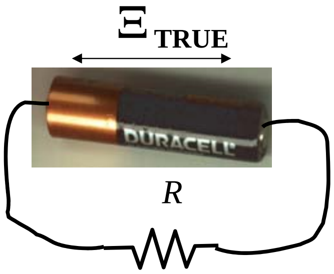
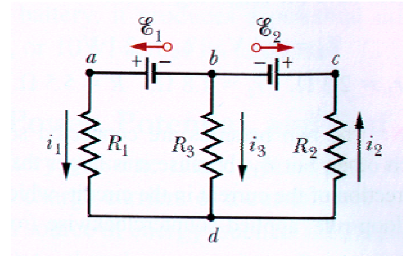


Lecture 12



Circuits

http://web.njit.edu/~sirenko/

Physics 103 Spring 20012

Today

- EMF devices: ideal vs. non-ideal
- Single loop circuits
- Multiloop circuits
- Examples for CQZ3

Resistance:

$$R = \rho \frac{L}{A}$$

Current:

$$i = \frac{dQ}{dt}$$

Ohm's laws

$$R \equiv \frac{V}{i} \qquad \text{and therefore: } i = \frac{V}{R} \quad \text{and } V = iR$$

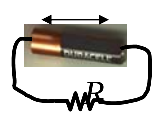
Georg Simon Ohm
(1789-1854)

$$\text{Units: [R]} = \frac{\text{Volt}}{\text{Ampere}} \equiv \text{Ohm (abbr. } \Omega)$$

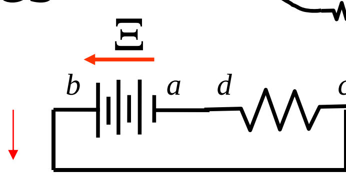
For many materials, R remains a constant for a wide range of values of current and potential.

Devices specifically designed to have a constant value of R are called resistors, and symbolized by 

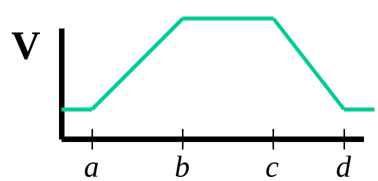
EMF Devices

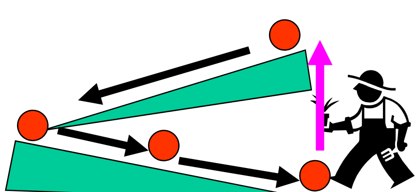


- "EMF devices": boost electric charge from a point of low electric potential energy to high electric potential energy
- This is done by converting various forms of energy (chemical, mechanical, light,…) into electrical energy (batteries, generators, solar cells,….)
- View battery as pump that pushes positive charge around a circuit.



Difference in electric potential between the terminals of an EMF device is called the "EMF" = Ξ

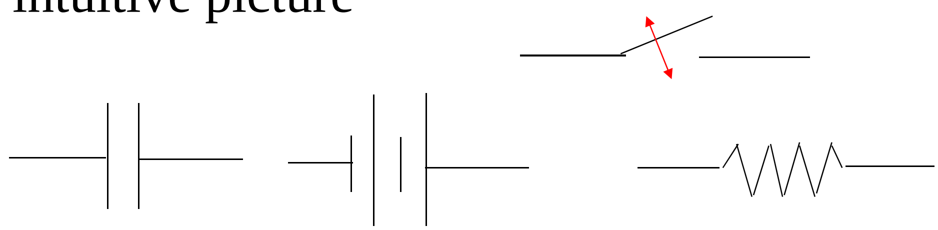




Emf devices:
- "dc" (Ξ fixed)
- "ac" (Ξ varies with time)
- Initial focus: dc circuits.

RC Circuits

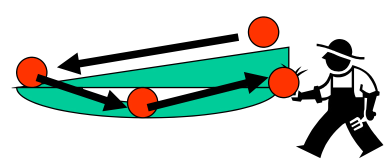
- Loop analysis of an RC circuit: doing the math
- Time constant
- Physical understanding of RC circuits: intuitive picture



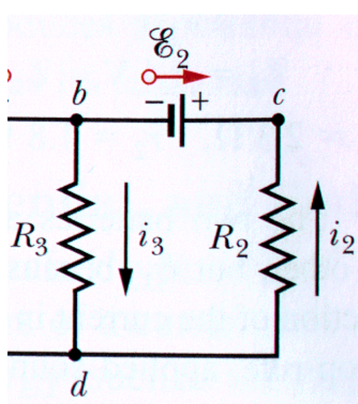
EMF Devices

- "EMF"

$$\Xi = \frac{dW}{dq}$$

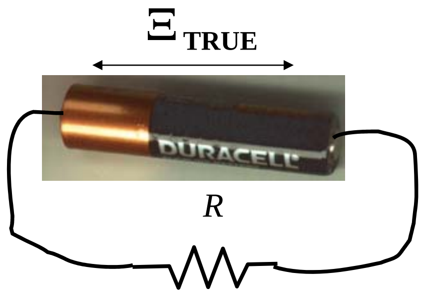


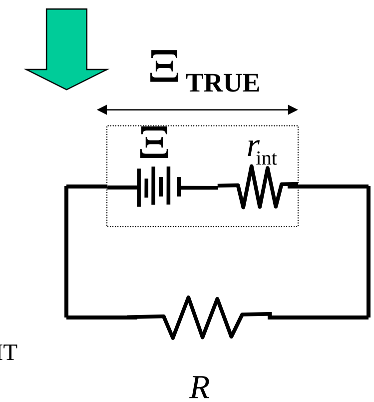
- Units: [V]



