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TURBULENT PAIR DISPERSION OF PHOTOSPHERIC BRIGHT POINTS

F. Lepreti[1,2], V. Carbone[1,3], V. I. Abramenko[4], V. Yurchyshyn[4], P. R. Goode[4], V. Capparelli[1], and A. Vecchio[1]
[1] Dipartimento di Fisica, Università della Calabria, Ponte P. Bucci 31/C, I-87036 Rende, Italy
[2] CNISM, Unità di Ricerca di Cosenza, Ponte P. Bucci 31/C, I-87036 Rende, Italy
[3] IPCF CNR, Liquid Crystal Lab, I-87036 Arcavacata Di Rende, Italy
[4] Big Bear Solar Observatory, 40386 North Shore Lane, Big Bear City, CA 92314, USA
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ABSTRACT

Observations of solar granulation obtained with the New Solar Telescope of Big Bear Solar Observatory are used to study the turbulent pair dispersion of photospheric bright points in a quiet-Sun area, a coronal hole, and an active region plage. In all the three magnetic environments, it is found that the pair mean-squared separation $\Delta^2(t)$ follows a power-law timescaling $\Delta^2(t) \sim t^\eta$ in the range 10 s $\lesssim t \lesssim$ 400 s. The power-law index is found to be $\eta \simeq 1.5$ for all the three investigated regions. It is shown that these results can be explained in the same framework as the classical Batchelor theory, under the hypothesis that the observed range of timescales corresponds to a non-asymptotic regime in which the photospheric bright points keep the memory of their initial separations.

Key words: Sun: photosphere – Sun: surface magnetism – turbulence

Online-only material: color figures

1. INTRODUCTION

Understanding the diffusion and transport of magnetic fields within the turbulent motions in the solar convection zone and atmosphere is of fundamental importance for several solar physics problems, such as dynamo, magnetoconvection, and energy release processes in the atmosphere. In this context, the motions of local magnetic flux concentrations and magnetic bright points (BPs) represent one of the main sources of information.

These magnetic elements represent the photospheric signature of strong, i.e., kilogauss, magnetic flux tubes and undergo random walk motions driven by turbulent convection. These motions have been studied extensively as a diffusion process and in this framework they have been investigated through the analysis of scaling properties of mean-square displacements $\langle(\Delta l)^2\rangle$ with time. For normal diffusion (in two dimensions), $\langle(\Delta l)^2\rangle = 4Kt$, where K is the diffusion coefficient. When $\langle(\Delta l)^2\rangle \sim t^\gamma$, with $\gamma \neq 1$, the diffusion is called anomalous. The cases $\gamma < 1$ and $\gamma > 1$ are called sub-diffusion and super-diffusion, respectively. Of course, in the case of anomalous diffusion, the quantity $K(t) = \langle(\Delta l)^2\rangle/4t$ is not a constant anymore as in the case of normal diffusion, but it scales with a power law $K(t) \sim t^{\gamma-1}$ with time. This is due to the violation of the central limit theorem arising form the presence of long-range correlations in the velocity field.

Previous studies of the diffusion properties of magnetic elements and photospheric G-band BPs have shown significant discrepancies. In some cases, normal diffusion has been reported, but with different values of the diffusion coefficient. Wang (1988) derived a value of $K \approx 150$ km^2 s^{-1} for magnetic elements in network regions using cross-correlation techniques on videomagnetograms. Hagenaar et al. (1999) studied magnetic flux concentrations in *SOHO*/MDI magnetograms and found two time ranges of nearly constant diffusion coefficients, $K = 70$–90 km^2 s^{-1} for $t < 10^4$ s and $K = 200$–250 km^2 s^{-1} for $t > 3 \times 10^4$ s. Other authors reported anomalous diffusion. Lawrence & Schrijver (1993) found $\gamma = 0.89 \pm 0.20$ for $t > 0.5$ days by studying magnetic elements in and around an active region (AR). Berger et al. (1998a) tracked G-band BPs and found that $K(t)$ decreases approximately as $K(t) \sim t^{-0.47}$ for times less than 1400 s, while for larger times they identified two regions of constant diffusion coefficient, namely $K \approx$ 50 km^2 s^{-1} in the range 26–40 minutes and $K \approx 79$ km^2 s^{-1} in the range 42–57 minutes. Cadavid et al. (1999) analyzed the diffusion properties of G-band BPs in the granulation network, finding sub-diffusion with $\gamma = 0.76 \pm 0.04$ in the range 0.3–22 minutes and nearly normal diffusion ($\gamma = 1.10 \pm 0.24$) in the range 25–57.5 minutes. In a later paper, Lawrence et al. (2001) reanalyzed the same observations in the framework of continuous time random walk and reported evidence of super-diffusion.

