Lecture 10

Electricity

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Physics 103   Spring 2012

The 4 Fundamental Forces

- Electromagnetic
- Force

Big Bang
(many years ago)

- WEAK
- FORCE
- STRONG
- FORCE
- GRAVITY

GRAND UNIFIED THEORIES:

Standard model: E.M. and WF are Manifestations of the same phenomenon

➢Electromagnetic ➢Force

Interactions between electric charges Newton's laws are valid in electrostatics

➢WEAK ➢FORCE

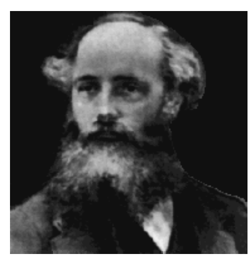
Interactions between particles with and without charge (neutrons, neutrinos)

➢STRONG ➢FORCE

Charge-independent Interactions between strongly interacting particles: hadrons and quarks

➢GRAVITY

MAXWELL's EQUATIONS:



$$\oint_S \vec{E}\cdot\hat{n}dA = \frac{Q}{\varepsilon_0} \qquad \oint_C \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint_S \vec{B}\cdot\hat{n}dA = 0 \qquad \oint_C \vec{B}\cdot d\vec{l} = \mu_0 i + \varepsilon_0\mu_0\frac{d\phi_E}{dt}$$

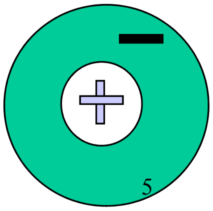
Electric Charge

- **Electric charge is fundamental characteristic of elementary particles**

- **Two types of charges: positive/negative**

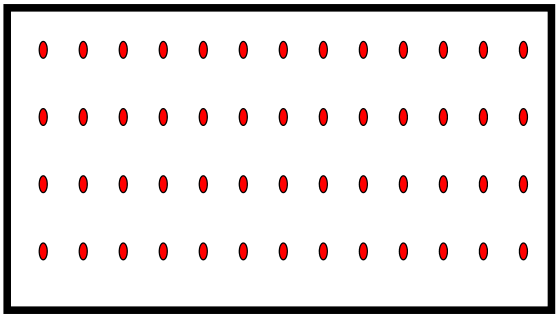
- **Labels are simply a convention**

- **Atomic structure :**
  - **negative electron cloud**
  - **nucleus of positive protons, uncharged neutrons**



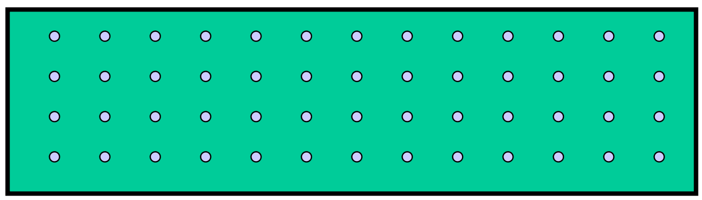
Electric Charge in Solids

- In macroscopic solids, protons often arrange themselves into a stiff regular pattern called a "lattice".
- Electrons move around this lattice. Depending on how they move the solid can be classified by its "electrical properties" as an insulator or a conductor.



Conducting solids

- In a conductor, electrons move around freely, forming a "sea" of electrons. This is why metals conduct electricity.
- Note that in solids it is ALWAYS electrons (negative charge) that actually move!!

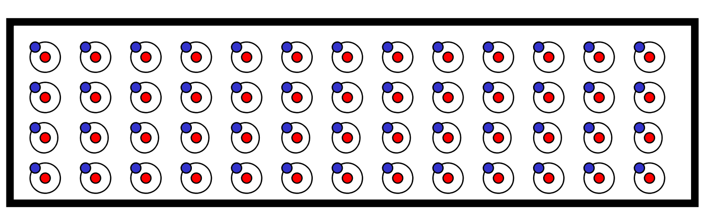


background = mobile electrons

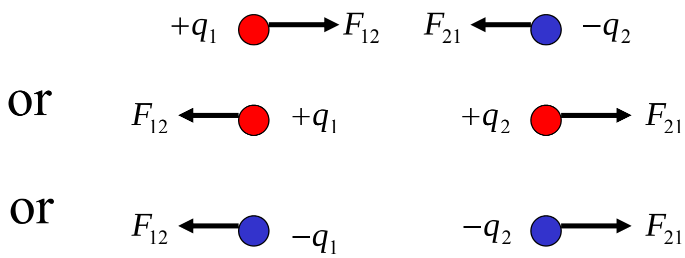
Circles = static positive charge (nuclei)

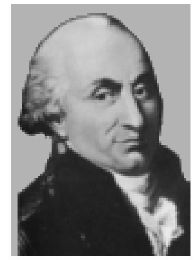
Insulating solids

- In an insulator, each electron cloud is tightly bound to the protons in a nucleus. Wood, glass, rubber.
- Note that the electrons are not free to move throughout the lattice, but the electron cloud can "distort" locally.



Force between pairs of point charges



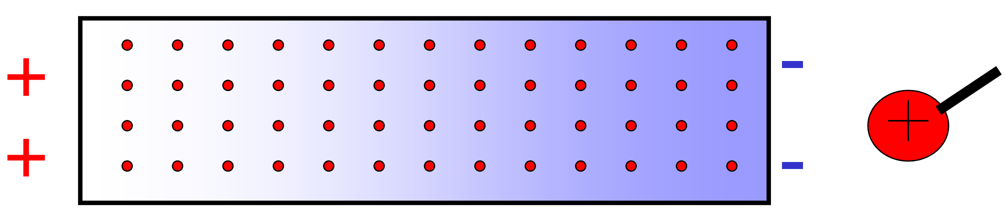


Coulomb's law -- the force between point charges:

- Lies along the line connecting the charges.
- Is proportional to the magnitude of each charge.
- Is inversely proportional to the distance squared.
- Note that Newton's third law says $|F_{12}| = |F_{21}|$!!

