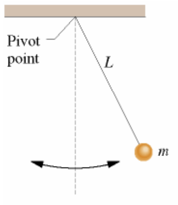
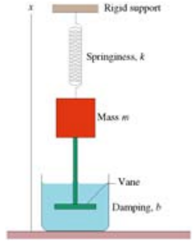
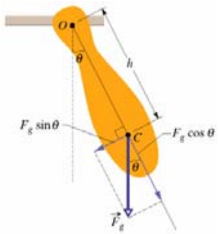


Lecture 12-13

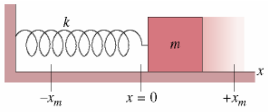
Physics 106
Fall 2006





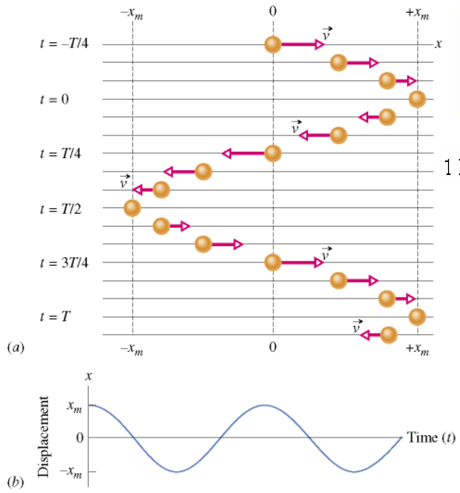
•Physical Pendulum
•Oscillations

HW&R



http://web.njit.edu/~sirenko/

Simple Harmonic Motion



$$T = \frac{1}{f}.$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1}

Displacemnt at time t

Phase

$$x(t) = x_m \cos(\omega t + \phi)$$

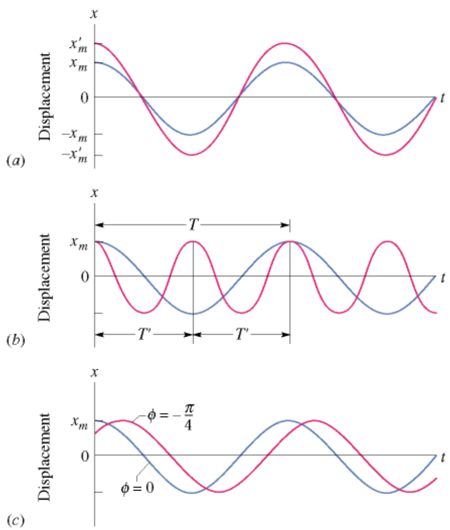
Amplitude

Time

Angular frequency

Phase constant or phase angle

Simple Harmonic Motion (SHM)



1. Amplitude is different
2. Period (or frequency) is different.
3. Phase is different.

Displacemnt at time t

Phase

$$x(t) = x_m \cos(\omega t + \phi)$$

Amplitude

Time

Angular frequency

Phase constant or phase angle

Displacement, Velocity, and Acceleration of SHM

$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement)},$$

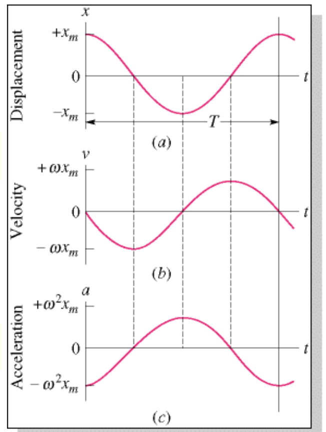
$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity)}.$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

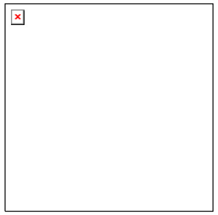
$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \qquad \text{(acceleration)}.$$