Ideal & non-ideal EMF Devices

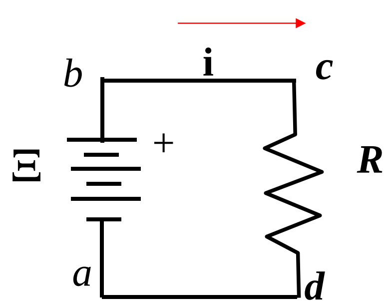
- **Ideal EMF devices**: no matter what you connect across the EMF source, the emf Ξ **is constant**.
- **Non-ideal EMF device**: the emf Ξ depends on what else you connect in the circuit
- Devices are non-ideal because of energy dissipation -- we model this is as an ideal emf Ξ + "internal resistance"





Single Loop DC circuit

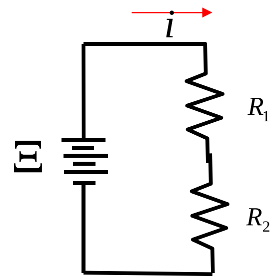
- Recall that electric potential at any given point is uniquely defined once you arbitrarily decide where $V = 0$.
- If you go around a circuit in a complete loop, the net change in potential = 0!
- RULES:
  - When walking through an EMF device, **add + Ξ if you go from - to + terminal** or - Ξ otherwise.
  - When walking through a resistor, **add -*iR* if flowing with the current** or +***iR*** otherwise. "Current flows downhill thru resistors"



- Start at **"a"**, walk clockwise, end at **"a"**
- $+ \Xi - iR = 0$
- $\Xi = iR$

Example 1: Resistors in Series

- "Series" Resistors:
  - connected in a "chain"
  - carry the SAME CURRENT
  - total potential difference across resistors in series = sum of potential difference across each resistor



$$+\Xi - iR_1 - iR_2 = 0$$

$$\Rightarrow \quad \Xi = i(R_1 + R_2) = iR_{\text{tot}}$$

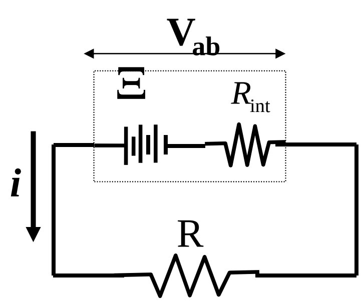
$$R_{\text{tot}} = R_1 + R_2$$

If you have n resistors in series : $R_{\text{tot}} = \sum_{i=1}^{n} R_i$

Series R is always greater than individual R

Example 2: non-ideal battery

- Non-ideal battery of emf Ξ is connected across a resistor R.
- Internal resistance of battery = R_{int}
- What is the potential difference across the terminals of the battery?



$$+\Xi - iR - iR_{\text{int}} = 0$$

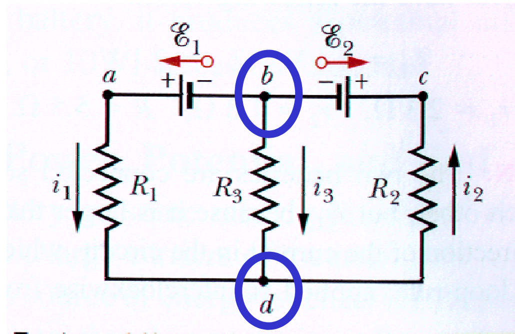
$$i = \frac{\Xi}{R + R_{\text{int}}}$$

$$V_{ab} = -iR_{\text{int}} + \Xi = iR$$

$$= \frac{\Xi R}{R + R_{\text{int}}}$$

Example 3: Multiloop Circuits

- Assign a different current to each branch of the circuit -- choose ARBITRARY directions for currents.
- At junctions, net current in = net current out. Each junction gives one equation.
- Take a walks around each loop in the circuit: sum of potential difference = 0. Each walk gives an equation.
  - Choose enough equations to solve for the unknowns!
  - Make sure equations are INDEPENDENT!

