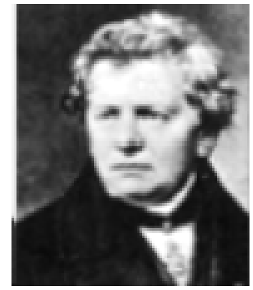
Lecture 11



Georg Simon Ohm (1789-1854)

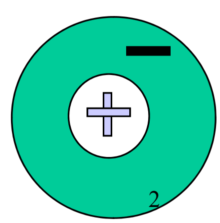
Capacitance, current, and resistance

http://web.njit.edu/~sirenko/

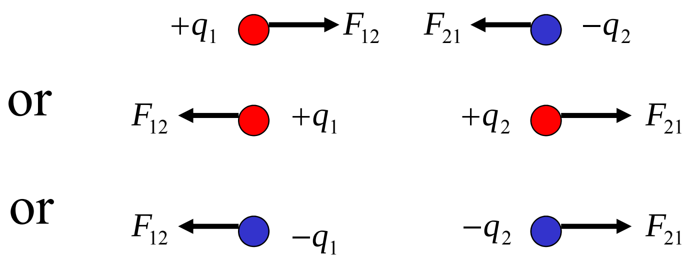
Physics 103 Spring 2012

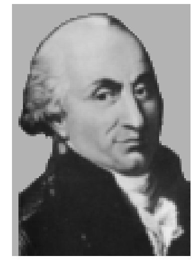
Electric Charge

- Electric charge is fundamental characteristic of elementary particles
- Two types of charges: positive/negative
- Labels are simply a convention
- Atomic structure :
  - negative electron cloud
  - nucleus of positive protons, uncharged neutrons



Force between pairs of point charges

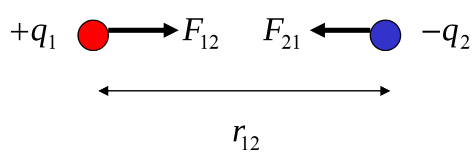




Coulomb's law -- the force between point charges:

- Lies along the line connecting the charges.
- Is proportional to the magnitude of each charge.
- Is inversely proportional to the distance squared.
- Note that Newton's third law says $|F_{12}| = |F_{21}|$!!

Coulomb's law



$$|F_{12}| = \frac{k\ |q_1||q_2|}{r_{12}^2}$$

For charges in a VACUUM

$$k = 8.99 \times 10^9\ \frac{N\,m^2}{C^2}$$

Often, we write k as:

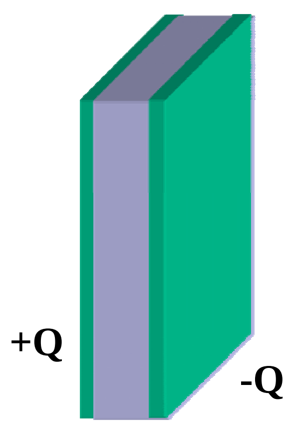
$$k = \frac{1}{4\pi\varepsilon_0} \text{ with } \varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\,m^2}$$

Quantization of Charge

- Charge is always found in **INTEGER** multiples of the charge on an electron/proton
- Electron charge $e^- = -1.6 \times 10^{-19}$ Coulombs [C]
- Proton charge $p = e = +1.6 \times 10^{-19}$ Coulombs
- Unit of charge: Coulomb (C) in SI units
- One cannot ISOLATE FRACTIONAL CHARGE (e.g., 0.8×10^{-19} C, $+1.9 \times 10^{-19}$ C, *etc.*)
- $e = \Sigma$ quarks, $(\pm 2/3e, \pm 1/3e)$
- Charge: Q, q, -q, -5q, …., 7q, *etc.*
- $Q = 1$ C is OK, it means $Q = (1 \pm 1.6 \times 10^{-19})$ C

Capacitance

- Capacitance depends only on GEOMETRICAL factors and on the MATERIAL that separates the two conductors
- e.g. Area of conductors, separation, whether the space in between is filled with air, plastic, etc.



(We first focus on capacitors where gap is filled by vacuum or air !)