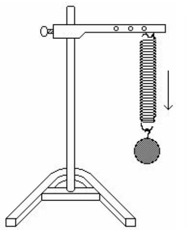
$$a(t) = -\omega^2 x(t)$$



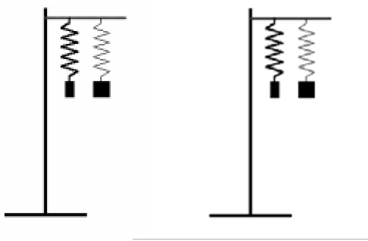
Click on the image to start the simulation

Examples of SHM





Hanging Mass



1. Pendulum
2. Spring+weight

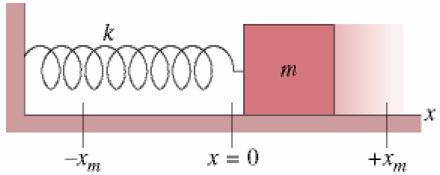
$$a(t) = -\omega^2 x(t)$$

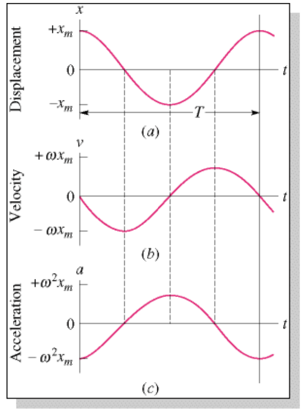
Which parameters of the system are important ?

Displacement, Velocity, and Acceleration of Simple Harmonic Motion

$$x(t) = x_m \cos(\omega t + \phi) \quad \text{(displacement)},$$
$$v(t) = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)},$$
$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}.$$

$$a(t) = -\omega^2 x(t)$$





Click on the image to start the simulation

$$F = -kx \qquad \boxed{\omega = \sqrt{\frac{k}{m}}} \quad \text{(angular frequency)} \qquad T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$

The Force Law for SHM

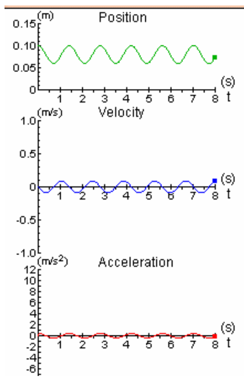
$$a(t) = -\omega^2 x(t)$$

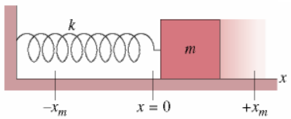
<u>Force is proportional to displacement with a negative constant of proportionality</u>

$$F = ma = -(m\omega^2)x.$$

$$F = -kx,$$

$$k = m\omega^2.$$



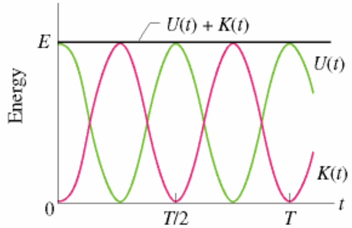


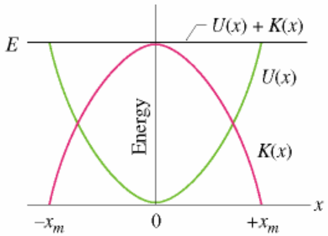
$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}$$

Force does not depend on the amplitude

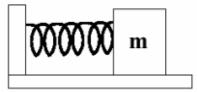
Energy of SHM





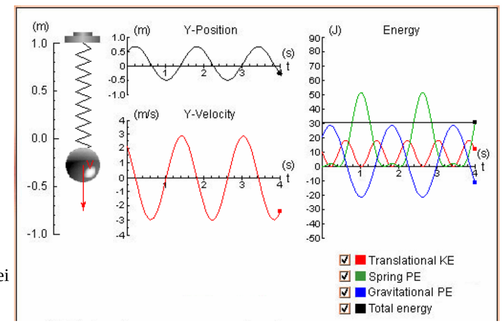
Total Energy is a constant

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

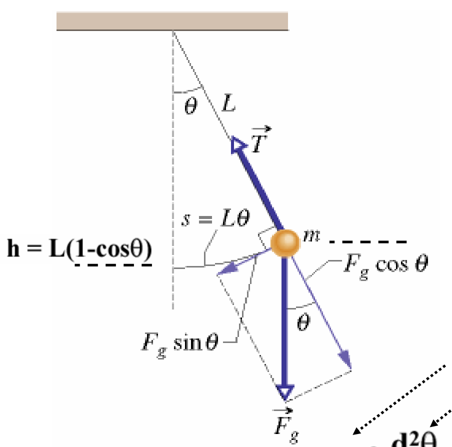


$$\boxed{E = \tfrac{1}{2}k(x_m)^2}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgx$$