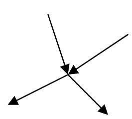


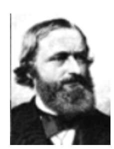
Given: Ξ_1, Ξ_2, R_1, $R_{2,}$, R_3
Find: i_1, i_2, i_3
3 unknowns; need 3 <u>independent</u> simultaneous equations:
• 1 junction equation
• 2 loop equations

Kirchoff's Laws

- Kirchhoff's law for currents

$$\sum_{j=1}^{K} i_j = 0$$



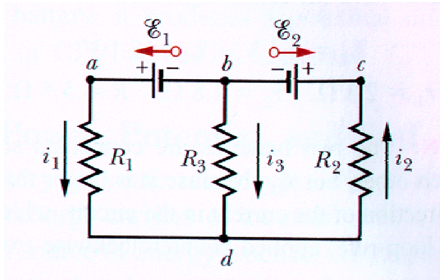


Gustav Robert Kirchhoff (1824-1887)

- Kirchhoff's law for EMF

$$\sum_{i=1}^{K} \Xi_i = \sum_{j=1}^{N} i_j R_j$$

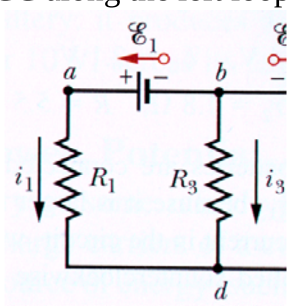
Let's do it!



Junction Rule (1st Kirchhoff law):

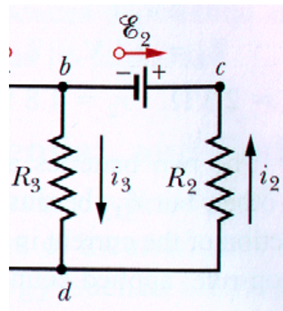
$$i_1 + i_3 = i_2$$

Walk CC along the left loop:



$$i_1R_1 - i_3R_3 = \Xi_1$$

Walk CC along the right loop:

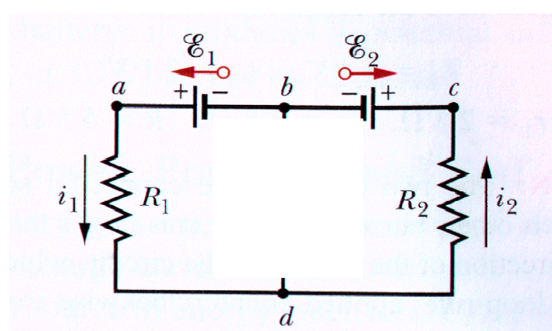


$$-i_3R_3 - i_2R_2 = \Xi_2$$

Given the values of the EMF's and the resistors, we are left with three equations with three unknowns, i.e., the three currents.

Kirchoff's Laws

- Procedure just followed: Kirchoff's Laws
- Is the procedure unique? No! For instance, we could have taken the left loop and the "exterior loop".
- Note that the end result will be unique!

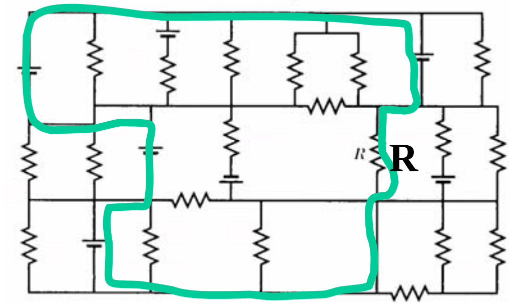


$$-i_1R_1 - i_2R_2 - \Xi_2 + \Xi_1 = 0$$

Example 3 -- A Loop Puzzle: HRW Q28.12

- All resistors are equal (4 Ω) and all batteries are IDEAL with emf = 4 V.
- What is the current through resistor R?
- Hint: potential difference across the terminals of an **ideal** battery is ALWAYS = emf!

12. *Res-monster maze.* In Fig. 28-21, all the resistors have a resistance of 4.0 Ω and all the (ideal) batteries have an emf of 4.0 V. What is the current through resistor *R*? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



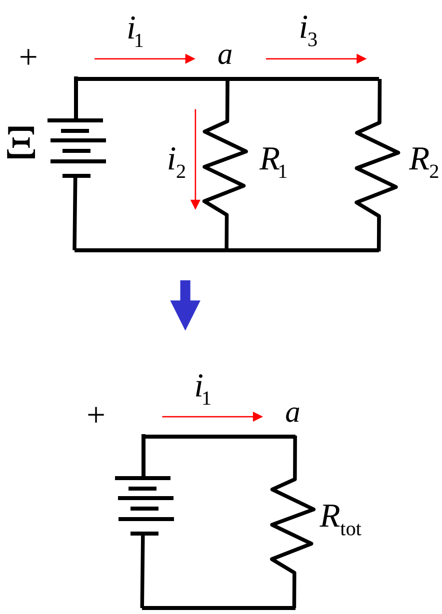
$$+V - V - V - V - iR = 0$$

$$-8V - i4\Omega = 0$$

$$|i| = 2A$$

Example 5: Resistors in Parallel

- Resistors in parallel:
  - SAME POTENTIAL DIFFERENCE
  - Total CURRENT thru resistors in parallel = sum of currents thru each resistor
- What is the equivalent resistance? -- let's use Kirchoff's Laws.