Parallel Plate Capacitor

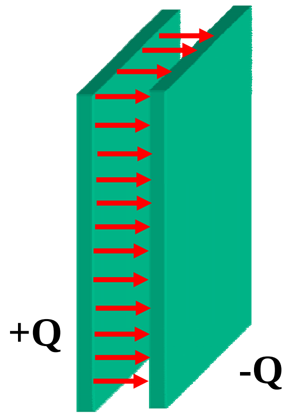
What is the electric field in between the plates? (Gauss' Law)

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Relate E to potential difference V between the plates:

What is the capacitance C?

$$C = Q/V = \frac{\varepsilon_0 A}{d}$$



Area of each plate = A
Separation = d
charge/area = σ = Q/A

$$\varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{F/m}$$

Isolated Parallel Plate Capacitor

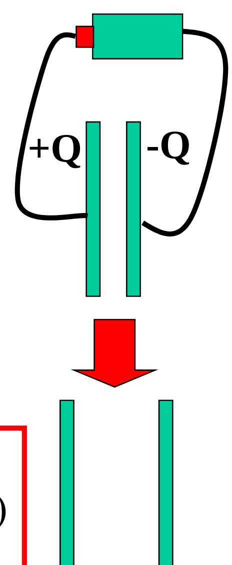
- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Battery is then disconnected.
- If the plate separation is INCREASED, does potential difference V:

(a) Increase?

(b) Remain the same?

(c) Decrease?

- Q is fixed!
- C decreases (=ε_0A/d)
- Q=CV; V increases.



Parallel Plate Capacitor & Battery

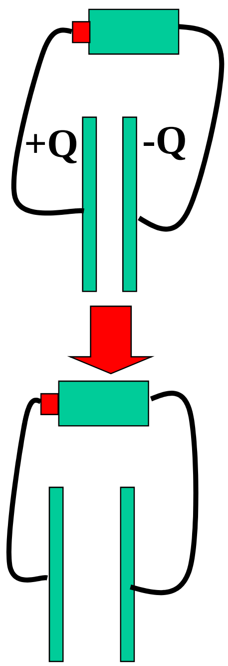
- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Plate separation is INCREASED while battery remains connected.
- Does the electric field inside:

(a) Increase?

(b) Remain the same?

(c) Decrease?

- V is fixed by battery!
- C decreases ($=\varepsilon_0 A/d$)
- Q=CV; Q decreases
- $E = Q/\varepsilon_0 A$ decreases

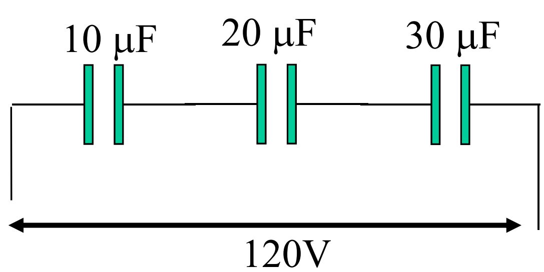


Example 2

What is the potential difference across each capacitor?

- Q = CV; Q is same for all capacitors
- Combined C is given by:

$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$



- C_{eq} = 5.46 μF
- Q = CV = (5.46 μF)(120V) = 655 μC
- $V_1 = Q/C_1$ = (655 μC)/(10 μF) = 65.5 V
- $V_2 = Q/C_2$ = (655 μC)/(20 μF) = 32.75 V
- $V_3 = Q/C_3$ = (655 μC)/(30 μF) = 21.8 V

Note: 120V is shared in the ratio of INVERSE capacitances i.e.1:(1/2):(1/3) (largest C gets smallest V)

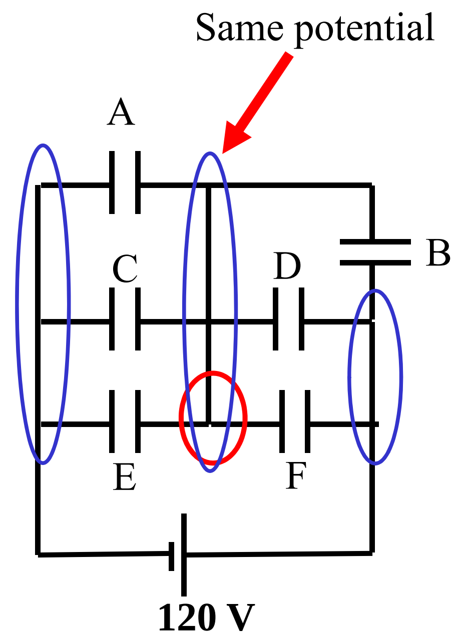
Example 3

Which of the following statements is FALSE?

(a) B, D and F are in PARALLEL.