Simple Pendulum



$$I = mL^2$$

$$F_T = -mg\sin(\theta)$$

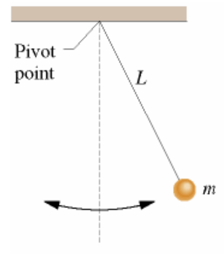
$$\tau = -mgL\sin(\theta)$$

$$\cong -mgL\theta \quad \text{(for small } \theta\text{)}$$

$$I \cdot \alpha = \tau$$

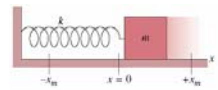
$$mL^2 \frac{d^2\theta}{dt^2} = -mgL\theta \longrightarrow \frac{d^2\theta}{dt^2} = -(g/L)\theta$$

Simple Pendulum



Simple pendulum follows SHM

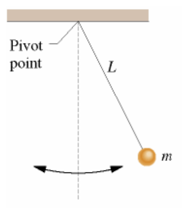
$$\frac{d^2\theta}{dt^2} = -(g/L)\theta \qquad \text{Looks like spring} \qquad \frac{d^2x}{dt^2} = -(k/m)x$$



Solution by analogy

Spring	Pendulum
$x = x_m\cos(\omega t+\phi)$	$\theta = \theta_m\cos(\omega t+\phi)$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

Simple Pendulum: Questions



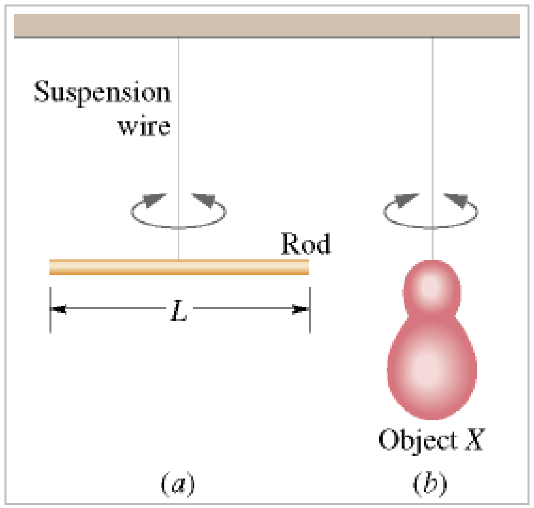
Q1. If we double θ_m the energy:

a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

Q2. If we double θ_m the period:

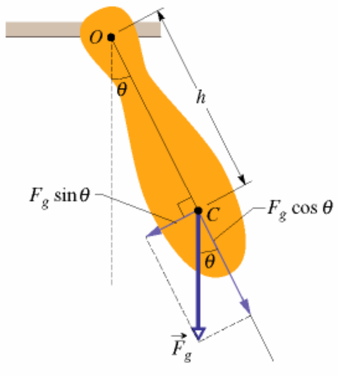
a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

An Angular Simple Harmonic Oscillator



$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

The Physical Pendulum



Any rigid body behaves like SHO close to stable equilibrium

$$\tau = I\alpha$$

$$\tau = -mgh\sin(\theta) \cong -mgh\theta$$

$$I\frac{d^2\theta}{dt^2} = -mgh\theta$$

We know the solution

$$\omega = \sqrt{\frac{mgh}{I}}$$

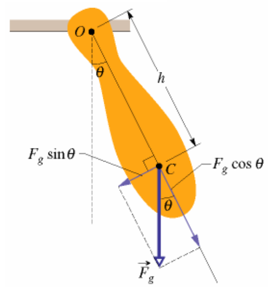
$$\theta = \theta_m\cos(\omega t + \phi)$$

$T = 2\pi\sqrt{\frac{I}{mgh}}$ (physical pendulum, small amplitude).