In a recent paper, Abramenko et al. (2011) used the very high resolution data on solar granulation obtained with the New Solar Telescope (NST; Goode et al. 2010) of the Big Bear Solar Observatory (BBSO) to study diffusion properties of BPs. They reported super-diffusion with $\gamma = 1.48$ in the AR plage area, $\gamma = 1.53$ in the quiet-Sun (QS) area, and $\gamma = 1.67$ in the coronal hole (CH). The turbulent diffusion coefficient, which follows the power-law scaling $K(t) \sim t^{\gamma-1}$ as previously mentioned, was found to be 22 and 19 km^2 s^{-1}, respectively, in the CH and QS areas at the smallest timescale considered (10 s), whereas it was about 12 km^2 s^{-1} in the AR plage at the smallest timescale of 15 s.

The aim of this Letter is to study the pair dispersion of BPs by using the same NST observations as in Abramenko et al. (2011). A first motivation for this work stems from the fact that the scaling properties of single particle Lagrangian diffusion in turbulent flows, even in the long-time asymptotic range, depends on the local, detailed structure of the velocity field (see, e.g., Crisanti et al. 1991; Castiglione et al. 1999). On the contrary, the two-particle dispersion reflects the diffusivity properties arising from the inertial range of turbulence. Another important point to be stressed is that the pair dispersion problem in turbulent flows is closely linked to fluid mixing and transport processes (see, e.g., Bourgoin et al. 2006; Salazar & Collins 2009), which, in the case under study here, concern the photospheric magnetic field. Therefore, the study of pair BP separation properties allows us to access more direct information about turbulence properties of the solar photosphere. To our knowledge, the pair separation of BPs has been investigated so far only by Berger et al. (1998a), who analyzed the mean separation of randomly selected pairs of BPs as a function of time. Here, we study this process by analyzing the mean-square separation in the framework of two-particle dispersion theory in turbulent flows.

The theory of pair dispersion in turbulent flows is briefly reviewed in Section 2. Section 3 presents the observations, the algorithm used for the detection of BPs, and the results on BP pair dispersion statistics. The results are summarized and discussed in Section 4.

2. PAIR DISPERSION OF PASSIVE FLUID TRACERS IN TURBULENT FLOWS

The relative motion of pairs of fluid particles in isotropic turbulence was first studied by Richardson (1926), who established the theoretical foundations of pair dispersion. For fluid parcels that are small enough and neutrally buoyant so as to be considered passive tracers, the kinematic equation governing the separation vector $\boldsymbol{r}(t) = \boldsymbol{X}_i(t) - \boldsymbol{X}_j(t)$ ($\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ being the positions of the two particles) is

$$\boldsymbol{r}(t) = \boldsymbol{r}_0 + \int_0^t \boldsymbol{u}(t')dt', \quad (1)$$

where $\boldsymbol{u}(t)$ is the relative velocity between the two particles and $\boldsymbol{r}_0$ is the initial separation vector. In the case of turbulence, $\boldsymbol{r}(t)$ is a stochastic quantity and it is interesting to investigate the statistical behavior of the mean-square separation $\Delta^2(t) = \langle[\boldsymbol{r}(t) - \boldsymbol{r}_0]^2\rangle$, where brackets indicate the average over all particle pairs.

Richardson (1926) investigated the dynamics of particle pairs at spatial scales within the inertial range of turbulence. In this range of scales, in the framework of the Kolmogorov theory (Kolmogorov 1941), statistical quantities must depend solely on the scale of turbulent fluctuations $\ell \sim \Delta$ and on the energy dissipation rate ϵ, assumed to be finite as the kinematic viscosity tends to zero. Under these assumption, since $[K] = [L^2T^{-1}]$ and $[\epsilon] = [L^2T^{-3}]$, it can be conjectured that $K = K_0\Delta^\alpha\epsilon^\beta$ (K_0 is a non-dimensional constant). Dimensional analysis gives $\alpha = 4/3$ and $\beta = 1/3$, that is,

$$K(\Delta, t) = K_0\epsilon^{1/3}\Delta(t)^{4/3} . \quad (2)$$

This is the famous Richardson 4/3-law. Introducing (under the hypothesis of isotropy) the definition of an effective eddy diffusivity $K_{\rm eff}$,

$$K_{\rm eff} = \frac{1}{2d}\frac{d\Delta^2}{dt} , \quad (3)$$

where d represents the space dimension, by using Equation (2) and integrating Equation (3), it follows that, asymptotically,

$$\Delta^2(t) = g\epsilon t^3, \quad (4)$$

(g being a constant) which is the Richardson law for the pair diffusion in turbulence.