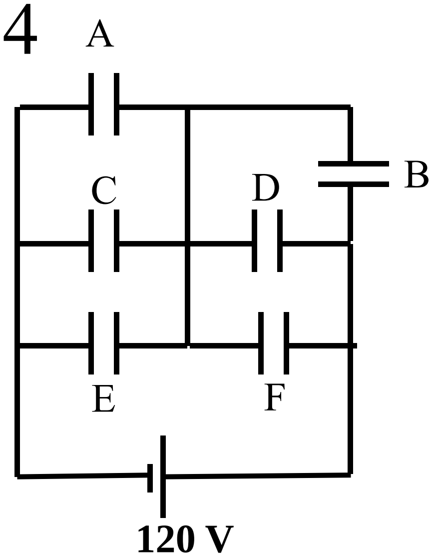
(b) E and F are in SERIES.

(c) A, C and E are in PARALLEL.

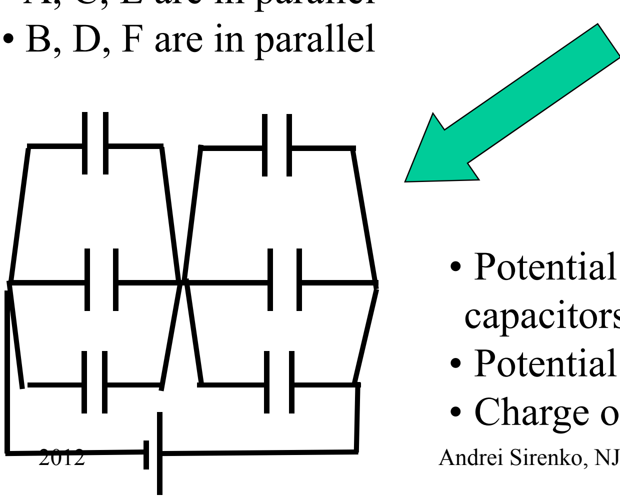


Example 4

If each capacitor = 10 μF, what is the charge on capacitor A?



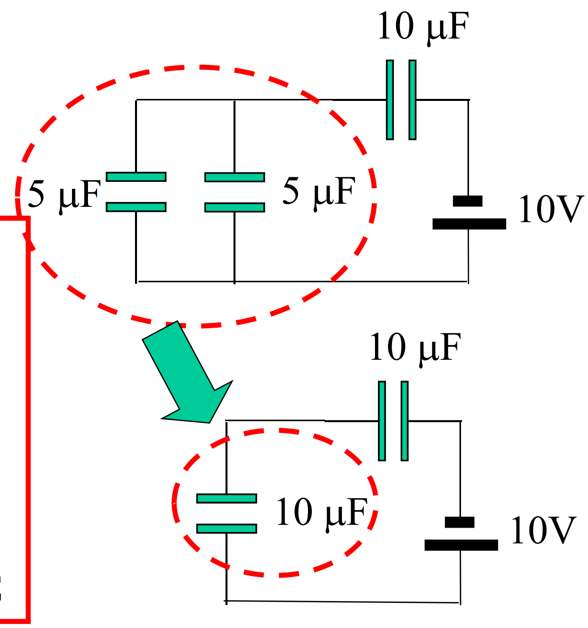
- A, C, E are in parallel
- B, D, F are in parallel



- Potential difference across each set of capacitors = 120V/2 = 60V
- Potential difference across A = 60V
- Charge on A = **(10 μF)(60 V) = 600 μC**

Example 5

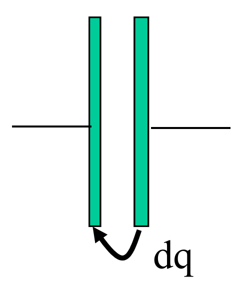
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Energy Stored in a Capacitor

- **Start out with uncharged capacitor**
- **Transfer small amount of charge *dq* from one plate to the other until charge on each plate has magnitude Q**
- **How much work was needed?**

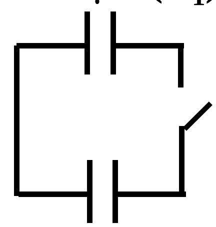


$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \boxed{\frac{Q^2}{2C}} = \frac{CV^2}{2}$$

Example 6

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**

10μF (C_1)



20μF (C_2)

Initial charge on 10μF = (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2$ = 1200μC

Also, V_{final} is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ⇨ $Q_1 = \frac{Q_2}{2}$ ⇨

- Q_1 = 400μC
- Q_2 = 800μC
- $V_{final} = Q_1/C_1$ = 40 V

Initial energy stored = $(1/2)C_1V_{initial}^2 = (0.5)(10\mu F)(120)^2$ = 72mJ

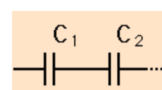
Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2$ = 24mJ

Energy lost (dissipated) = 48mJ

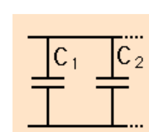
Question: where did the energy go??

