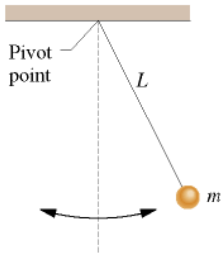
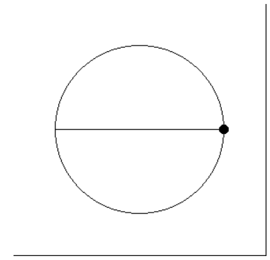
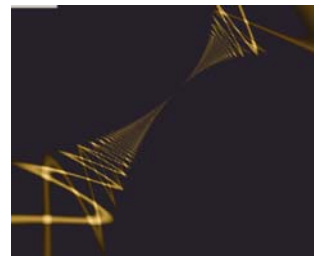


Lecture 7

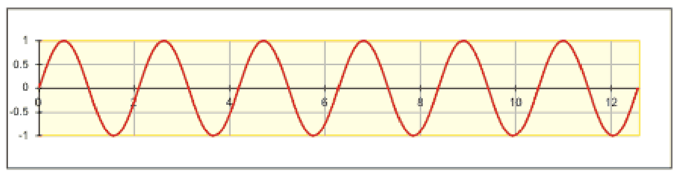
Oscillations



http://web.njit.edu/~sirenko/



Physics 103   Spring 2012



Acceleration: a=F/m [m/s^2]

1. Positive: free fall
2. Zero: constant speed   (65 mph on a highway)
3. Negative   motion under friction

Acceleration

- Average acceleration

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

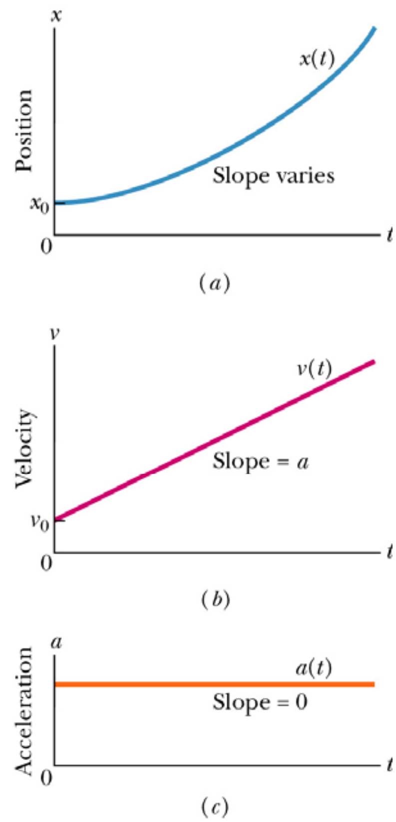
- Instantaneous acceleration

$$a = \frac{dv}{dt}$$

- Constant acceleration

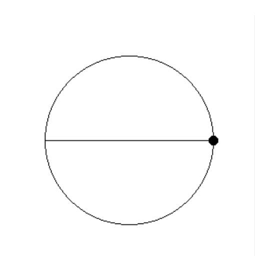
$$v = v_0 + at$$

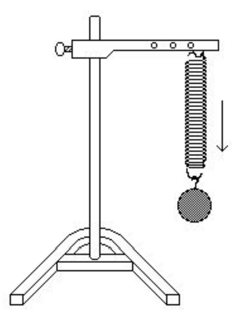
$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