Induction of charge in conductors

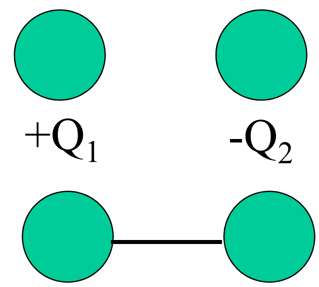
In conductors, electrons can move freely ("like water in the sea"). So, one can alter their distribution just by bringing a charge nearby the conductor. This phenomenon is called "induction".



Conservation of Charge

- Total amount of charge in an isolated system is fixed ("conserved")
- $e^+ + e^- \rightarrow \gamma + \gamma$ (annihilation)
- $\gamma \rightarrow e^+ + e^-$ (pair production)

- Example: identical metal spheres have charges $+Q_1$ and $-Q_2$.
- You connect these together with a metal wire; what is the final charge distribution?



$(Q_1-Q_2)/2$ on each sphere

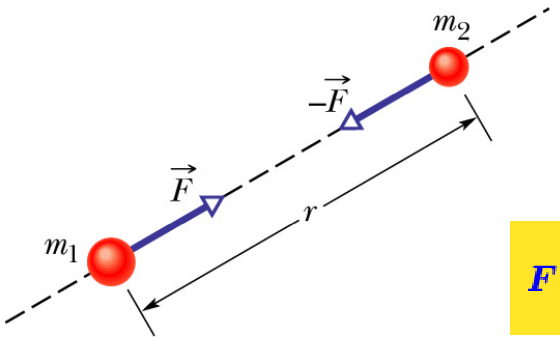
Quantization of Charge

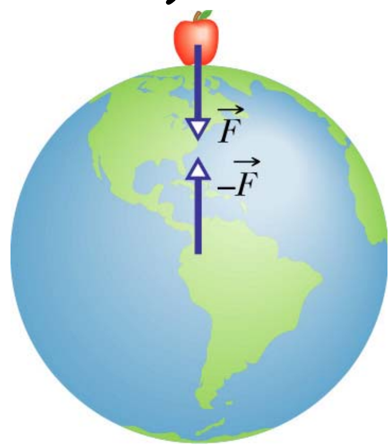
- Charge is always found in **INTEGER** multiples of the charge on an electron/proton
- Electron charge $e^- = -1.6 \times 10^{-19}$ Coulombs [C]
- Proton charge $p = e = +1.6 \times 10^{-19}$ Coulombs
- Unit of charge: Coulomb (C) in SI units
- One cannot ISOLATE FRACTIONAL CHARGE (e.g., 0.8×10^{-19} C, $+1.9 \times 10^{-19}$ C, *etc.*)
- $e = \Sigma$ quarks, ($\pm 2/3e$, $\pm 1/3e$) beyond Phys103
- Charge: Q, q, -q, -5q, …., 7q, *etc.*
- Q = 1 C is OK, it means $Q = (1 \pm 1.6 \times 10^{-19})$ C

Newton's Law of Gravitation (known since 1665)

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$

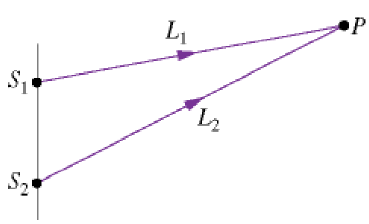




$$F = G\frac{Mm}{r^2} = ma_g \quad\Rightarrow\quad a_g = \frac{GM}{r^2}$$

Earth surface (0 km, 9.83 m/s^2), Mt. Everest (8.8 km, 9.80 m/s^2), Space Shuttle orbit (400 km, 8.70 m/s^2)

INTERFERENCE of the SOUND WAVES:



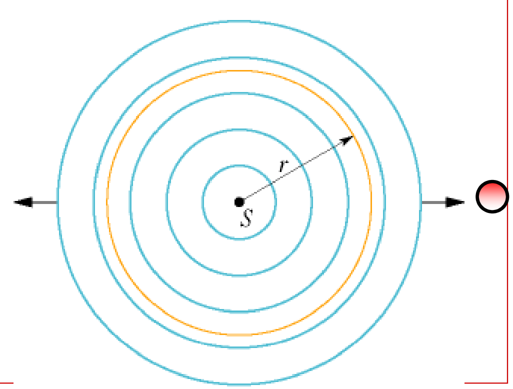
$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \quad \text{(fully constructive interference).}$$

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \quad \text{(fully destructive interference).}$$

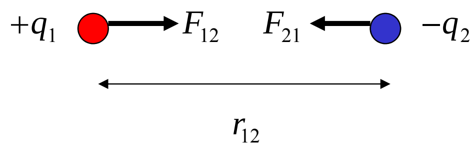
INTENSITY of the SOUND WAVES:

$$I = \frac{P_s}{4\pi r^2},$$

$$I = \tfrac{1}{2}\rho v \omega^2 s_m^2.$$



Coulomb's law



For charges in a VACUUM

$$k = 8.99 \times 10^9 \ \frac{N\,m^2}{C^2}$$

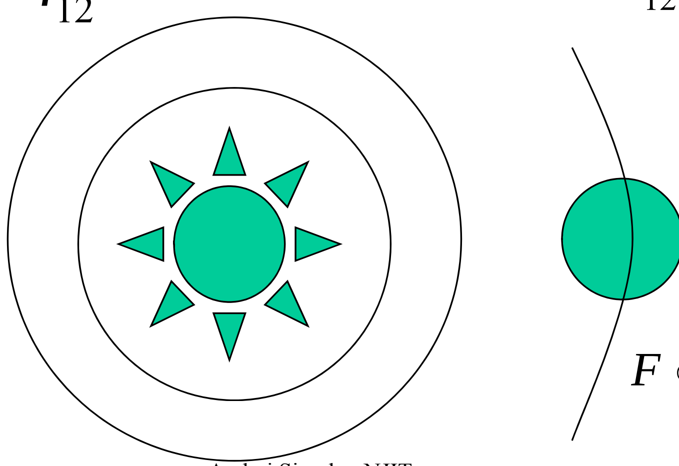
$$|F_{12}| = \frac{k\ |q_1||q_2|}{r_{12}^2}$$