Summary for Capacitors

- Capacitors in SERIES: (same Q) $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$



- Capacitors in PARALLEL: $C_{eq} = C_1 + C_2 + \ldots$ (same *V)*



- Capacitors store energy: $U = (1/2)CV^2 = Q^2/2C$

Georg Simon Ohm
(1789-1854)

"a professor who preaches such heresies is unworthy to teach science." Prussian minister of education 1830

Current and resistance

Microscopic view of charge flow: current density, resistivity and drift speed

Macroscopic view: current and resistance

Relating microscopic and macroscopic

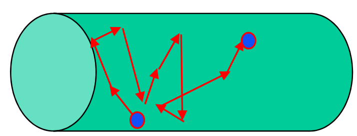
Conductors, semiconductors, insulators, superconductors

Conductors in electrostatic equilibrium

Suppose a piece of copper wire is placed in a static electric field and not connected to anything else. What happens?

• **Naïve view**: electrons are free to move; they arrange themselves in such a way that E = 0 inside the wire; no more electron motion!

• **Reality**: electrons inside the wire keep moving all the time, but on average they arrange themselves so that E = 0 inside!



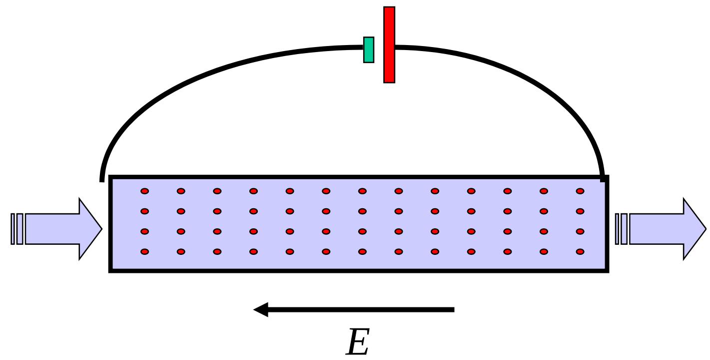
Random motion of electrons:
• Movement is random because of "collisions" with vibrating nuclei
• In between collisions, electrons move VERY FAST $\sim 10^6$ m/s!

Conductors in absence of equilibrium

Non-equilibrium -- imagine that you attach a battery across a copper wire so that electrons can be put into the wire and extracted from the wire:

- Now: E is NOT ZERO inside the conductor
- Electrons "drift" because of the non-zero electric field ("electric current")
- Drift is much, much SLOWER than random motion: typically < mm/s

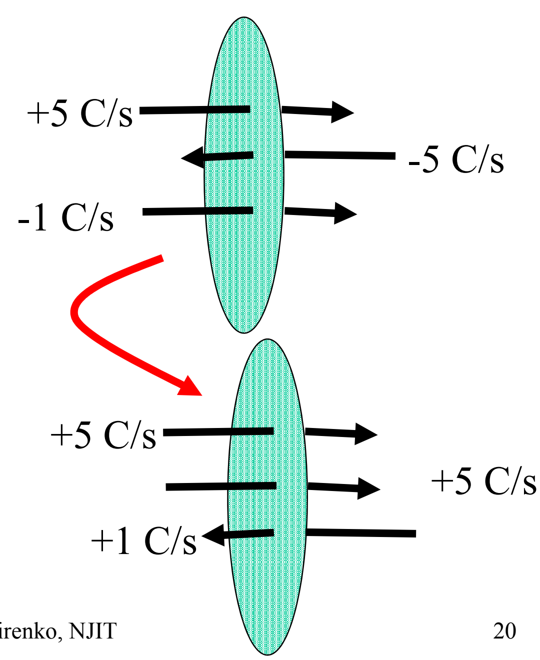
$$I = \frac{dQ}{dt} \quad [A = C/s]$$



- Drift of electrons creates a net flow of charge -- CURRENT measured in Amperes = C/s
- Convention: "current" is viewed as flow of POSITIVE charge