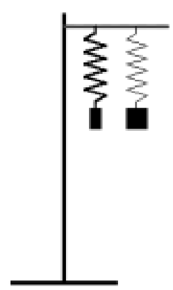


Examples of SHM

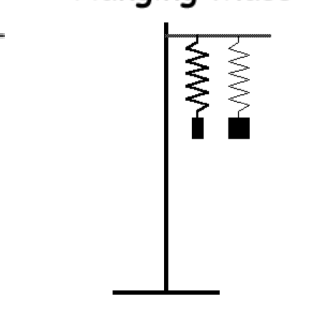
$$a(t) = -\omega^2 x(t)$$



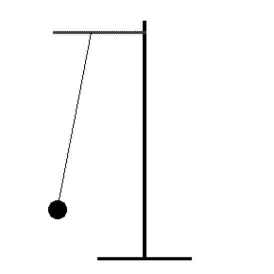


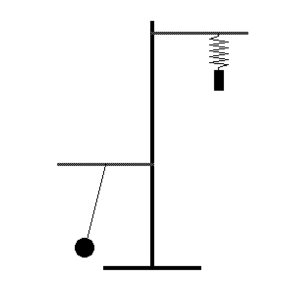


Hanging Mass

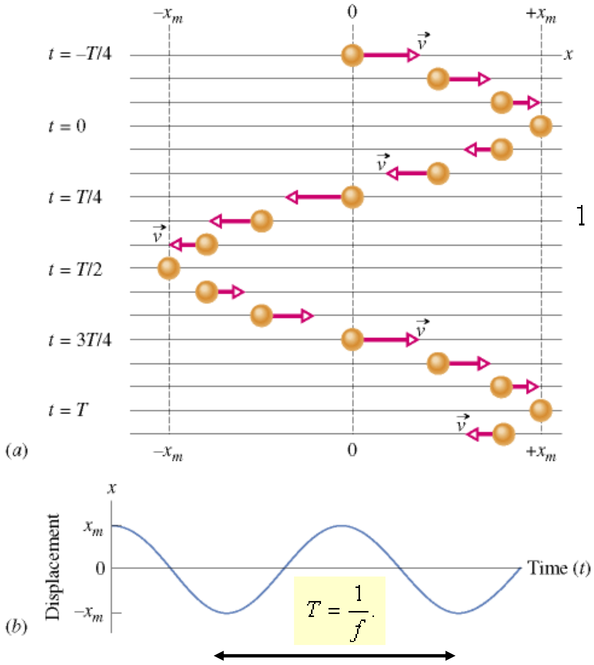


1. Projection of the Circular motion.
2. Pendulum
3. Spring+weight





Simple Harmonic Motion



$$T = \frac{1}{f}.$$

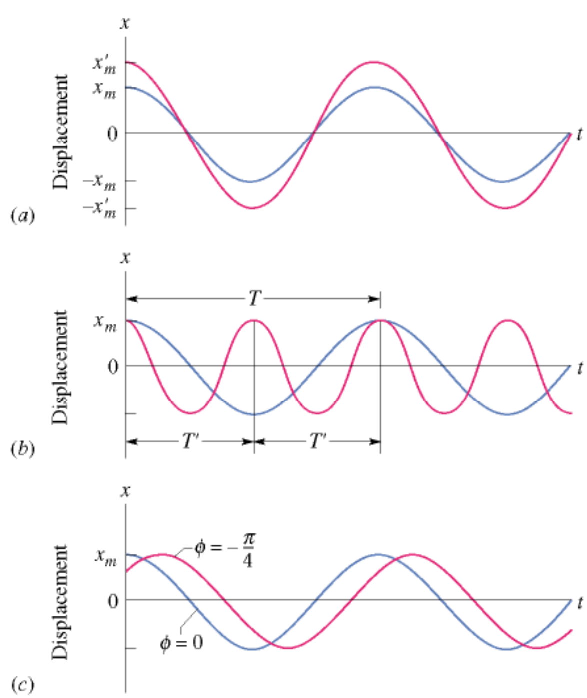
$$\omega = \frac{2\pi}{T} = 2\pi f$$

1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1}

$$x(t) = x_m \cos(\omega t + \phi)$$

Displacemnt at time t — $x(t)$; Amplitude — x_m; Phase — $\omega t + \phi$; Angular frequency — ω; Time — t; Phase constant or phase angle — ϕ

Simple Harmonic Motion (SHM)



1. Amplitude is different
2. Period (or frequency) is different.
3. Phase is different.

$$x(t) = x_m \cos(\omega t + \phi)$$

Displacemnt at time t — $x(t)$; Amplitude — x_m; Phase — $\omega t + \phi$; Angular frequency — ω; Time — t; Phase constant or phase angle — ϕ

Displacement, Velocity, and Acceleration of SHM

$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement)},$$

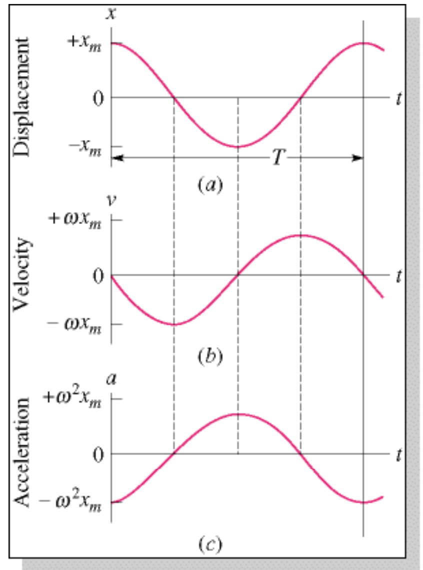
$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity)}.$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \qquad \text{(acceleration)}.$$