Often, we write k as:

$$k = \frac{1}{4\pi\varepsilon_0} \text{ with } \varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\,m^2}$$

Coulomb's law and Newton's law

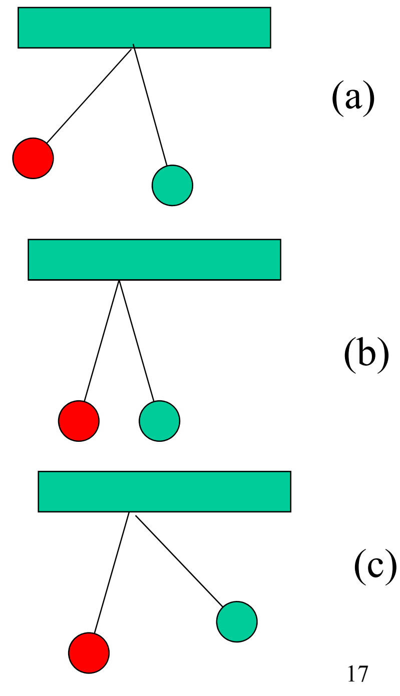
$$|F_{12}| = \frac{k\ |q_1||q_2|}{r_{12}^2} \qquad |F_{12}| = \frac{G\ m_1 m_2}{r_{12}^2}$$



$$F \propto \frac{N}{4\pi\, r^2}$$

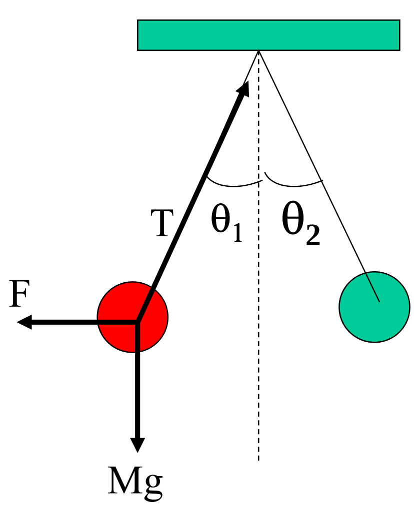
Example 1

- Two small uniformly charged spheres of equal mass M are suspended by string of equal length.
- Left sphere: +5 C
- Right sphere: +10 C
- **Which picture correctly represents equilibrium?**



Solution to Example 1

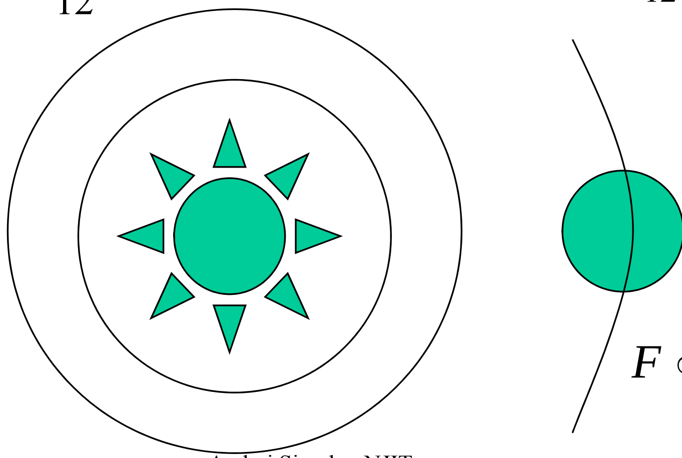
- F -- Coulomb force
- T -- Tension in string
- $T \sin\theta_1 = F$
- $T \cos\theta_1 = Mg$
- $\tan \theta_1 = F/Mg$
- Newton's 3rd Law: spheres exert equal & opposite forces on each other



$\theta_1 = \theta_2$

Coulomb's law and Newton's law

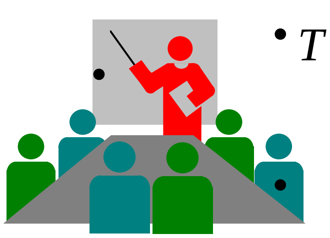
$$|F_{12}| = \frac{k\ |q_1||q_2|}{r_{12}^2} \qquad |F_{12}| = \frac{G\ m_1 m_2}{r_{12}^2}$$

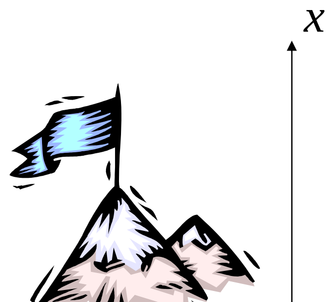


$$F \propto \frac{N}{4\pi\, r^2}$$

Scalar Fields:

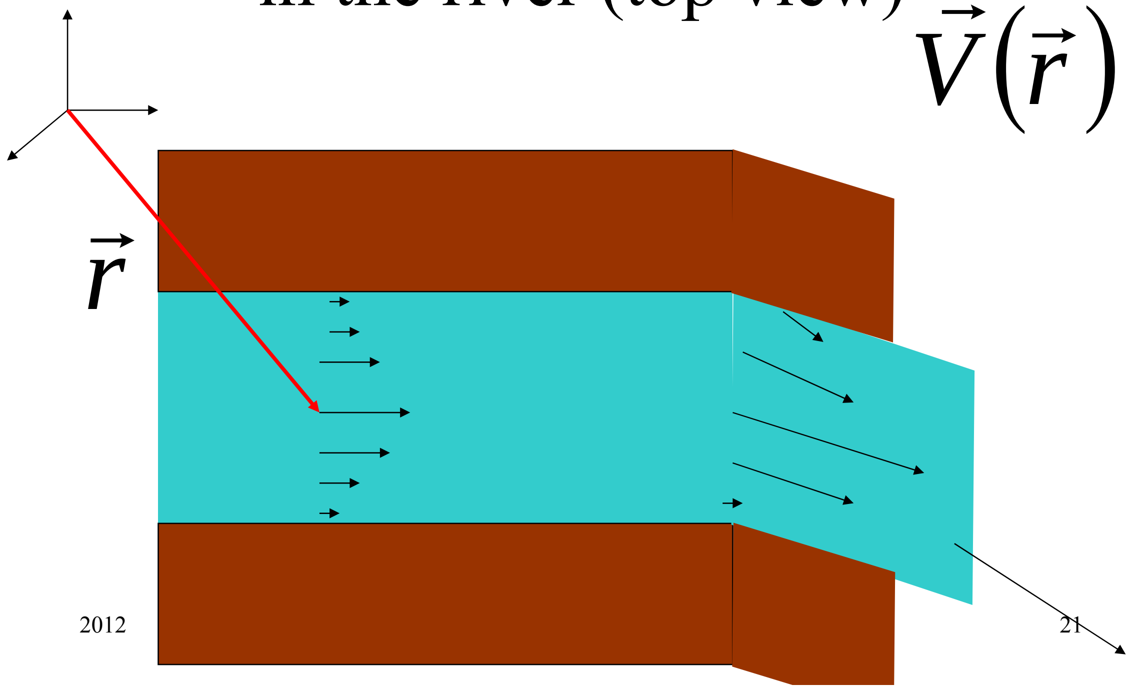
- Temperature distribution in the classroom (3D) $\quad T(\vec{r}) = T(x, y, z)$
- Population density (2D) $\quad N(x, y)$
- Atmospheric pressure as a function of height (1D) $\quad P(x)$





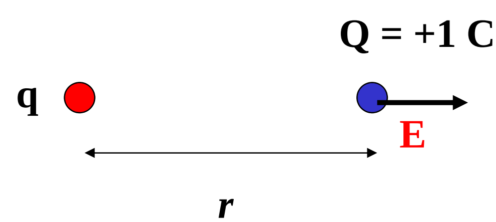
Vector Fields: velocity of water in the river (top view)

$$\vec{V}(\vec{r})$$



Electric Field (EF)

- **Electric field E at some point in space is defined as the force experienced by a point charge of +1 C**
- **Note that E is a VECTOR:** $\vec{E}(\vec{r})$
- **Since E is the force per unit charge, it is measured in units of N/C or Volt/meter**



$$|E| = \frac{|F|}{Q} = \frac{|F|}{+1C} = k\frac{|q|}{r^2}$$

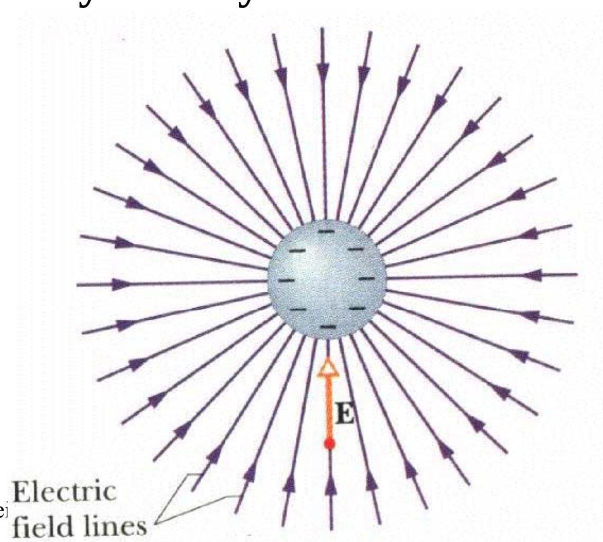
Vector form of EF at *r* created by a charge *q* located at *r = 0*

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{\vec{r}}{|r|}$$

Electric Field Lines

- **Field lines: useful way to visualize electric field E**
- **Field lines start at a positive charge, end at negative charge**
- **E at any point in space is tangential to field line**
- **Field lines are closer where E is stronger**

Example: a negative point charge -- note spherical symmetry



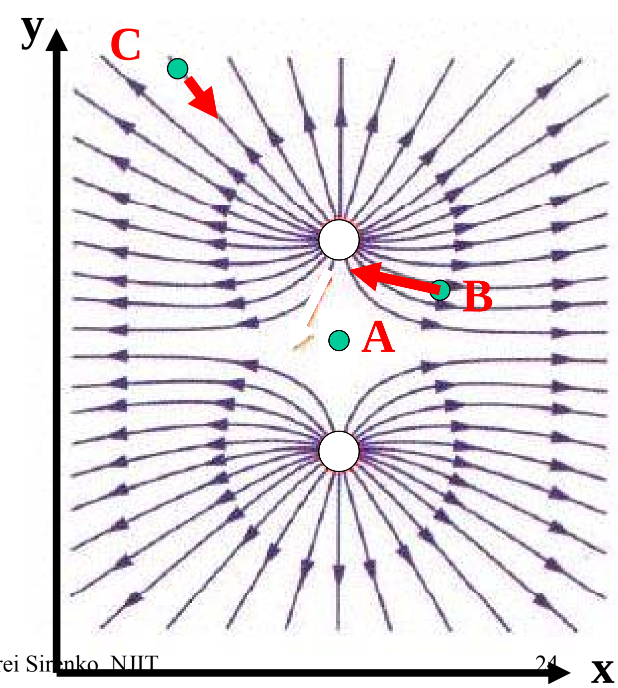
Example 2

- **Figure shows electric field lines in some region.**
- **An ELECTRON is released from rest at points A, B and C.**
- **Rank the instantaneous acceleration in order of increasing magnitude.**

