\{Q, \sqrt{\frac{\left(\left(2\alpha^2+\beta^2\right)P+\sigma_1^2\right)\sigma_2^2}{2\mu^2(2\eta^2+\gamma^2)}}\right\}. \qquad (26)$$

Finally, we mention that Proposition 5 and the results that follow, also apply for a setup with finite number of cells $M > 4$.

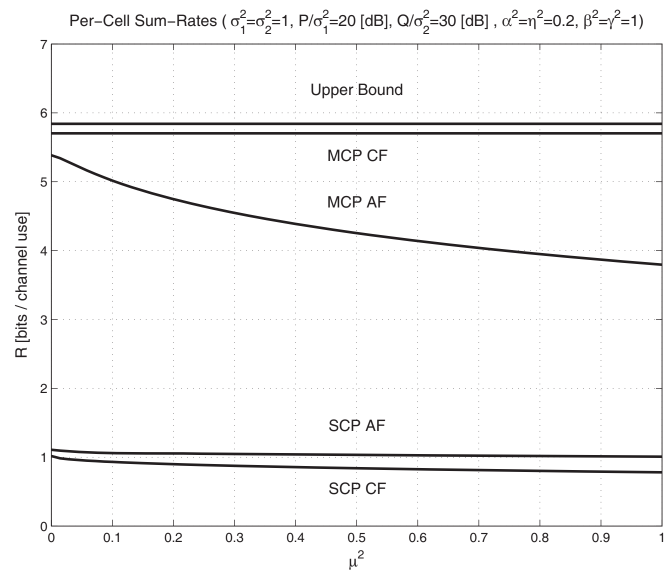

Fig. 6. Rates with optimal MTs' power vs. the inter-relay interference factor μ for $P/\sigma_1^2 = 20$ [dB] and $Q/\sigma_2^2 = 30$ [dB].

VI. Numerical Results

Figures 4 and 5 present the CF and AF rate curves and the upper-bound for $\rho_1 = P/\sigma_1^2 = 5$, 20 [dB], respectively, as functions of the RTs' power to the MTs' power ratio Q/P. The curves in the three figures are plotted for $\sigma_1^2 = \sigma_2^2 = 1$, $\beta^2 = \gamma^2 = 1$, $\alpha^2 = \eta^2 = 0.2$, and $\mu^2 = 0.1$, where the RTs using their *full* power (solid line) and *optimal* power (dashed line) for transmission. Examining the figures the benefits of MCP over the conventional SCP approach are evident. It is observed that the MCP CF scheme performs well (within one bit per channel use of the upper bound) over the entire displayed range of RTs' power Q. The MCP AF scheme also performs fairly, and even outperforms the MCP CF scheme below a certain threshold of the RTs' power Q, which is approximately $Q/\sigma_2^2 = 10$ [dB] for this setup. On the other hand, the SCP schemes perform poorly and demonstrate, as expected, an interference limited behavior. It can be seen that the SCP AF scheme outperforms the SCP CF scheme under all tested conditions. The latter may be explained noting that in the SCP CF scheme signals stemming for adjacent cells are pure interferences, while in the SCP AF scheme some of the inter-cell signals combine coherently with the useful signal, thus improving the performance via beamforming gains (e.g. the signal path traveling from the local MT to it adjacent cells' RTs and back to the local BS). In addition, the fact that using the full RTs' power for the SCP schemes is not always optimal is observed as well (especially in Fig. 4).

In Figure 6 the per-cell sum-rates of the CF and AF schemes are plotted along with the upper bound (3), as functions of the inter-relay interference factor μ for $\rho_1 = P/\sigma_1^2 = 20$ [dB], $\rho_2 = Q/\sigma_2^2 = 30$ [dB], $\sigma_1^2 = \sigma_2^2 = 1$, $\alpha^2 = \eta^2 = 0.2$, and $\beta^2 = \gamma^2 = 1$. It is noted that the all curves are plotted with *optimal* relay power (resulting in a full usage of the RTs' power Q for the MCP curves and partial RTs' power for the SCP curves). Examining the figure, the benefits of the MCP CF scheme are evident in view of the deleterious effect of increasing inter-relay interference μ on the MCP AF rate; the MCP CF scheme provides about two bits more per channel use than the MCP AF scheme, for strong inter-relay interference levels. Also visible from the figure is the proximity of the MCP CF rate to the upper bound (less than 0.2 bits per channel use) which for this setting is dominated by the rate of the first "MT-RT" lag.