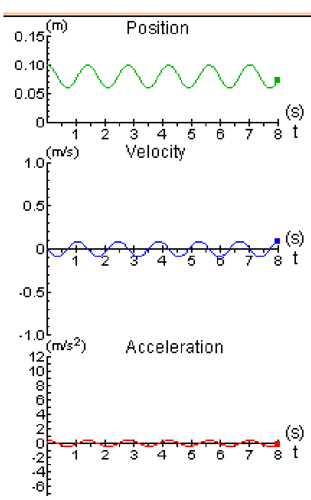
$$a(t) = -\omega^2 x(t)$$

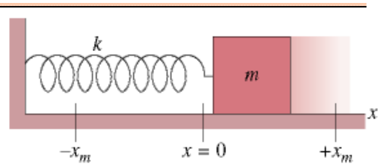


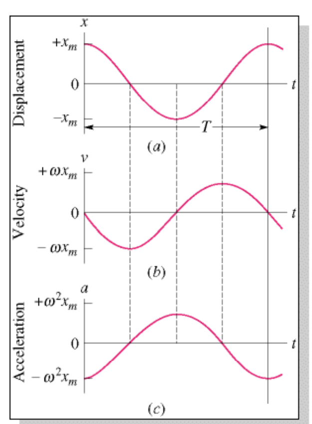
Click on the image to start the simulation

Displacement, Velocity, and Acceleration of SHM

$$a(t) = -\omega^2 x(t)$$







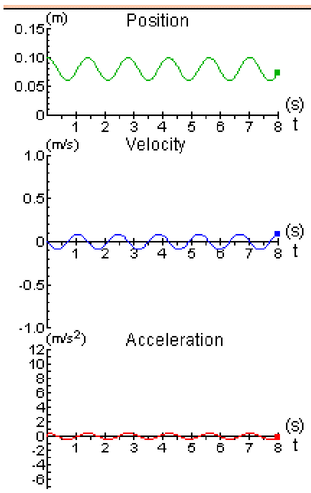
Click on the image to start the simulation

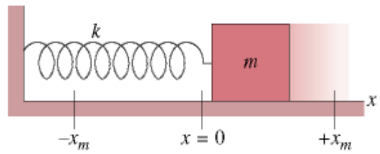
The Force Law for SHM

$$a(t) = -\omega^2 x(t)$$

Force is proportional to displacement with a negative constant of proportionality

$$F = ma = -(m\omega^2)x.$$





$$F = -kx,$$

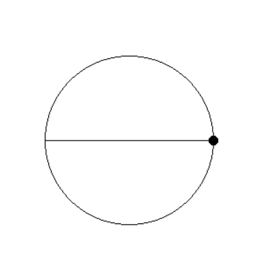
$$k = m\omega^2.$$

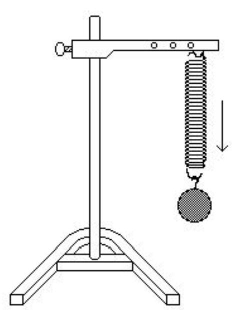
$$\omega = \sqrt{\frac{k}{m}} \qquad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(period)}$$

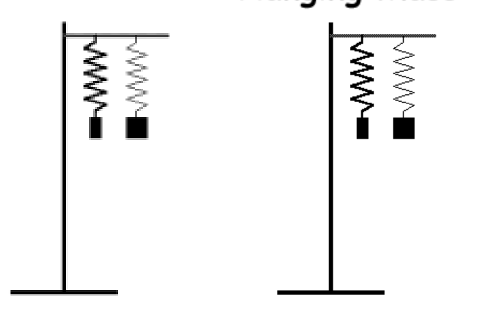
Examples of SHM

$$a(t) = -\omega^2 x(t)$$

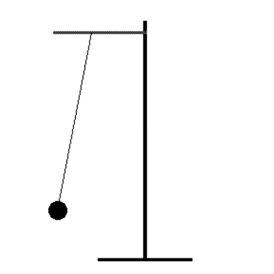


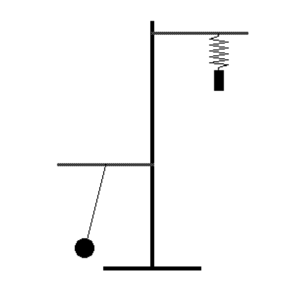


Hanging Mass



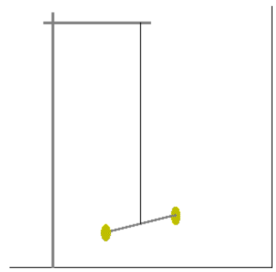
1. Projection of the Circular motion.
2. Pendulum
3. Spring+weight