(a) A, B, C

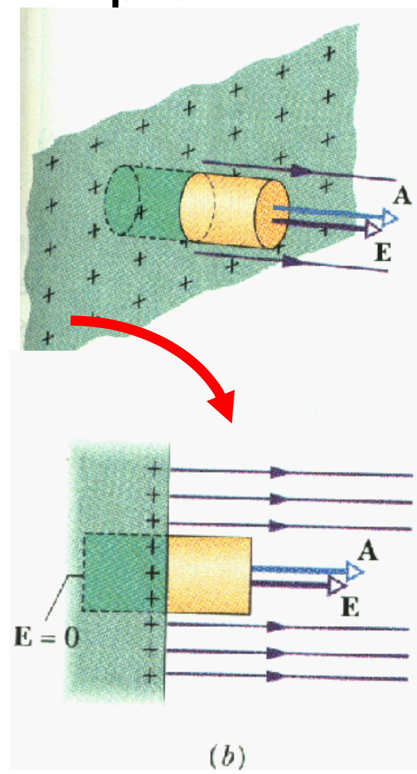
(b) A, C, B

(c) C, A, B



Gauss' Law: Example

- Infinite CONDUCTING plane with uniform area charge density σ
- E is NORMAL to plane
- Note that E = 0 inside conductor

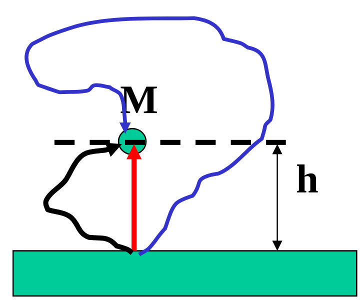


Applying Gauss' law, we have, $\frac{A\sigma}{\varepsilon_0} = AE$

Solving for the electric field, we get $E = \frac{\sigma}{\varepsilon_0}$

Potential & Potential Energy: Recall Gravity

- Mass **M,** height **h** above ground
- Assume: uniform gravitational field
- Define "gravitational potential" = 0 on ground (ARBITRARY!!)
- Gravitational potential = work needed to lift 1 kg to height h from ground
  V= **gh**
- Gravitational potential energy of mass M = work needed to lift mass M to height **h**: U = Mgh
- Equipotential surface: any plane at constant height -- same potential



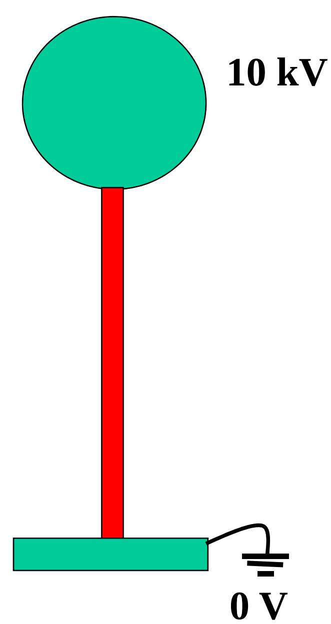
NOTE:
CONSERVATIVE FORCE!
• Potential and potential energy do NOT depend on path taken!!

Electric Potential

- Electric potential at any point in space = ***V*** = work needed to bring a charge of **+1 C** from infinity to that point
- DEFINE: electric potential at infinity = 0.
- Electric potential is measured in **VOLTS (V)**
- If potential at some point is 1 V, then you have to do 1 J of work to bring a **+1 C** charge from infinity to that point: **1 V = 1 J/C**
- Force on **+1 C** = "electric field"; since VOLTS = [*E*] x METERS, *E* can also be measured as Volts per meter *i.e.* **1 N/C = 1 V/m**

Understanding Electric Potential - I

A Van de Graaf generator builds up a large potential on a dome by using a motor-driven belt to remove electrons from the dome and transfer them to "ground" (a point of low potential). What is the work done by the machine in moving an electron from the dome (**V_A = 10 kV**) down to "ground" (**V_B = 0 V**)?



In general, work done by an external agent to move a charge Q from potential V_A to potential V_B $W = Q(V_B - V_A)$
So: Work done by generator
$= (-1.6 \times 10^{-19}\ \mathrm{C})(0\ \mathrm{kV} - 10\ \mathrm{kV})$
$= +1.6 \times 10^{-15}\ \mathrm{J}$

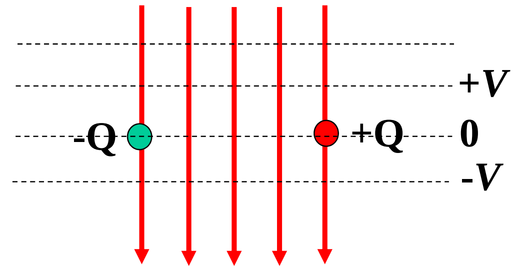
Equipotential Surfaces & Electric Field

- Uniform electric field *E* as shown
- Equipotentials are PLANES or SURFACES normal to field lines (why??)
- In a gravitational field, a free mass moves from high to low potential.
- In an electric field:

(a) Positive charge moves to lower *V*, negative charge moves to higher *V*

(b) Positive charge moves to higher *V*, negative charge moves to lower *V*

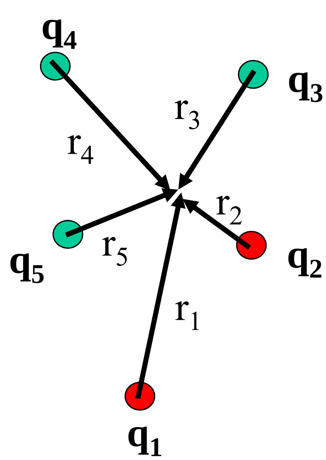
(c) All charge moves to lower *V*.



Note: all charges freely move to regions of lower potential ENERGY! Don't confuse potential with potential energy!

Electric Potential of Many Point Charges

- Electric potential is a SCALAR
- Just calculate the potential due to each individual point charge, and add together! (Make sure you get the SIGNS correct!)



$$V = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

Electric Field & Potential: A Neat Relationship!

Notice the following:

- Point charge:
  $E = kQ/r^2$
  $V = kQ/r$
- Dipole (far away):
  $E \sim kp/r^3$
  $V \sim kp/r^2$
- E is given by a DERIVATIVE of V !

