

Lecture 9

Energy Work and Kinetic Energy Kinetic and Potential Energy

Physics 105 Fall 2009



ENERGY

Energy is a property of the state of an object: hard to define precisely

Energy is a scalar quantity. It does not have a direction associated with it

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule = $\text{kg} \cdot \text{m}^2/\text{s}^2$

Kinetic Energy

Kinetic Energy \equiv Energy of motion

$K = \frac{1}{2}mv^2$ for object moving with velocity v

$$K = \frac{1}{2}mv^2 \quad \left[J = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

Kinetic Energy: Orders of Magnitude

$K = \frac{1}{2}mv^2$ for object moving with velocity v



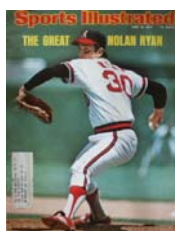
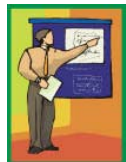
Earth orbiting sun: 2×10^{29} J

Car at 60 mph: 100,000 J

Nolan Ryan pitch: 300 J

Professor walking: 40 J

Angry bee: 0.005 J



Why $K = \frac{1}{2}mv^2$?



Why $K = \frac{1}{2}mv^2$?

Special case: Constant Acceleration

Remember result eliminating t :

$$v^2 - v_0^2 = 2a(x - x_0)$$

Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0) = ma\Delta x$$

But $F=ma!$

$$\Delta\left(\frac{1}{2}mv^2\right) = F\Delta x$$

Energy and Work

Kinetic energy

$$K = \frac{1}{2}mv^2 \quad \left[J = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

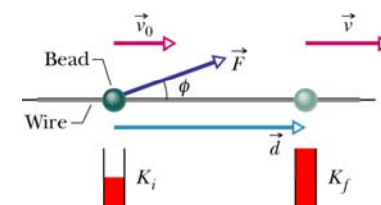
Units of Work and Energy: Joule

Work done by a constant force

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

Work-kinetic energy theorem

$$\Delta K = K_f - K_i = W$$



Work

Work \equiv Energy transferred by a force

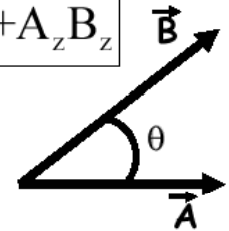
Work done on an object is the energy transferred to/from it

$W > 0 \rightarrow$ energy added
 $W < 0 \rightarrow$ energy taken away

$W = \vec{F} \cdot \vec{r} \equiv$ Work done on an object by a constant force \vec{F} while moving through a displacement \vec{r}

Dot Product: Physical Meaning

$$\vec{A} \cdot \vec{B} = \underline{AB\cos\theta} = A_x B_x + A_y B_y + A_z B_z$$

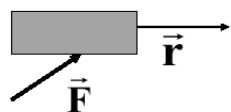


$$\theta = 0 \rightarrow \vec{A} \cdot \vec{B} = \underline{AB}$$

$$\theta = 90^\circ \rightarrow \vec{A} \cdot \vec{B} = \underline{0}$$

Dot product measures how much vectors are along each other

What does $W = \vec{F} \cdot \vec{r}$ mean?



$$W = \vec{F} \cdot \vec{r} \\ = F_x r_x + F_y r_y \\ = Fr\cos\theta$$

$W > 0$ if $\theta < 90^\circ \rightarrow$ force is adding energy to object

$W < 0$ if $\theta > 90^\circ \rightarrow$ force is reducing energy of object



$W = 0$ if $r = 0$ or $F = 0$ or $\vec{F} \perp \vec{r}$

Work Examples

Push on a wall

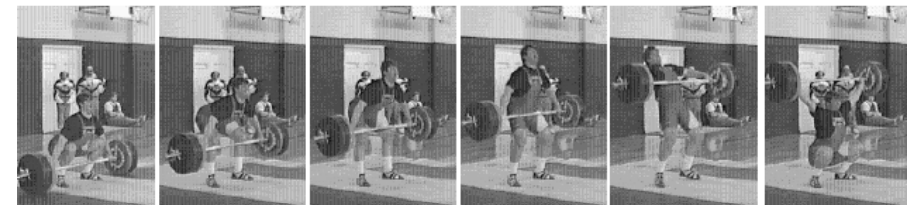
$W = 0$ since wall does not move ($\vec{r} = 0$)

Work due to Gravity

A weightlifter does work when lifting a weight

$$W = mgh$$

(h is the vertical drop)

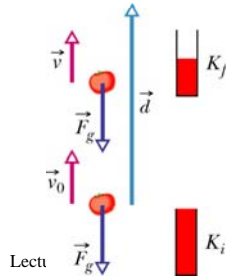


Work Done by a Gravitational Force

Work done by gravitational force

$$W_g = mgd \cos \theta$$

Tomato thrown upward



$$W_g < 0$$

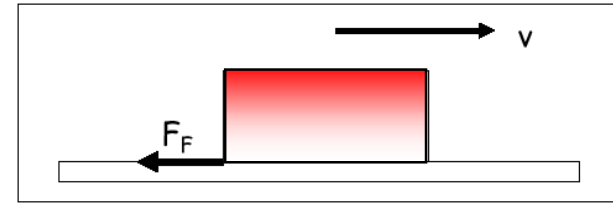
$$W_g > 0$$

Lifting/lowering an object

Change in kinetic energy:

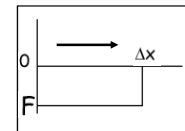
$$\Delta K = K_f - K_i = W_a + W_g$$

Work due to Friction



The frictional force always opposes the motion

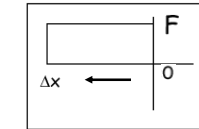
Moving to the right



$$W = -|F| \Delta x$$

$$\Delta x > 0$$

Moving to the left

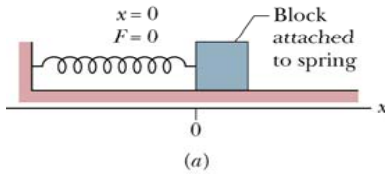


$$W = +|F| \Delta x$$

$$\Delta x < 0$$

W negative in both cases

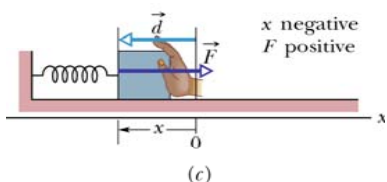
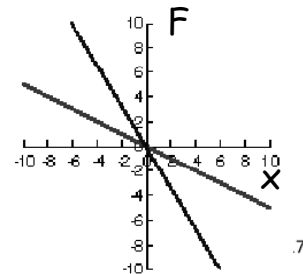
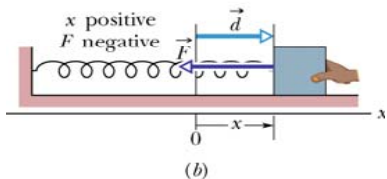
Restoring Force



Equilibrium - no force
Stretched - force towards equilibrium point

$$F = -kx$$

Hooke's Law

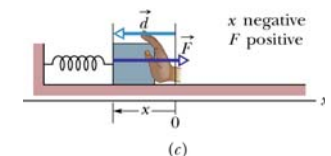
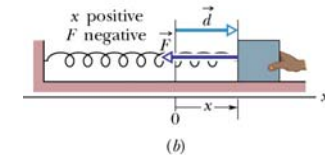
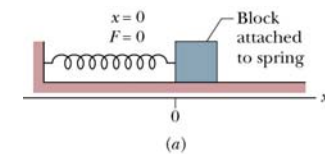


Work Done by a Spring Force

Hooke's law: $\vec{F} = -k\vec{d}$

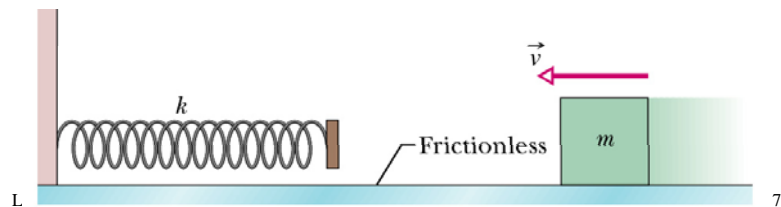
Work done by a spring force:

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$



Sample Problem 7-8

A block of mass $m = 0.40 \text{ kg}$ slides across a horizontal frictionless counter with a speed of $v = 0.50 \text{ m/s}$. It runs into and compresses a spring of spring constant $k = 750 \text{ N/m}$. When the block is momentarily stopped by the spring, by what distance d is the spring compressed?

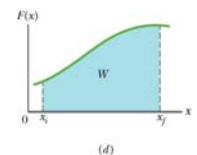
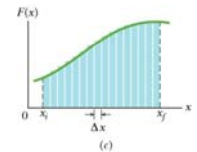
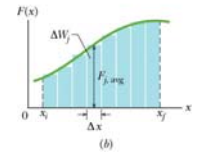
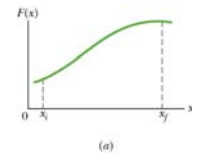


Work Done by a General Variable Force

Work: variable force

$$W = \int_{x_i}^{x_f} F(x) dx$$

- Calculus
- Divide area under curve
- Add increments of W (numerically)
- Analytical form?
- Integration!!!



Lecture 9

Andrei Sirenko, NJIT

Power

Work doesn't depend on the time interval

Work to climb a flight of stairs ~ 3000 J

10 s
1 min
1 hour

Power is work done per unit time

Average Power $P_{avg} = \frac{W}{\Delta t}$

Instantaneous Power $P = dW/dt = F dx/dt = Fv$

Units $\frac{\text{Work}}{\text{time}} \quad \frac{1 \text{ J}}{1 \text{ s}} = 1 \text{ Watt} \quad 1 \text{ hp} = 746 \text{ W}$

$$P = \frac{1}{2} * 50 \text{ kg} * (5 \text{ m/s})^2 / 1 \text{ s}$$

Lecture 9

Andrei Sirenko, NJIT

19

Power

Average Power

$$P_{avg} = \frac{W}{\Delta t}$$

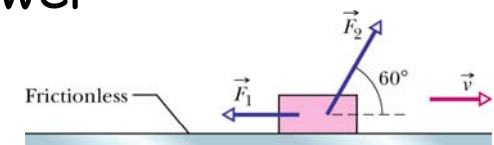
Units: Watts

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = \frac{dE}{dt}$$



Sample Problem 7-10: Two constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude 2.0 N , force F_2 is angled upward by 60° to the floor and has a magnitude of 4.0 N . The speed v of the box at a certain instant is 3.0 m/s .

- a) What is the power due to each force acting on the box? Is the net power changing at that instant?
- b) If the magnitude F_2 is, instead, 6.0 N , what is now the net power, and is it changing?

Andrei Sirenko, NJIT

20

Quiz 9

1. As a sled is pulled by dogs across a flat, snow-covered field at a constant velocity, net work done on the sled is _____,

2. ...and work done by the air resistance and friction is _____,

3. ... and the work done by dogs is _____.

- a) Positive
- b) Zero
- c) Negative



4. A 200-kg sled is moving along a flat road with initial velocity 5 m/s. A 10-N friction force is acting on the sled (dogs take some rest). The sled has traveled 100 m. What is its final kinetic energy?