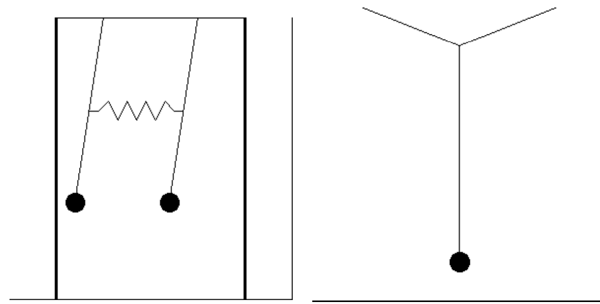


Examples of nonSHM

1. Torsion Pendulum with the moving weights



2. Coupled Pendulums

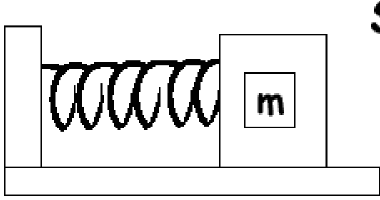


What sort of force gives SHM?

$a(t) = -\omega^2 x(t) \longrightarrow F_{tot} = ma = -m\omega^2 x$

Force is proportional to displacement with a negative constant of proportionality

Spring Force!

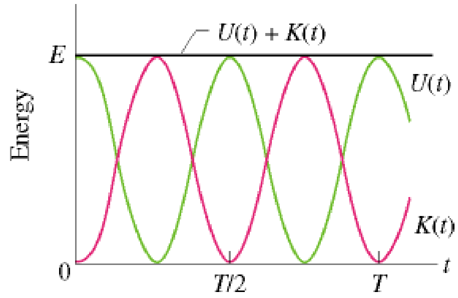


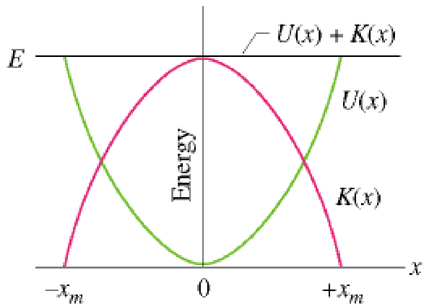
$F = -kx \longrightarrow \omega = (k/m)^{\frac{1}{2}}$