VII. Concluding Remarks

In this work, we have considered the uplink of a simple two-hop cellular system aided by full-duplex relaying. The analysis has focused on non-regenerative relaying schemes, amplify-and-forward (AF) and a compress-and-forward (CF), and has included the impact of inter-cell and inter-relay interference. For both protocols, we have studied achievable rates with both multicell processing (MCP) and single cell processing (SCP) schemes. The proposed MCP CF scheme, which is a form of distributed CF with decoder side information scheme, has been shown to totally eliminate the inter-relay interference. This is unlike the achievable rate of the MCP AF, which decreases with the inter-relay interference. Numerical results have complemented the analysis and illustrated the benefits of MCP over SCP for both AF and CF. Moreover, MCP CF is seen to outperform MCP AF for sufficiently large relay terminal (RT) power (where the first hop limits the performance) while the situation is reversed for low RTs' power. It is also found that using the full transmission power at the RTs is optimal if MCP is employed, while it might reduce the rates with SCP. To conclude and put this work in a broader prospective, we remark that the proposed non-regenerative relaying techniques are excellent alternatives to regenerative techniques, and generally outperform the latter when MCP is deployed. This is because regenerative methods tend to have performance limited by the need for the RTs to decode the mobile terminals messages, and may thus not be able to fully leverage the benefits of MCP.

Appendix

A. Proof of Proposition 2

It is easily verified that the RT output signal $X_{m,n}^{(2)}$ (4) is a WSS complex Gaussian 2D process with zero mean. Hence, its power can be expressed by

$$\begin{aligned}\sigma_r^2(g) &= \mathbb{E}\left\{\left|X_{m,n}^{(2)}\right|^2\right\} \\ &= \int_0^1\int_0^1 \frac{\left(P\left|H_1\right|^2+\sigma_1^2\right)\left|H_r\right|^2}{\left|1-H_rH_3\right|^2}\,d\varphi\,d\theta \\ &= \int_0^1\int_0^1 \frac{(P(\beta+2\alpha\cos 2\pi\theta)^2+\sigma_1^2)g^2}{1-4g\mu\cos 2\pi\theta\cos 2\pi\lambda\varphi+4g^2\mu^2\cos^2 2\pi\theta}\,d\varphi\,d\theta\ ,\end{aligned} \tag{27}$$

where the third equality is achieved by substituting (7). Examining (27), it is clear that in order for the relay to transmit finite power (or for the whole system to be stable) the poles of the integrand must lie inside the unit circle. Assuming that g is real this condition implies that $g < 1/(2\mu)$. It is also verified by differentiating the integrand of (27) with respect to g that $\sigma_r^2(g)$ is an increasing function of g with $\sigma_r^2(0) = 0$. By making a change of variable $\varphi' = \lambda\varphi$, and integrating (27) over φ' we get

$$\sigma_r^2(g) = \int_0^1 \frac{(P(\beta+2\alpha\cos 2\pi\theta)^2+\sigma_1^2)g^2}{1-4g^2\mu^2\cos^2 2\pi\theta}\,d\theta\ , \tag{28}$$

where the last equality is achieved by using formula 3.616.2 of [23] and some algebra. It is noted that (28) implies that the power of the relay signal is *independent* of the actual relay delay duration. Expression (28) can be further simplified into its final closed form of (13), by applying formulas 3.653.2 and 3.682.2 of [23] and some additional algebra.

To derive the per-cell sum-rate expression for an arbitrary RT gain g, we substitute (10) and (11) into (9) to obtain

$$R_{\mathrm{af-mcp}} = \int_0^1\int_0^1 \log\left(1+\frac{P\left|H_1H_rH_2\right|^2}{\sigma_1^2\left|H_rH_2\right|^2+\sigma_2^2\left|1-H_rH_3\right|^2}\right)d\varphi\,d\theta\ . \tag{29}$$

It is easily verified by differentiating the integrand of (29) with respect to g, that the rate is an increasing function of the RT gain g for $0 \le g < 1/(2\mu)$. We can conclude that, since $\sigma_r^2(g)$ is also an increasing function of g, the rate is maximized when the RTs use their full power by setting their gain to g_o which is the unique solution to $\sigma_r^2(g) = Q$. Finally, by substituting (7), applying formula 4.224.9 of [23] twice to (29), and using some algebra we obtain (12).

B. Proof of Proposition 3

First, we express the three PSDs of interest in terms of the system signal and noise 2D transfer functions $H_S(\theta,\varphi)$ and $H_N(\theta,\varphi)$. Starting with the noise component, it is easily verified that its PSD is given by

$$\mathcal{S}_N(\varphi) = \int_0^1 \mathcal{S}_N(\theta,\varphi)d\theta = \sigma_1^2\int_0^1 \left|H_N(\theta,\varphi)\right|^2 d\theta + \sigma_2^2\ ,$$

where the 2D filter $H_N(\theta,\varphi)$ is defined in (11).