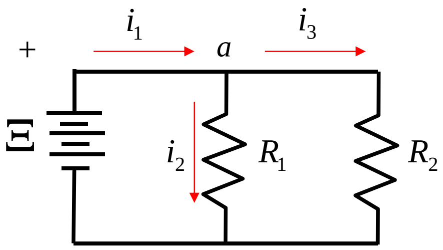
- First, use junction rule at “a”
- Next, use left loop:
- Then, use outer loop:

$$\text{Node a}: \; i_1 = i_2 + i_3$$

$$\text{Left loop}: \; \Xi - i_2 R_1 = 0 \Rightarrow \; i_2 = \frac{\Xi}{R_1}$$

$$\text{Outer loop}: \; \Xi - i_3 R_2 = 0 \; \Rightarrow \; i_3 = \frac{\Xi}{R_2}$$

$$i_1 = \frac{\Xi}{R_1} + \frac{\Xi}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Xi$$



$$R_{tot} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)}$$

Same as capacitors in series.

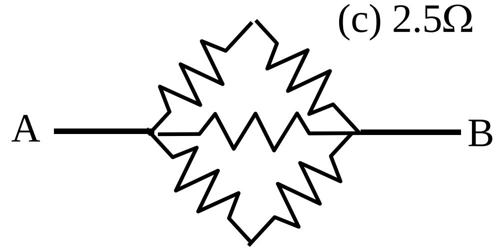
$$\text{For } n \text{ resistors}: \qquad R_{tot}^{-1} = \sum_{i=1}^{n} R_i^{-1}$$

Parallel R is always SMALLER than each resistor involved!

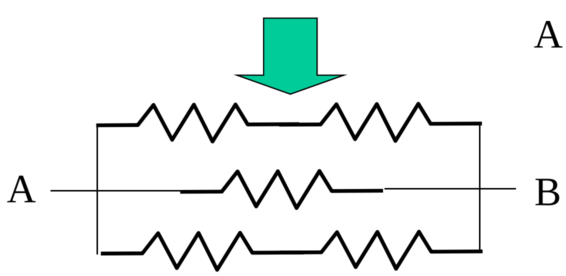
Example 1

Each resistor = 5Ω
What is the resistance between A and B?

(a) 5Ω
(b) 25Ω
(c) 2.5Ω



Equivalent to:



NOTE:
- **R in series with R = 2R**
- **R in parallel with R = R/2**

R_{total} = 5Ω|| 10Ω|| 10Ω = 5Ω|| 5Ω = 2.5Ω

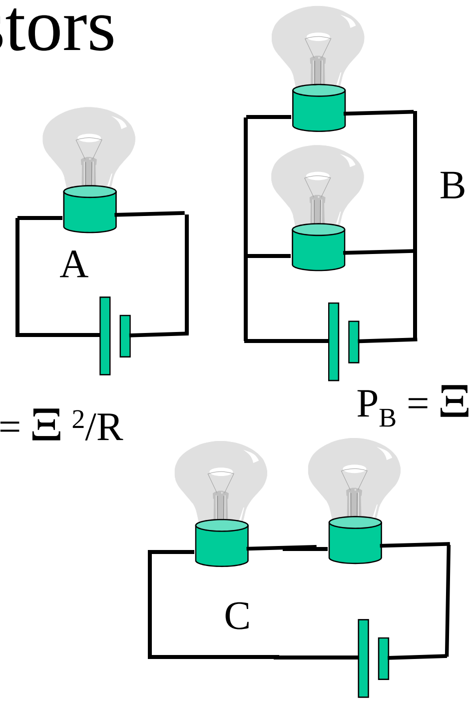
Example 3: Series & Parallel Resistors

- Assume that battery is IDEAL & all bulbs are identical & that brightness is proportional to power dissipated ($P = I^2R = V^2/R$)
- Rank the 3 circuits in order of <u>increasing</u> bulb brightness

(a) ABC

(b) C, [A&B same]

(c) [B&C same], A



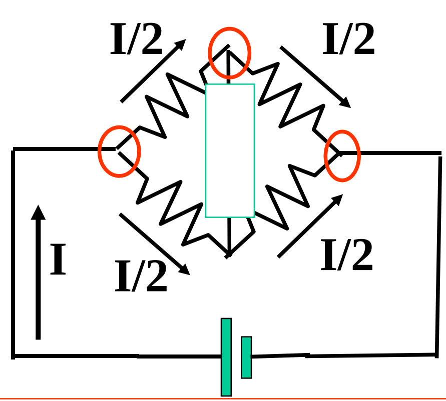
$P_A = \Xi^2/R$

$P_B = \Xi^2/R$

$P_C = \Xi^2/(4R)$

Example 8: Using Symmetry

- All resistors are equal = R;
- Ideal battery has emf Ξ
- What is the current I?



- Symmetry: no current goes thru the middle resistor!
- So, you can throw it away!
- Equivalent resistance = (2R||2R) = R
- So, $\mathbf{I = \Xi / R}$

Summary:

For n parallel resistors: $R_{\text{tot}}^{-1} = \sum_{i=1}^{n} R_i^{-1}$

For n resistors in series: $R_{\text{tot}} = \sum_{i=1}^{n} R_i$

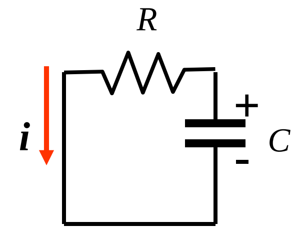
- To solve a circuit, start at any point and "take a walk" around it.
- Add potential every time you traverse an EMF from - to +.
- Deduct potential every time you move along a resistor "with the flow".
- Take as many walks along INDEPENDENT loops as needed to solve the circuit.