ω is the frequency of oscillation of the mass

ω does not depend on amplitude of motion

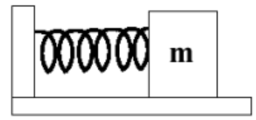
Energy of SHM





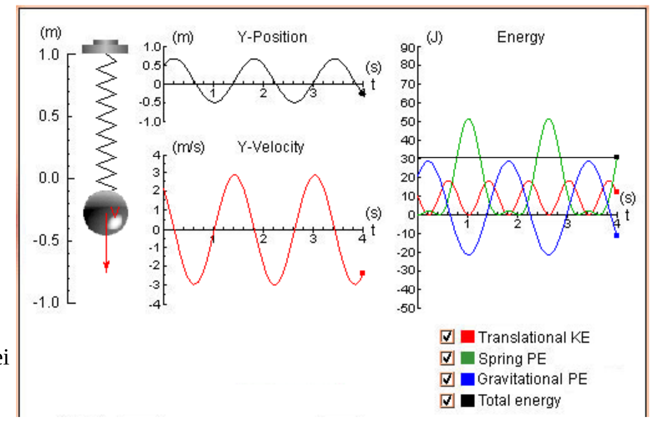
Total Energy is a constant

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

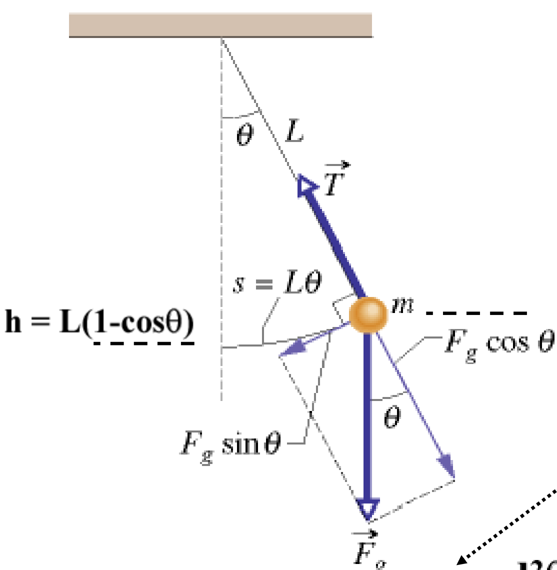


$$E = ½k(x_m)^2$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgx$$



Simple Pendulum



$$I = mL^2$$

$$F_T = -mg\sin(\theta)$$

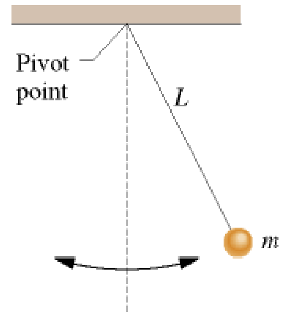
$$\tau = -mgL\sin(\theta)$$

$$\cong -mgL\theta \quad \text{(for small } \theta\text{)}$$

$$I \cdot \alpha = \tau$$

$$mL^2 \frac{d^2\theta}{dt^2} = -mgL\theta \longrightarrow \frac{d^2\theta}{dt^2} = -(g/L)\theta$$

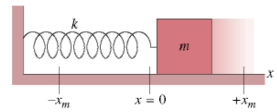
Simple Pendulum



Simple pendulum follows SHM

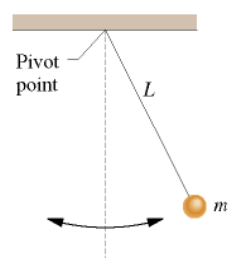
$\frac{d^2\theta}{dt^2} = -(g/L)\theta$ Looks like spring $\frac{d^2x}{dt^2} = -(k/m)x$

Solution by analogy



Spring	Pendulum
$x = x_m\cos(\omega t+\phi)$	$\theta = \theta_m\cos(\omega t+\phi)$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

Simple Pendulum: Questions



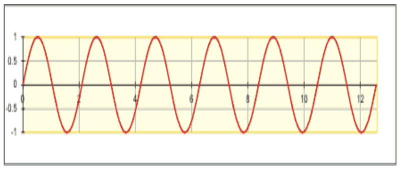
Q1. If we double θ_m the energy:

a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

Q2. If we double θ_m the period:

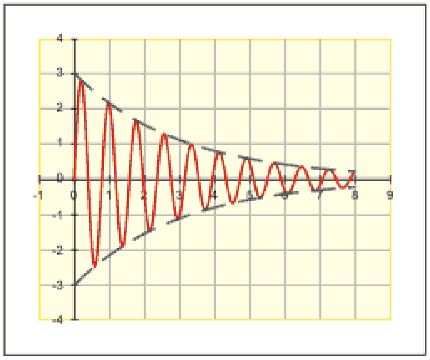
a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

Damping of harmonic oscillations



Simple Harmonic Motion is an Idealization

Energy is constant → Motion never decays



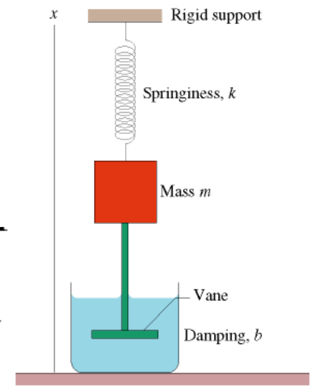
In real life the motion eventually stops

Friction

Air Resistance

….

Mechanical Energy → 0

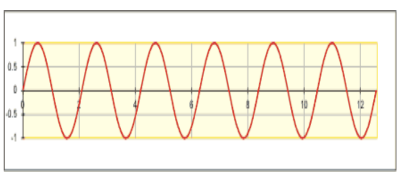


F_d = -bv   Air resistance, etc.

Direction opposite to motion

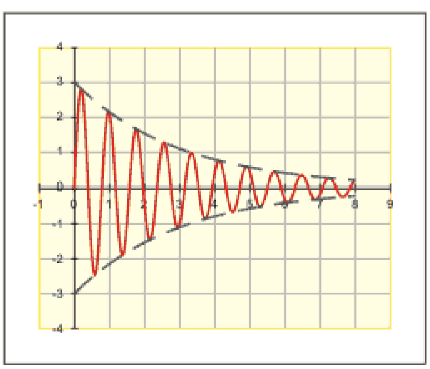
Magnitude proportional to velocity

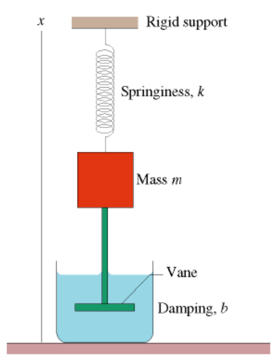
Damping of harmonic oscillations



$x = x_0\cos(\omega t)$ $v = -x_0\omega\sin(\omega t)$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2$$





Damping Force

$$F_d = -bv$$

$$-bv - kx = ma.$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$