Current: example 1

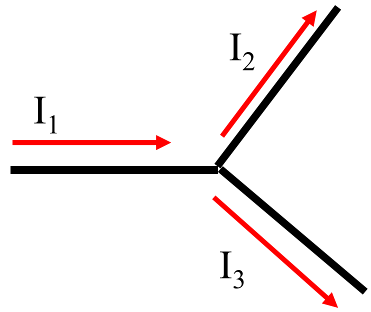
- The figure shows charges moving at the given rates
- What is the total current flowing through the area shown?
- Remember that current is the flow of POSITIVE charge!
- Total current = 5A + 5A - 1A = 9A (towards the right)

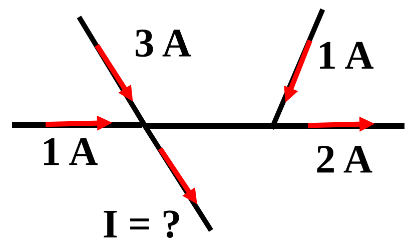


Current: Example 2

- Charge is CONSERVED: total current into junction = total current out of junction
- In figure shown, e.g. $I_1=I_2+I_3$
- Useful analogy: think of water flow!
- Example: what is the current I in the wire shown at right?

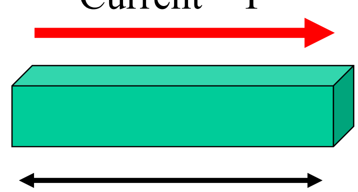
- Answer: I = 3A





Resistance

- Since electrons collide with vibrating nuclei, there is "resistance" to the free flow of charge.
- Define "Resistance" = R = V/I (i.e. "how much charge flow do I get for an applied potential difference?")
- Units: Ohm (Ω)
- Macroscopic property -- e.g. we talk about "resistance" of an object such as a specific **piece** of wire. This is not a **local** property

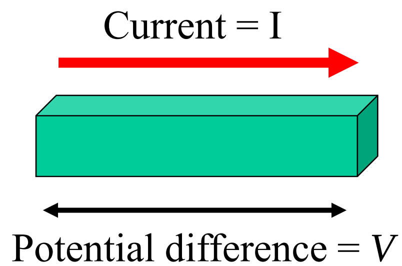


$$R = \frac{V}{I}$$

Circuit symbol for R

Relating the Macro- and Microscopic views

- Conductor of UNIFORM cross-sectional area A, length L
- Potential difference *V* applied across ends
- Note: what if the area was not uniform?? (HW problem)



$$R = \frac{V}{I} = \frac{EL}{JA} = \frac{\rho L}{A}$$

Resistance:

How much potential do I need to apply to a device to drive a given current through it?

$$R = \rho \frac{L}{A} = \frac{E}{J}\frac{L}{A}$$

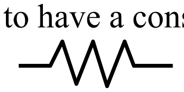
Ohm's laws

$$R \equiv \frac{V}{i} \quad \text{and therefore}: \; i = \frac{V}{R} \quad \text{and} \; V = iR$$

Georg Simon Ohm (1789-1854)

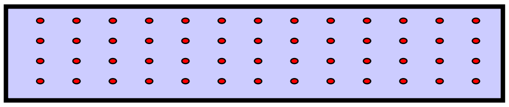
$$\text{Units}: [\text{R}] = \frac{\text{Volt}}{\text{Ampere}} \equiv \text{Ohm (abbr. } \Omega)$$

For many materials, R remains a constant for a wide range of values of current and potential.

Devices specifically designed to have a constant value of R are called resistors, and symbolized by 

Variation of resistance (resistivity) with temperature

R, ρ change with temperature in a complicated, material-dependent way.



Why does it change? Nuclei vibrate due to thermal agitation, and scatter electrons as they pass.

For many conductors, it can be approximated by a linear temperature dependence (for a small range of temperatures),

$$\frac{\rho - \rho_0}{\rho_0} = \alpha (T - T_0)$$

With α determined empirically and listed in tables.

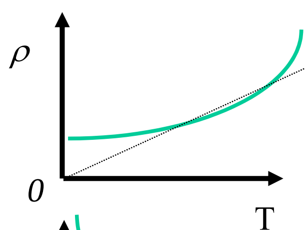
Trivia: why do light bulbs mostly die at the moment of switch-on?

Answer: when the filament is cold it has less resistance, therefore it is the moment when the current is maximum.