Reminder

- Capacitors: Q = CV
- Current: I=dQ/dt
- DC circuits: going around a loop, the net potential difference = 0

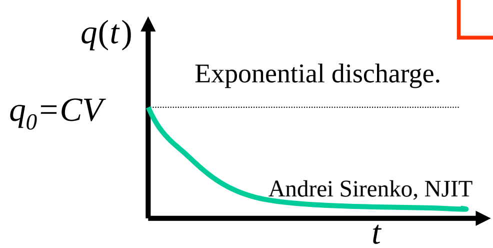
Discharging a capacitor

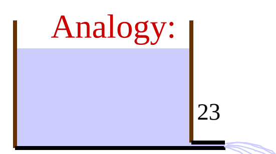
- Capacitor C is fully charged by an emf Ξ -- then we discharge the capacitor thru the resistor R.
- Use loop analysis at any instant in time
- Discharging a capacitor using an RC circuit: $q_0 = V_0C$ at $t = 0$.
- Note that i = -dq/dt *i.e.* decreasing charge gives a positive current.



$$-i(t)R + \frac{q(t)}{C} = 0$$

Solution: $q(t) = CV_0 e^{-\frac{t}{RC}}$



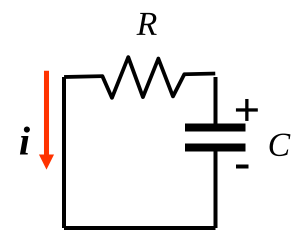


Discharging a capacitor

Kirchhoff's 2[nd] law

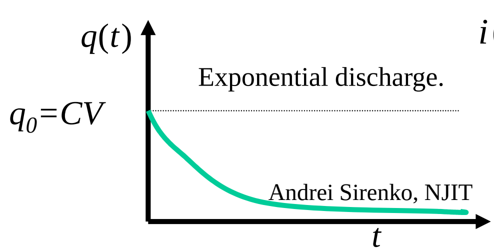
$$-iR + \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} + \frac{q}{C} = 0$$

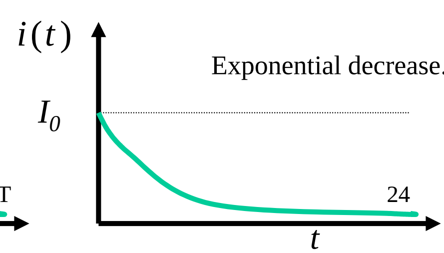
$$\frac{dq(t)}{dt} = -\frac{q_0}{RC} \qquad q(t=0) = q_0 = CV_0$$



$$i(t) = -\frac{dq}{dt} = I_0 e^{-\frac{t}{RC}} = \frac{q_0}{RC} e^{-\frac{t}{RC}} = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

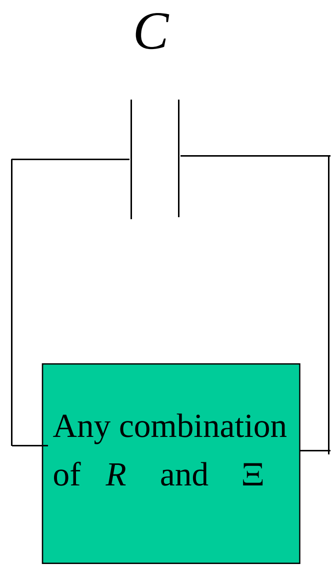
$$q(t) = q_0 e^{-\frac{t}{RC}} = CV_0 e^{-\frac{t}{RC}}$$





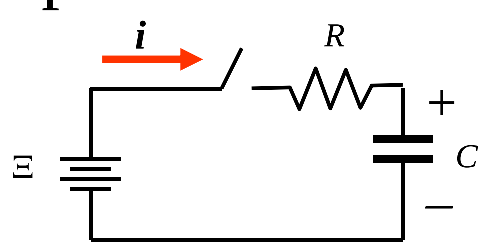
Current through a capacitor

- When t→∞
  $i = 0$
- V = Q/C



Charging a capacitor - I

- Capacitors are NOT conductors of current!
- Current can only flow through wires connected to capacitors to "build up charge" on the plates
- Eventually, this current stops.
- Loop analysis can be used for an RC circuit at any instant in time
- Charging a capacitor using an RC circuit: q = 0 at t =0.

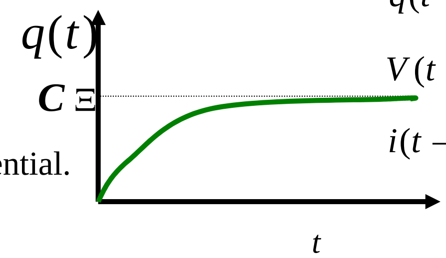


$$\Xi - iR - \frac{q}{C} = 0 \qquad i = \frac{dq}{dt}$$

$$\Xi - \frac{dq}{dt}R - \frac{q}{C} = 0$$

Solution: $q(t) = C\Xi\left(1 - e^{-\frac{t}{RC}}\right)$

$$q(t) = C\Xi\left(1 - e^{-\frac{t}{RC}}\right)$$



$$q(t \to \infty) = C\Xi$$

$$V(t \to \infty) = \frac{q}{C} = \Xi$$

$$i(t \to \infty) = 0$$

The capacitor charges as an exponential.

Proof that *q(t)* is a solution:

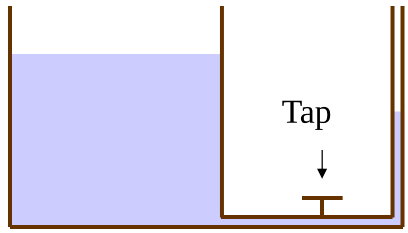
$$i = \frac{dq}{dt} = C\Xi\frac{e^{-\frac{t}{RC}}}{RC} = \Xi\frac{e^{-\frac{t}{RC}}}{R}$$

$$Ri = \Xi e^{-\frac{t}{RC}} \qquad \frac{q}{C} = \Xi\left(1 - e^{-\frac{t}{RC}}\right)$$

So, $\Xi - iR - \dfrac{q}{C} = 0$

Analogy:

Water tank



When tap is opened, the water level in the thin pipe will rise, until it equals the level of the big tank. The approach is actually exponential as well.

The Time Constant τ = RC

- Time constant = τ = RC
- For capacitor that is being charged, at t = τ, q(t) = (1-e^{-1}) C Ξ = 0.63C Ξ
- τ is the time required for a charging capacitor to reach ~63% of maximum charge
- Discharge: q(t) = (1/e) C Ξ = 0.37C Ξ
- Discharge: τ is time required to reach ~37% of maximum charge

Charging:

$$q(t) = C\Xi\left(1 - e^{-\frac{t}{RC}}\right)$$

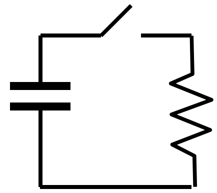
$$q(t) = C\Xi\left(1 - e^{-\frac{t}{\tau}}\right)$$

Discharging:

$$q(t) = C\Xi e^{-\frac{t}{\tau}}$$

Example 1

- The capacitor discharged, starting with a potential of 100 V.
- At *t=0*, close switch (on). At *t=10s*, potential=1V
- What is the time constant τ?



$$q(t) = q_0 e^{-\frac{t}{\tau}} = CV_0 e^{-\frac{t}{\tau}}$$

$$V(t) = \frac{q(t)}{C} = V_0 e^{-\frac{t}{\tau}}$$

$$-\frac{t}{\tau} = \ln\left[\frac{V(t)}{V_0}\right]$$

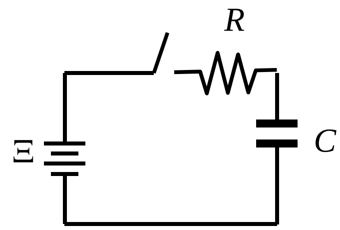
$$\tau = \frac{t}{\ln\left[\frac{V_0}{V(t)}\right]}$$

$$= \frac{(10s)}{\ln\left[\frac{100V}{1V}\right]} = 2.17 \text{ s}$$

RC Circuits: intuitive approach

- You cannot change the voltage (potential difference) across a capacitor SUDDENLY! It takes time.
- So, just after a switch is closed, the V across a capacitor is the SAME as just before the switch was closed.
- A capacitor is NOT A CONDUCTOR; so after a steady state is reached (i.e. long time after a switch is closed), there is NO CURRENT in a branch containing a capacitor.

Example 2



- The capacitor is initially uncharged
- Immediately after the switch is closed, which of the following quantities are **NOT** zero?