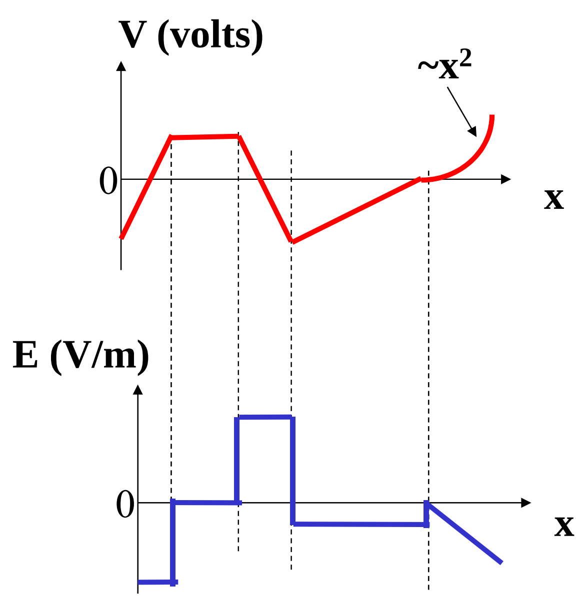
Focus only on a simple case: electric field that points along $+x$ axis but whose magnitude varies with x.

$$E_x = -\frac{dV}{dx}$$

Note:
- **MINUS sign!**
- **Units for E -- VOLTS/METER (V/m)**

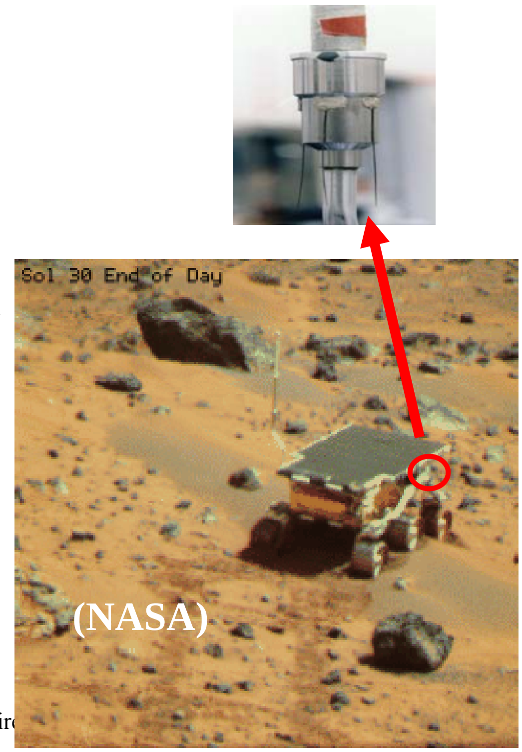
Electric Field & Potential: Example 1

- Given that electric potential varies along x axis as shown in graph.
- How does electric field vary with x?
- Recall: $E_x = -\frac{dV}{dx}$



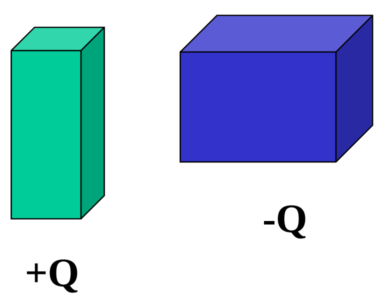
Applications of "sharp"conductors

- Charge density is higher at conductor surfaces that have small radius of curvature
- $E = \sigma/\varepsilon_0$ for a conductor, hence STRONGER electric fields at sharply curved surfaces!
- Used for attracting or getting rid of charge:
  - lightning rods
  - Van de Graaf -- metal brush transfers charge from rubber belt
  - Mars pathfinder mission -- tungsten points used to get rid of accumulated charge on rover (electric breakdown on Mars occurs at ~100 V/m)



Capacitors and Capacitance

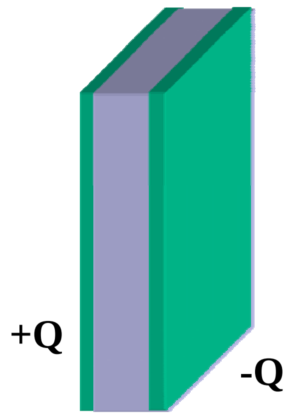
- Capacitor: any two conductors, one with charge +Q, other with charge -Q
- Potential DIFFERENCE between conductors = V
- $Q = CV$ → C = capacitance
- Units of C: Farad (F) = Coulomb/Volt



Uses: storing and releasing electric charge/energy.
Most electronic capacitors: micro-Farads (μF), pico-Farads (pF) -- 10^{-12} F
Earth: ~ 0.001 F

Capacitance

- Capacitance depends only on GEOMETRICAL factors and on the MATERIAL that separates the two conductors
- e.g. Area of conductors, separation, whether the space in between is filled with air, plastic, etc.



(We first focus on capacitors where gap is filled by vacuum or air !)

Parallel Plate Capacitor

What is the electric field in between the plates? (Gauss' Law)

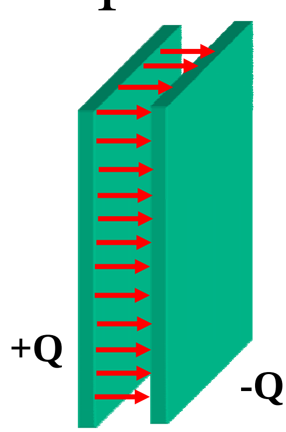
$$\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}}$$

Relate E to potential difference V between the plates:

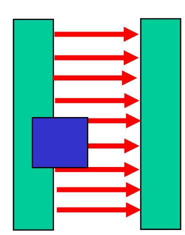
$$V = \int_0^d \vec{E} \cdot d\vec{x} = \int_0^d \frac{Q}{\varepsilon_0 A} dx = \frac{Qd}{\varepsilon_0 A}$$

What is the capacitance C?