To calculate the useful signal PSD, let us define the following 2D filter $\hat{h}_{U\,m,n} \triangleq \delta_m h_{S\,m,n}$. It is easily verified that

$$S_{U\,m,n} = \sum_{l_1=-\infty}^{\infty}\sum_{l_2=-\infty}^{\infty} \hat{h}_{U\,l_1-m,l_2-n}X_{l_1,l_2}^{(1)}\ ,$$

and that the 2D Fourier transform of $\hat{h}_{U\,m,n}$ is given by

$$\begin{aligned}\hat{H}_U(\theta,\varphi) &= \mathcal{F}\{h_{S\,m,n}\} ** \mathcal{F}\{\delta_m\} = H_S(\theta,\varphi) ** \delta(\varphi) \\ &= \int_0^1 H_S(\theta,\varphi)d\theta\ ,\end{aligned}$$

where $**$ denotes a 2D cyclic convolution operation, and $\delta(\varphi)$ denotes the *Dirac* delta function. Hence, the useful signal PSD becomes

$$\begin{aligned}\mathcal{S}_U(\varphi) &= P\int_0^1 \left|\hat{H}_U(\theta,\varphi)\right|^2 d\theta \\ &= P\int_0^1 \left|\int_0^1 H_S(\theta',\varphi)d\theta'\right|^2 d\theta \\ &= P\left|\int_0^1 H_S(\theta,\varphi)d\theta\right|^2 .\end{aligned}$$

To calculate the interference PSD, let us define the following 2D filter $\hat{h}_{Im,n} \triangleq (1-\delta_m)h_{Sm,n}$. Then we have that

$$S_{Im,n} = \sum_{l_1=-\infty}^{\infty}\sum_{l_2=-\infty}^{\infty} \hat{h}_{Il_1-m,l_2-n}X^{(1)}_{l_1,l_2} ,$$

and that the 2D Fourier transform of $\hat{h}_{Sm,n}$ is given by

$$\begin{aligned}\hat{H}_I(\theta,\varphi) &= \mathcal{F}\{h_{Sm,n}\} * * \mathcal{F}\{1-\delta_m\} \\ &= H_S(\theta,\varphi) * * (\delta(\theta)\delta(\varphi) - \delta(\varphi)) \\ &= H_S(\theta,\varphi) - \int_0^1 H_S(\theta,\varphi)d\theta .\end{aligned}$$

Hence, the interference PSD is given by

$$\begin{aligned}\mathcal{S}_I(\varphi) &= P\int_0^1 \left|\hat{H}_I(\theta,\varphi)\right|^2 d\theta \\ &= P\int_0^1 \left|H_S(\theta,\varphi) - \int_0^1 H_S(\theta',\varphi)d\theta'\right|^2 d\theta \\ &= P\int_0^1 |H_S(\theta,\varphi)|^2 d\theta - P\left|\int_0^1 H_S(\theta,\varphi)d\theta\right|^2 .\end{aligned}$$

C. Proof of Proposition 4

We focus on the decoding stage at the CP for the nth block (recall Fig. 3). Since the rate of the channel codebooks used by the RTs on the second lag is equal to the per-cell capacity R_{w} of the corresponding Wyner channel (see Sec. III-A), the CP is able to correctly decode $W^{(2)}_{\mathcal{M},n-1}$ from the previous block and $W^{(2)}_{\mathcal{M},n}$ from the current with high probability. Based on the former, it can also build an accurate estimate $\hat{\boldsymbol{T}}_{\mathcal{M},n-1}$. As per Fig. 3, the CP then attempts to decode the messages $W^{(1)}_{\mathcal{M},n-1}$. In the following, the variables of interest are $Y^{(1)}_{m,n-1}$, $U_{m,n}$ and $X^{(1)}_{m,n-1}$ which are denoted for simplicity as Y_m, U_m and X_m. To elaborate, we note that, due to the quantization rule (19), the following Markov relation holds $\{X_{\mathcal{M}}, U_{\mathcal{M}\backslash m}, T_{\mathcal{M}}\} - Y_m - U_m$. Recall also that the CP decodes $X_{\mathcal{M}}$ by looking for jointly typical sequences $\{\mathbf{X}_{\mathcal{M}}, \mathbf{U}_{\mathcal{M}}, \hat{\mathbf{T}}_{\mathcal{M}}\}$, where $X_{\mathcal{M}}$ belong to the MTs codebooks (each of size $2^{NR_{\text{cf}-\text{mcp}}}$) and $U_{\mathcal{M}}$ are within the bins (of size $2^{N(\hat{R}-R_{\text{w}}(\eta,\gamma,\rho_2))}$) whose indices are given by $W^{(2)}_{\mathcal{M},n}$.

Assuming $\hat{R} \geq I(Y_m;U_m)$, for large block length N, the probability of error is dominated by the events where a set with erroneous $\mathbf{X}_{\mathcal{L}}$ and $\mathbf{U}_{\mathcal{S}}$, for any subsets $\mathcal{L},\mathcal{S} \subseteq \mathcal{M}$, is found that is jointly typical in the sense explained above (see [15]). Using the union bound, we found that the error probability is bounded

$$P_e \leq \sum_{\mathcal{L},\mathcal{S}\subseteq\mathcal{M}} 2^{NR_{\text{cf}-\text{mcp}}|\mathcal{L}|+N(\hat{R}-R_{\text{w}})|\mathcal{S}|} \cdot 2^{N(h(X_{\mathcal{L}},U_{\mathcal{S}}|X_{\mathcal{L}^c},U_{\mathcal{S}^c},T_{\mathcal{M}})-|\mathcal{L}|h(X)-|\mathcal{S}|h(U))} . \quad (30)$$

It follows that, in order to drive the probability of error to zero, it is sufficient that

$$\begin{aligned}&|\mathcal{L}|R_{\text{cf}-\text{mcp}} + |\mathcal{S}|(\hat{R}-R_{\text{w}}) \leq \\ &-h(X_{\mathcal{L}},U_{\mathcal{S}}|X_{\mathcal{L}^c},U_{\mathcal{S}^c},T_{\mathcal{M}}) + |\mathcal{L}|h(X_m) + |\mathcal{S}|h(U_m).\end{aligned} \quad (31)$$

Now, defining $\tilde{Y}_m = Y_m - T_m$ and $\tilde{U}_m = \tilde{Y}^{(1)}_m + V_m$, and using the Markov properties of the compression scheme, we have that

$$\begin{aligned}I(Y_m;U_m) &= h(U_m) - h(U_m|Y_m) \\ &= h(U_m) - h(U_m|Y_m,X_{\mathcal{M}},T_{\mathcal{M}}) \\ &= h(U_m) - h(\tilde{U}_m|\tilde{Y}_m,X_{\mathcal{M}}) ,\end{aligned} \quad (32)$$

and

$$\begin{aligned}&h(X_{\mathcal{L}},U_{\mathcal{S}}|X_{\mathcal{L}^c},U_{\mathcal{S}^c},T_{\mathcal{M}}) = \\ &= h(X_{\mathcal{L}}|X_{\mathcal{L}^c},U_{\mathcal{S}^c},T_{\mathcal{M}}) + h(U_{\mathcal{S}}|X_{\mathcal{M}},U_{\mathcal{S}^c},T_{\mathcal{M}}) \\ &= h(X_{\mathcal{L}}|X_{\mathcal{L}^c},U_{\mathcal{S}^c},T_{\mathcal{M}}) + |\mathcal{S}|h(U_m|X_{\mathcal{M}},T_{\mathcal{M}}) \\ &= h(X_{\mathcal{L}}|X_{\mathcal{L}^c},\tilde{U}_{\mathcal{S}^c}) + |\mathcal{S}|h(\tilde{U}_m|X_{\mathcal{M}}) ,\end{aligned} \quad (33)$$

and it is also easy to prove that

$$|\mathcal{L}|h(X_m) = h(X_{\mathcal{L}}) = h(X_{\mathcal{L}}|X_{\mathcal{L}^c}) . \quad (34)$$

Using (32)-(34) in (31) and dropping the subscript denoting the cell index for symmetry, we get

$$|\mathcal{L}|R_{\text{cf}-\text{mcp}} \leq |\mathcal{S}|(R_{\text{w}} - I(\tilde{U};\tilde{Y}|X_{\mathcal{M}})) + I(X_{\mathcal{L}};\tilde{U}_{\mathcal{S}^c}|X_{\mathcal{L}^c}) ,$$

which corresponds to the result in [15] by substitution of $\tilde{U}$ and $\tilde{Y}$ with U and Y, and the proof is completed by following [15].

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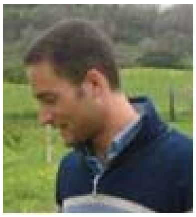

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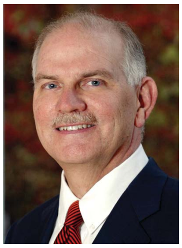

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