(a) Current I in the circuit

(b) Potential difference V_C across capacitor

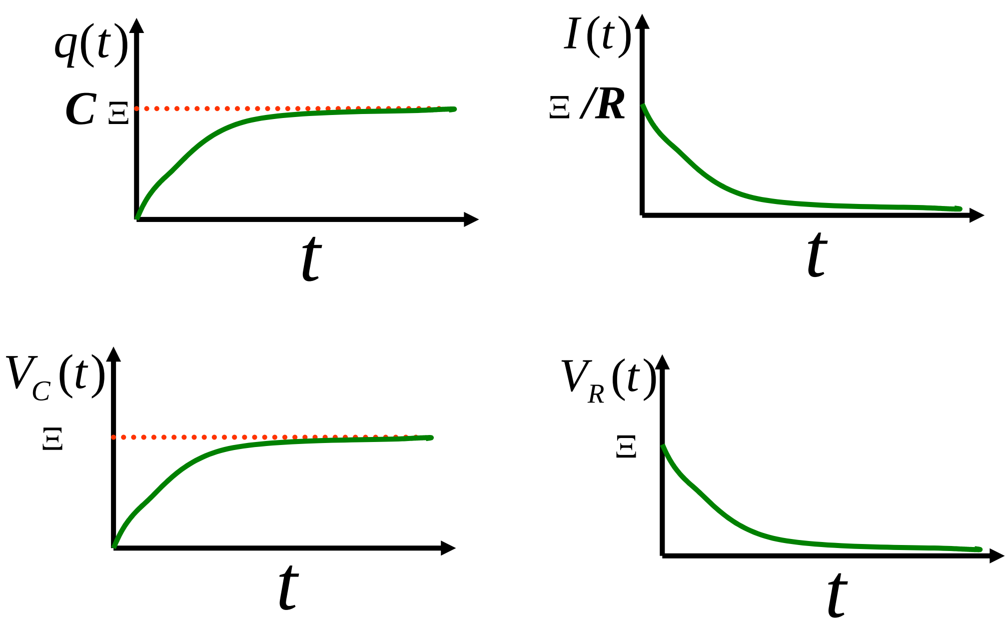
(c) Potential difference V_R across resistor

(d) Charge on capacitor

(e) Power dissipated by resistor

- Initially, q= 0, so $V_C = 0$;
- So, immediately after switch is closed, $V_C = 0$!
- So, V_R = Ξ ; and current I = Ξ /R (neither are zero!)
- Also, power dissipated by resistor = Ξ ²/R

Example 2 (continued)



Do the experiment: attach a light bulb across 1F capacitor and battery: Bulb lights up IMMEDIATELY and then gradually goes out!

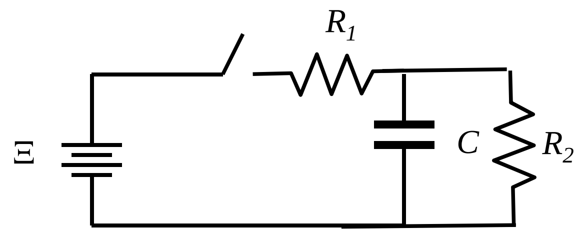
Example 3: using our intuition

- Capacitor initially UNCHARGED.
- At $t=0$, close switch.
- **Immediately** after the switch is closed, what is the power dissipated by the resistor R_2?

(a) ZERO.

(b) Ξ^2/R_2

(c) $\Xi^2 R_2/(R_1+R_2)^2$



• Before switch is closed, capacitor voltage = 0

• So, just after switch is closed, capacitor voltage = 0

• Potential difference across $R_2 = 0$

• Current thru $R_2 = 0$

• Power dissipated by $R_2 = 0$

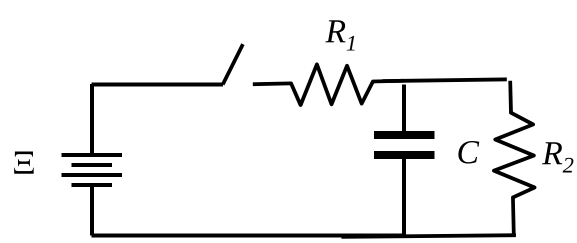
Example 4: using our intuition

- Capacitor initially UNCHARGED.
- At $t=0$, close switch.
- **Long after** the switch is closed, what is the energy dissipated by the resistor R_2?

(a) ZERO.

(b) $\Xi^2 R_2/(R_1+R_2)^2$

(c) Ξ^2/R_2



• Long time after switch is closed, NO current thru capacitor branch

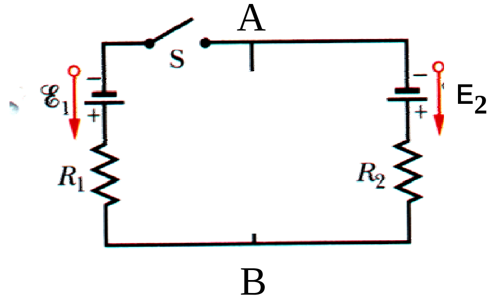
• Treat as if C wasn't there!

• Current = $I = \Xi/(R_1+R_2)$

• Power dissipated = $I^2R_2 = \Xi^2 R_2/(R_1+R_2)^2$

Example 5:HRW 76P

- The circuit has been connected for a LONG time with S kept open from the beginning. Suppose $\Xi_1 > \Xi_2$.
- S is now closed and, again, we wait for a long time. What is the charge on the capacitor?