C = Q/V = $\boxed{= \frac{\varepsilon_0 A}{d}}$



Area of each plate = A
Separation = d
charge/area = σ = Q/A



Parallel Plate Capacitor -- example

- A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm
- What is the capacitance?
- $C = \varepsilon_0 A/d =$

$(8.85 \times 10^{-12}\ \mathrm{F/m})(0.25\ \mathrm{m}^2)/(0.001\ \mathrm{m})$

$= 2.21 \times 10^{-9}\ \mathrm{F}$

(small!!)

Lesson: difficult to get large values of capacitance without special tricks!

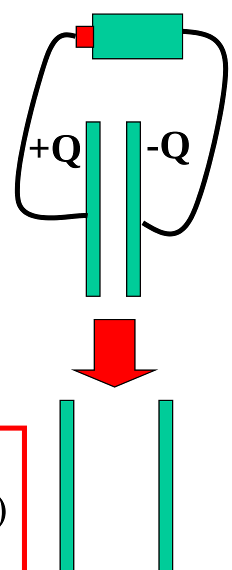
Isolated Parallel Plate Capacitor

- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Battery is then disconnected.
- If the plate separation is INCREASED, does potential difference V:

(a) Increase?

(b) Remain the same?

(c) Decrease?



- Q is fixed!
- C decreases ($=\varepsilon_0 A/d$)
- Q=CV; V increases.

Parallel Plate Capacitor & Battery

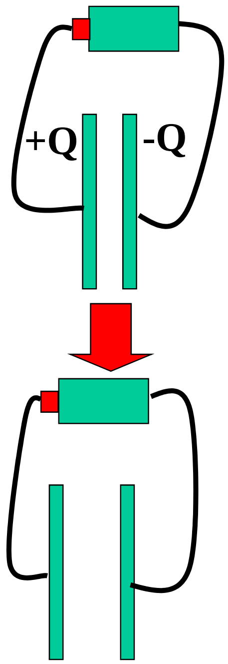
- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Plate separation is INCREASED while battery remains connected.
- Does the <u>electric field</u> inside:

(a) Increase?

(b) Remain the same?

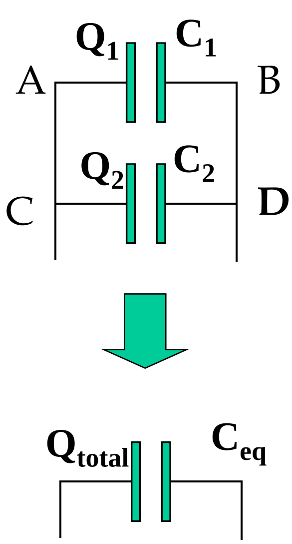
(c) Decrease?

- V is fixed by battery!
- C decreases ($=\varepsilon_0 A/d$)
- Q=CV; Q decreases
- $E = Q/ \varepsilon_0 A$ decreases



Capacitors in Parallel

- A wire is an equipotential!
- Capacitors in parallel have SAME potential difference but NOT ALWAYS same charge!
- $V_{AB} = V_{CD} = V$
- $Q_{total} = Q_1 + Q_2$
- $C_{eq}V = C_1V + C_2V$
- $C_{eq} = C_1 + C_2$
- Equivalent parallel capacitance = sum of capacitances



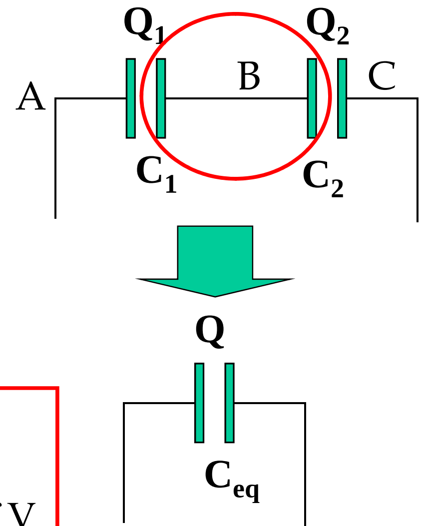
Series Capacitors

- $Q_1 = Q_2 = Q$ (WHY??)
- $V_{AC} = V_{AB} + V_{BC}$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

SERIES:
- Q is same for all capacitors
- Total potential difference = sum of V

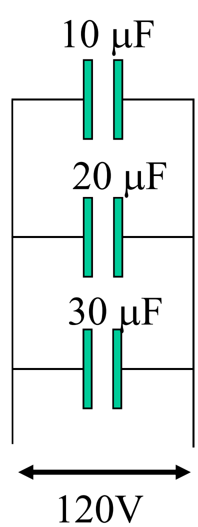


Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10\ \mu F)(120V) = 1200\ \mu C$
- $Q_2 = (20\ \mu F)(120V) = 2400\ \mu C$
- $Q_3 = (30\ \mu F)(120V) = 3600\ \mu C$

Note that:
- Total charge (7200 μC) is shared between the 3 capacitors in the ratio $C_1:C_2:C_3$ -- i.e. 1:2:3

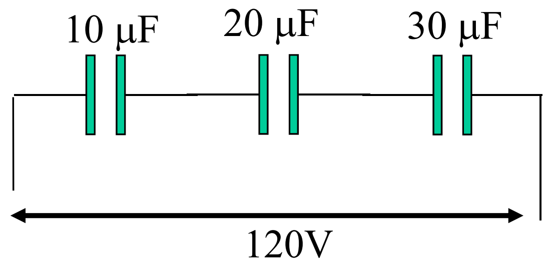


Example 2

What is the potential difference across each capacitor?

- Q = CV; Q is same for all capacitors
- Combined C is given by:

$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$



- C_{eq} = 5.46 μF
- Q = CV = (5.46 μF)(120V) = 655 μC
- V_1= Q/C_1 = (655 μC)/(10 μF) = 65.5 V
- V_2= Q/C_2 = (655 μC)/(20 μF) = 32.75 V
- V_3= Q/C_3 = (655 μC)/(30 μF) = 21.8 V

Note: 120V is shared in the ratio of INVERSE capacitances i.e.1:(1/2):(1/3) (largest C gets smallest V)

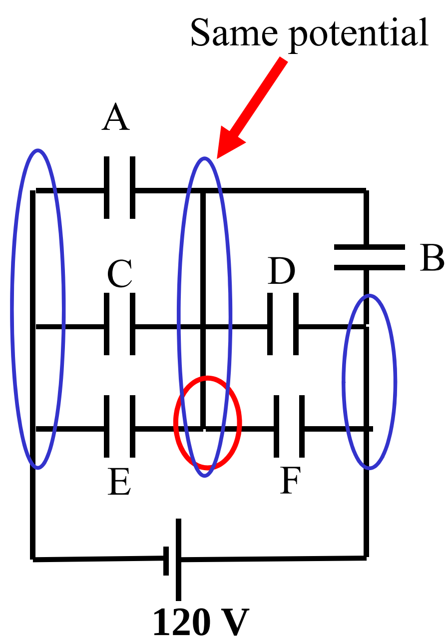
Example 3

Which of the following statements is FALSE?

(a) B, D and F are in PARALLEL.

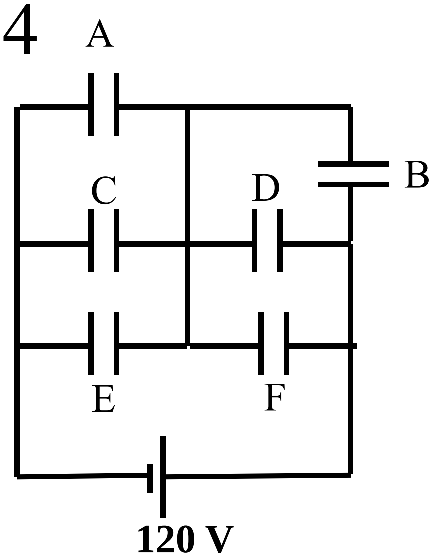
(b) E and F are in SERIES.

(c) A, C and E are in PARALLEL.

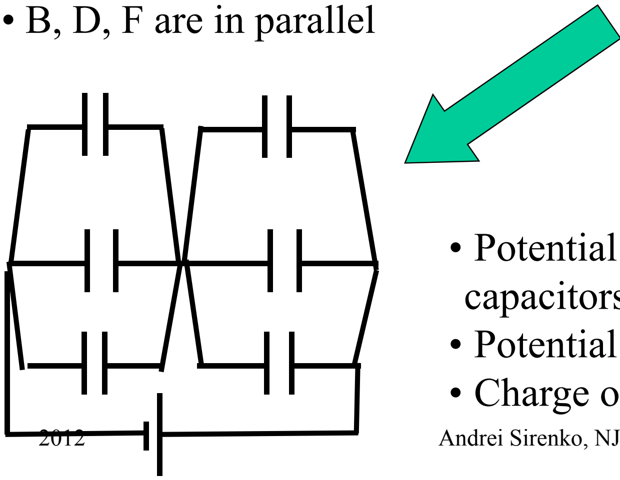


Example 4

If each capacitor = 10 μF, what is the charge on capacitor A?



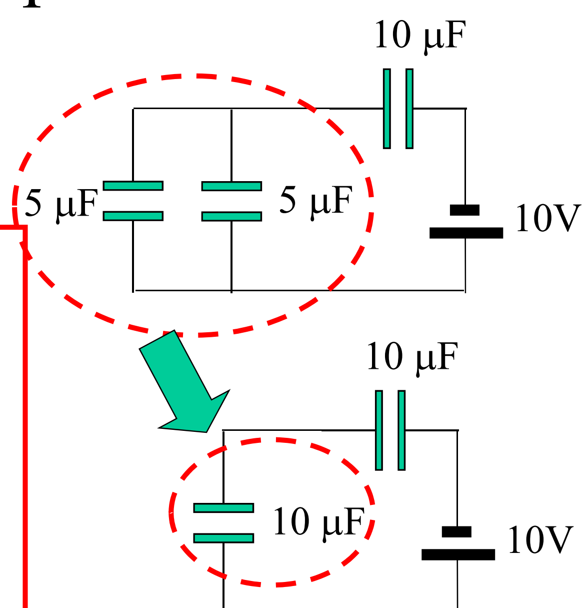
- A, C, E are in parallel
- B, D, F are in parallel



- Potential difference across each set of capacitors = 120V/2 = 60V
- Potential difference across A = 60V
- Charge on A = (**10 μF)(60 V) = 600 μC**

Example 5

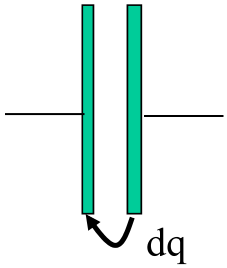
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Energy Stored in a Capacitor

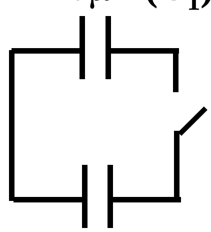
- **Start out with uncharged capacitor**
- **Transfer small amount of charge *dq* from one plate to the other until charge on each plate has magnitude Q**
- **How much work was needed?**



$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \boxed{\frac{Q^2}{2C}} = \frac{CV^2}{2}$$

Example 6

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**



Initial charge on 10μF = (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2 = 1200\mu C$

Also, V_{final} is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_1 = \frac{Q_2}{2} \Rightarrow$

- $Q_1 = 400\mu C$
- $Q_2 = 800\mu C$
- $V_{final} = Q_1/C_1 = 40$ V

Initial energy stored = $(1/2)C_1V_{initial}^2 = (0.5)(10\mu F)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2 = 24mJ$

Energy lost (dissipated) = 48mJ

Question: where did the energy go??

Summary

- Capacitors in SERIES: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$
  (same Q)
- Capacitors in PARALLEL: $C_{eq} = C_1 + C_2 + \ldots$
  (same *V)*
- Capacitors store energy: $U = (1/2)CV^2 = Q^2/2C$