Batchelor (1950) noted that, actually, the Richardson law is asymptotic, in the sense that K depends on time only through $\Delta(t)$, and pairs are distant enough so that they completely lose memory of the initial separation $\boldsymbol{r}_0$. Following the argument by Batchelor (1950), assuming that in the small t range K depends explicitly on time and on r_0, the following general scaling relation for K can be written as

$$K(\Delta, t) = K_0 r_0^\alpha \epsilon^\beta t^\zeta . \quad (5)$$

Dimensional analysis of Equation (5) gives $\alpha = 2 - 2(1+\zeta)/3$ and $\beta = (1 + \zeta)/3$. Therefore, several possible solutions are found depending on the value of the ζ parameter. Using Equation (5) to integrate Equation (3), the scaling relation for Δ^2 is (neglecting the constant of integration)

$$\Delta^2 = g_1 r_0^\alpha \epsilon^\beta t^{\zeta+1} , \quad (6)$$

with g_1 being constant.

Batchelor (1950) used the further assumption that the time dependence is linear ($\zeta = 1$). This leads to

$$K(\Delta, t) = K_0\epsilon^{2/3}r_0^{2/3}t , \quad (7)$$

from which, again using Equation (3) and integrating, it follows that

$$\Delta^2 = g_1(\epsilon r_0)^{2/3}t^2, \quad (8)$$

which is the Batchelor law for pair diffusion. Note that Equations (6) and (8) are valid for small times, when the pairs keep memory of their initial separations.

Due to the limited range of Reynolds numbers that can be achieved, the observation of the Richardson scaling in laboratory experiments and numerical simulations is a difficult task. The most convincing confirmation of the Richardson law has been found in two-dimensional turbulence experiments (Jullien et al. 1999; Rivera & Ecke 2005), but evidence of Richardson scaling has also been provided in three-dimensional turbulence experiments (Ott & Mann 2000; Berg et al. 2006) and in the direct numerical simulations of Navier–Stokes equations (see, e.g., Boffetta & Celani 2000; Boffetta & Sokolov 2002; Ishihara & Kaneda 2002; Biferale et al. 2005; Sawford et al. 2008). Relative dispersion consistent with the Batchelor regime has been found in three-dimensional fluid turbulence simulations (Yeung 1994), while a robust scaling in agreement with the Batchelor law has been reported in the three-dimensional turbulence experiments of Bourgoin et al. (2006).

3. OBSERVATIONS, ANALYSIS, AND RESULTS

3.1. Data and Data Processing

In this study, we analyzed the same data sets as we did in Abramenko et al. (2011). Solar granulation data were obtained with the NST (Goode et al. 2010) of BBSO in 2010 August–September. Series of speckle-reconstructed images taken with a TiO filter (centered at a wavelength of 705.7 nm, with a bandpass of 1 nm) for three magnetic environments on the Sun were utilized. Namely, we analyzed (1) the quiet-Sun internetwork/network area (QS, 648 images of 10 s cadence), (2) CH area (CH, 183 images of 10 s cadence), and (3) plage area inside an AR (ARP, 513 images of 15 s cadence).

Bright features, apparent inside dark inter-granule lanes, are called BPs and they are thought to be footpoints of magnetic flux tubes, e.g., Muller et al. (2000), Berger & Title (2001), and Ishikawa et al. (2007). Therefore, studying BPs makes it possible for us to measure the dynamics of the photospheric magnetic flux tubes. Only a fraction of magnetic elements (about 20%; de Wijn et al. 2008) are thought to be associated with BPs, therefore they allow us to study only a subset of the entire magnetic flux tube population.

BPs were automatically detected in all images and then tracked from one image to the next. We used the detection and tracking code previously described in Abramenko et al. (2010). In general, our method uses the same approach as that

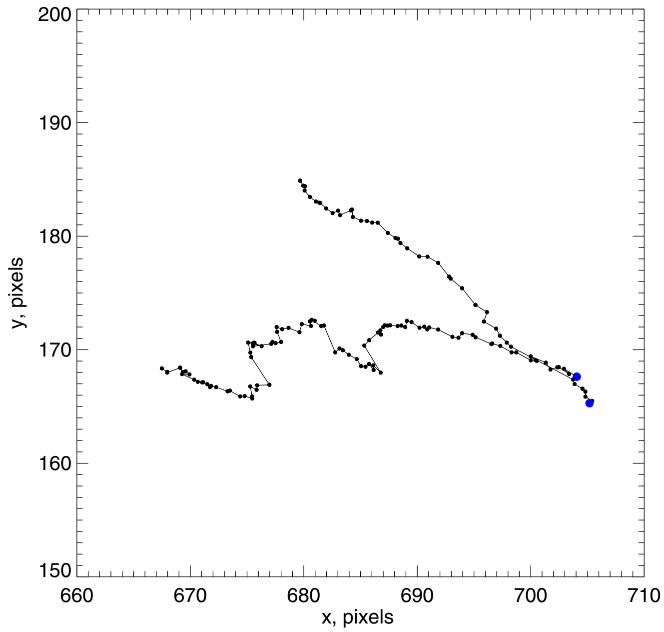

Figure 1. Trajectories of a pair of BPs separating from their initial positions, marked with the blue circles.

(A color version of this figure is available in the online journal.)

of Berger et al. (1998a, 1998b), i.e., BPs are first enhanced in the images and then selected by applying thresholding and masking. When two elements merged, the tracking of the smallest one was terminated. More details on our tracking code can be found in Abramenko et al. (2010, 2011). Examples of TiO images for each data set and trajectories of BPs are shown in Figures 1–4 in Abramenko et al. (2011).

3.2. Pair Dispersion Statistics

As mentioned in the introduction, a common approach used to study the dynamics of magnetic elements consists in considering the diffusion process of these elements as a result of the turbulent fluid motions occurring in the convection zone and in the photosphere (see, e.g., Lawrence & Schrijver 1993). In other words, it is assumed that magnetic flux concentrations are transported by turbulent flows and that they can be treated as Lagrangian "fluid particles" (see, e.g., van Ballegooijen et al. 1998; Abramenko et al. 2011). Following this idea, we utilize here the Lagrangian approach to investigate the turbulent pair dispersion of BPs in the solar photosphere.

Our first step is to compute the pair separations $\boldsymbol{r}_{ij}(t) = \boldsymbol{X}_i(t) - \boldsymbol{X}_j(t)$ of two BPs as a function of time interval, t, measured in seconds, where $\boldsymbol{X}_i(t) = (x_i(t), y_i(t))$ and $\boldsymbol{X}_j(t) = (x_j(t), y_j(t))$ are the coordinates of the ith and jth BPs at the time instant t. Figure 1 shows a typical example of a pair of BP trajectories separating with time. We then calculate the averages (over all pairs) $\Delta^2(t) = \langle[\boldsymbol{r}(t) - \boldsymbol{r}_0]^2\rangle$ as a function of time, where $\boldsymbol{r}_0$ are the initial pair separations.

This procedure is repeated for the QS, CH, and ARP data in order to compare the pair dispersion properties in these different areas (Figure 2). A power law $\Delta^2(t) \sim t^\eta$ is found for the mean-squared separation in the range 10 s $\lesssim t \lesssim$ 400 s with $\eta = 1.469 \pm 0.006$ for QS areas, $\eta = 1.469 \pm 0.009$ for CH areas, and $\eta = 1.487 \pm 0.004$ for ARP areas. This result can be compared with the analysis of the mean separation of randomly selected BP pairs performed by Berger et al. (1998a), who reported a power-law dependence with an exponent 0.67 ± 0.03. This corresponds to a mean-squared separation power-law scaling with $\eta \simeq 1.34$. For $t > 400$ s, it can be seen that the statistics is not sufficient to identify breaks and other scalings, such as for the Richardson law. The ARP data show the smallest

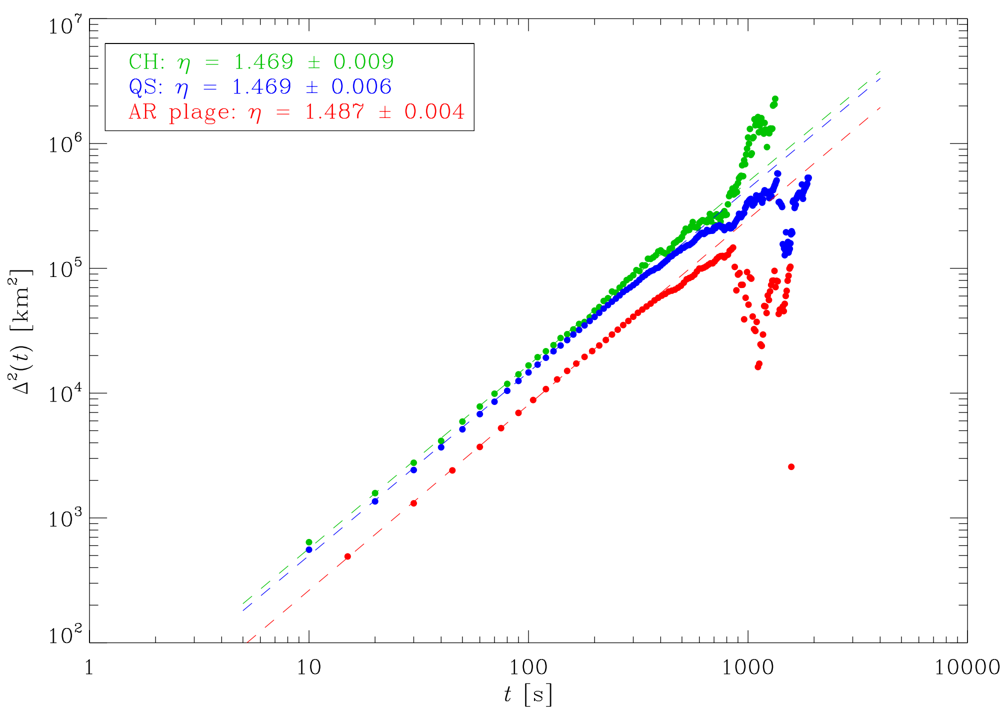

Figure 2. Mean-square BP separation $\Delta^2(t)$ determined for the CH data (green), QS area (blue), and AR plage area (red). The dashed lines represent power laws $\Delta^2(t) \sim t^\eta$, with the values of the power-law index η obtained from best fits in the range 10 s $\lesssim t \lesssim$ 400 s and shown in the inset.

(A color version of this figure is available in the online journal.)

BP separations as expected, since the BP surface density is larger in ARP areas compared to CH and QS (Abramenko et al. 2011).

The $\Delta^2(t)$ scaling of photospheric BPs is not compatible either with the Richardson or with the Batchelor model. However, the scaling exponent $\eta \simeq 1.5$ is consistent with the general scaling relations (5) and (6) when $\zeta = 1/2$. In this case, the diffusivity law (5) becomes

$$K(\Delta, t) = K_0 r_0 \epsilon^{1/2} t^{1/2} , \quad (9)$$

which yields the following scaling relation for the pair separation

$$\Delta^2(t) = g_1 \left(r_0^2 \epsilon\right)^{1/2} t^{3/2} . \quad (10)$$

We can conjecture that the pair dispersion with a smaller scaling exponent with respect to Batchelor theory can be related to the fact that BPs represent the photospheric mark of small-scale magnetic flux tubes, which oppose a stronger stiffness to dispersion with respect to ordinary fluid particles.

4. CONCLUSIONS

In this Letter, very high resolution observation of the solar granulation are used to investigate the turbulent pair dispersion of photospheric BPs. It is found that the pair mean-squared separation of BPs follows a power-law timescaling in the range $10 \text{ s} \lesssim t \lesssim 400$ s. The power-law index is $\eta \approx 1.469$ for quiet-Sun and coronal hole areas and $\eta \approx 1.487$ for AR plage. This result is shown to be consistent with scaling laws of turbulent pair dispersion obtained in a non-asymptotic regime in which the Lagrangian tracers keep the memory of their initial separations. Due to the lack of sufficient statistics of tracked BPs, it is currently not possible with the observations at our disposal to know if the observed scaling also extends for $t \gtrsim 400$ s or if other scaling regimes are present in the asymptotic range.

At variance with the case of single particle diffusion, for which the scaling index γ, and therefore the super-diffusion, is found to increase from the AR plage area to the QS and to the CH (Abramenko et al. 2011), the power-law index of the mean-squared pair separation $\Delta^2(t)$ has nearly the same value $\eta \approx 1.5$ for all the three regions. This difference can be attributed to the fact that single particle diffusion is significantly influenced by the detailed structure of the velocity field, while pair dispersion reflects the diffusivity properties arising from the local correlations in the inertial range of turbulence.

Further studies of BP pair dispersion can provide information about the magnetic field transport in the photosphere. Moreover, the transport properties of local magnetic field concentrations are closely related to the dynamics of solar atmosphere turbulence. The physical processes analyzed in the present work represent one of the possible ways to investigate the efficiency of the turbulent energy transfer to small scales, which may play an important role in the energy dissipation processes occurring in the upper atmospheric layers.

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