Compare to: $T = 2\pi\sqrt{\frac{L}{g}}$ $I = mL^2$

for SHO

The Physical Pendulum



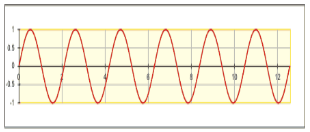
$T = 2\pi\sqrt{\frac{I}{mgh}}$ (physical pendulum, small amplitude).

QZ:

Three physical pendulums, of masses m_0, $2m_0$, and $3m_0$, have the same shape and size and are suspended at the same point. Rank the masses according to the periods of the pendulums, greatest period first.

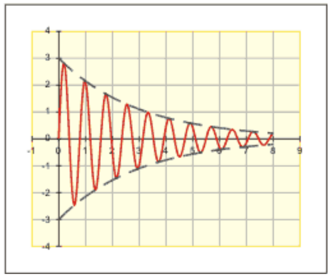
Hint: use the formula for the period and think about I (rotational inertia). Since the exact shape is not given to us in the text of the problem, then we can try to check a couple of different shapes; point mass $I = mL^2$ solid rod $I = 1/3\ mL^2$

Damping of harmonic oscillations



Simple Harmonic Motion is an Idealization

Energy is constant → Motion never decays



In real life the motion eventually stops

Friction

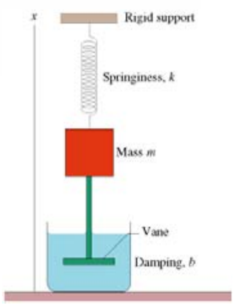
Air Resistance

….

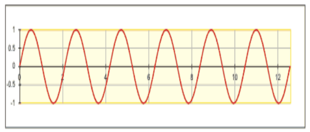
Mechanical Energy → 0

F_d = -bv Air resistance, etc.

Direction opposite to motion
Magnitude proportional to velocity

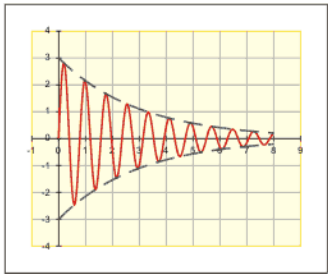


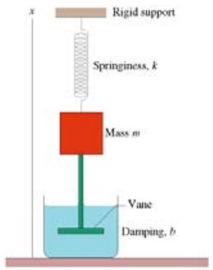
Damping of harmonic oscillations



$x = x_0\cos(\omega t)$ $v = -x_0\omega\sin(\omega t)$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2$$





Damping Force

$$F_d = -bv$$

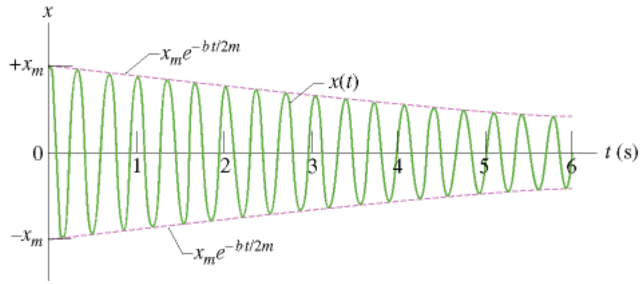
$$-bv - kx = ma.$$

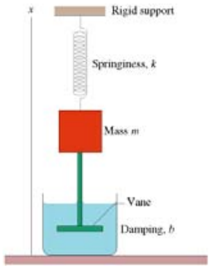
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Damping of harmonic oscillations





$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

$$E(t) \approx \frac{1}{2} k x_m^2 e^{-bt/m}.$$

Conclusion:

Amplitude X(t) and Mechanical energy E(t) decrease with time exponentially