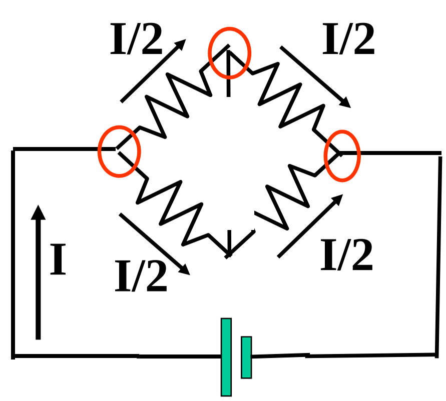


• If you wait for a LONG time, there is no current in capacitor branch!
• So: the only current is in the outer loop and you can ignore the capacitor! -- $I = (\Xi_1 - \Xi_2)/(R_1+R_2)$
• Now: potential difference across C = $V_A - V_B = \Xi_1 - IR_1$
• So, the charge on the capacitor = $Q = C(V_A - V_B)$

$$Q = C(\Xi_1 - iR_1) = C\left(\Xi_1 - R_1\frac{\Xi_1 - \Xi_2}{R_1 + R_2}\right) = C\left(\frac{\Xi_1 R_2 + \Xi_2 R_1}{R_1 + R_2}\right)$$

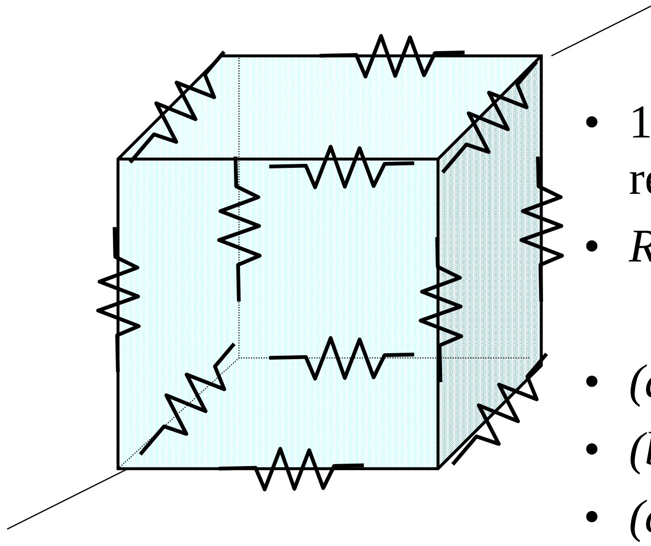
Example 8: Using Symmetry

- All resistors are equal = R;
- Ideal battery has emf Ξ
- What is the current I?



• Symmetry: no current goes thru the middle resistor!
• So, you can throw it away!
• Equivalent resistance = (2R||2R) = R
• So, **I = Ξ /R**

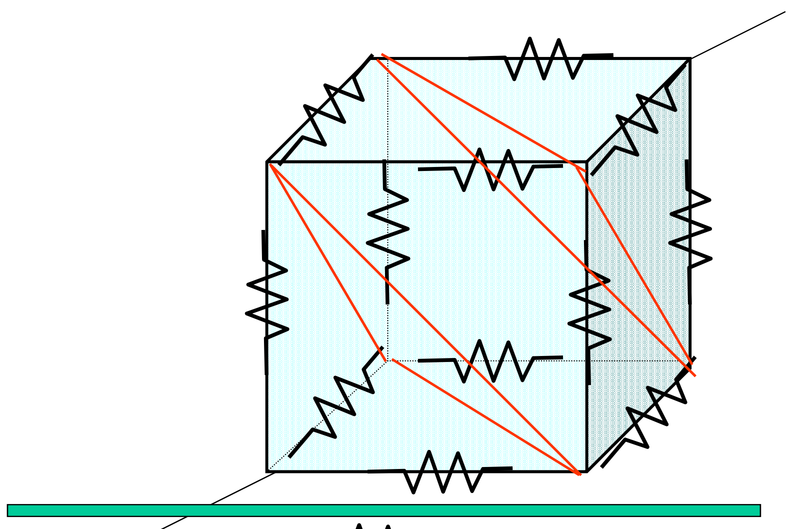
Resistors: Parallel or in Series ???



- 12 identical resistors *R*
- R_{eq} = *???*

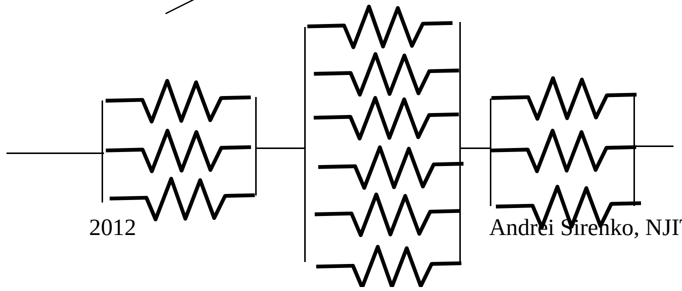
- *(a)* $R_{eq} = 12\,R$
- *(b)* $R_{eq} = R/12$
- *(c)* $R_{eq} = 10R/12$
- *(d)* $R_{eq} = 3R/12$

Resistors: Parallel or in Series ???



- 12 identical resistors *R*
- R_{eq} = *???*

- *(a)* $R_{eq} = 12\,R$
- *(b)* $R_{eq} = R/12$
- *(c)* $R_{eq} = 10R/12$
- *(d)* $R_{eq} = 3R/12$



R_{eq}=R/3+R/6+R/3=

=5R/6≡10R/12

Summary:

- Final Exam includes:
    - R-Ξ circuits
    - RC circuirs
    - RC- Ξ circuits
    - R and C connected parallel and in series