Various materials and temperature:

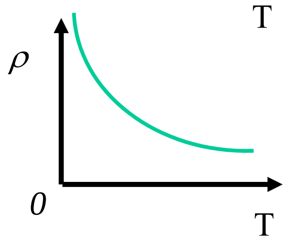
Conductors:

Examples: copper, aluminum, iron.

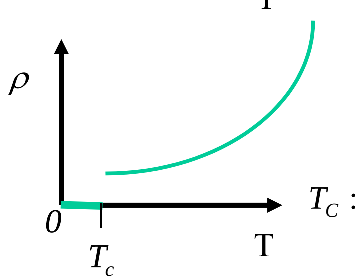


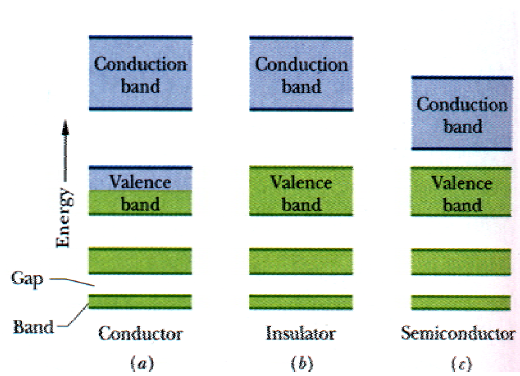
Semiconductors

Examples: Germanium, silicon.



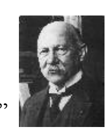
Superconductors





T_C :

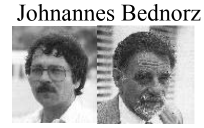
0.1 to 20K for "usual" materials, e.g. mercury (1911)



Heike Kammerlingh-Onnes

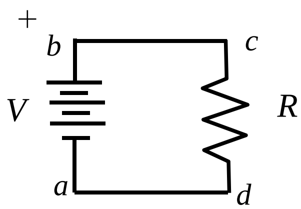
30 to 180K for "high temperature" materials, e.g. YBaCu (1986)

Karl Mueller
Johnannes Bednorz



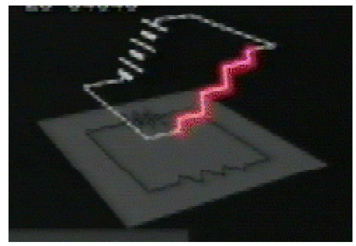
Power dissipation:

Resistance was a measure of the "cost" of establishing a current in a realistic conductor. The "cost" can be characterized in terms of the energy one needs to constantly input to a conductor in order to keep a current going.



Let us follow an amount of charge *dq* as it moves through the circuit, starting at a.

From *a* to *b*, through the battery, its potential energy is increased by *Vdq*.
From b to c its potential is constant, similarly from *c* to *d*.



When it is back at *a*, its potential energy should be the same as when it started. Therefore there must have been a loss of potential energy of amount -*Vdq* when moving through the resistance.

$$dU = Vdq \quad = V\,idt \quad \Rightarrow \quad \text{Power} = \frac{dU}{dt} = V\,i$$

Units: Watt

$$\text{Applying Ohm's laws:} \quad \text{Power} = (i\,R)\,i = i^2 R \qquad \text{Power} = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

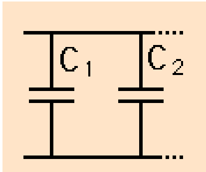
Summary:

- We saw that charges moving through conductors experience "resistance" to their motion.
- resistance is related to the electron drift speed
- We discussed microscopic and macroscopic view of electrical currents.
- Studied temperature and material dependence.
- Discussed how moving charges costs and delivers electrical power.

Lecture QZ

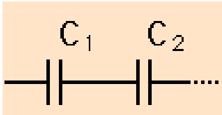
1. Capacitors C_1 and C_2 are connected in parallel. The equivalent capacitance is given by

(a) $C_1C_2/(C_1+C_2)$
(b) $(C_1+C_2)/C_1C_2$
(c) $1/(C_1+C_2)$
(d) C_1/C_2
(e) C_1+C_2



2. Capacitor s C_1 and C_2 are connected in series. The equivalent capacitance is given by

(a) $C_1C_2/(C_1+C_2)$
(b) $(C_1+C_2)/C_1C_2$
(c) $1/(C_1+C_2)$
(d) C_1/C_2
(e) C_1+C_2



Summary: