Lecture 2

Physics 106
Fall 2006

Rotational dynamics:

- Kinetic Energy of rotation,
- Rotational inertia,
- Torque,
- Cross product.

http://web.njit.edu/~sirenko/

Required Home Work

Specific information for the UT homework system:

UT Guest ID Registration: https://utdirect.utexas.edu/nlogon/eid_suite/essentials/create_eid.WBX?portal_role=O

UT HW Student Instructions:
https://hw.utexas.edu/bur/studentGuestEID.html

Student Login Page (Univ. of Texas):
https://utdirect.utexas.edu/security-443/UTEIDLogon.wb

UT EID Home Page (Forgotten Password): https://utdirect.utexas.edu/nlogon/eid_suite/general/

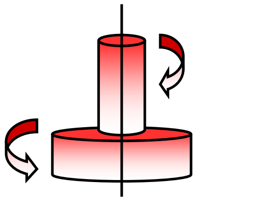
Your instructor will announce the 5 digit course number you need to use when you register for Physics 106 in the UT system.

If you already have a UT Guest login ID and password, you can continue to use it.

Fill out the following for your own future reference, and keep it someplace where you can find it:

- Unique course number to be announced by instructors: **41156**
- Your Login ID on the UT system (generated when you register with UT; case sensitive!):________
- Your own password (selected upon registration with UT; confidential!): ___________
  Note that NJIT instructors can not access your password.

Rotation:

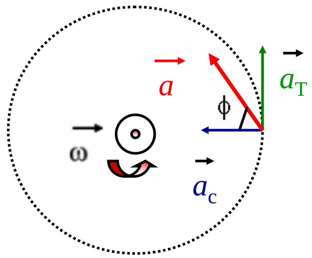


Angular Displacement
Angular Velocity
Angular Acceleration

$\theta, \quad \vec{\omega}, \quad \vec{\alpha}$

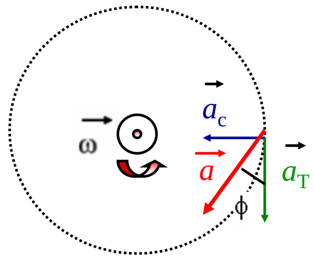
Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Acceleration in Circular Motion:



ω increases with time

$\alpha > 0$



ω decreases with time

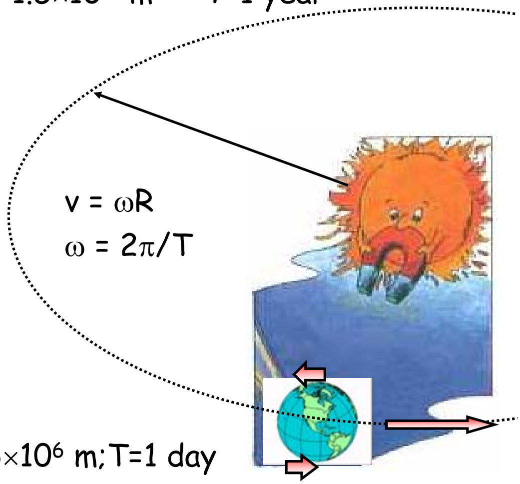
$\alpha < 0$

$a_T = r\alpha \qquad a_c = v^2/r = r\omega^2$

$\vec{a} = \vec{a}_c + \vec{a}_T\,; \quad a = (a_c^2 + a_T^2)^{1/2}, \ \tan\phi = a_T / a_c$

QZ: Our linear velocity with respect to the Sun

$R_{E\text{-}S} = 1.5\times10^{11}$ m T=1 year



$v = \omega R$

$\omega = 2\pi/T$

$R=6\times10^{6}$ m; T=1 day

When do we move faster ?

(a) Day
(b) Night

What is the velocity difference between Day and Night at the Equator line ?

$|(V_{day} - V_{night})|/V_{average}$

(a) 0.00008
(b) 0.015
(c) 0.03
(d) 0.3
(e) 100 %

Kinetic Energy of Rotation

$K = \frac{1}{2} mv^2$ Point mass (no rotation); v of the COM

$$K = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \cdots$$
$$= \sum \tfrac{1}{2}m_iv_i^2,$$

System of particles or an object

$v = \omega r$

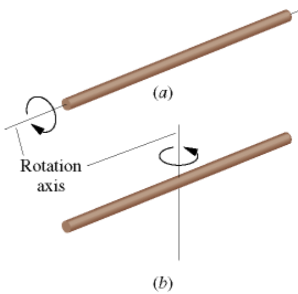
$\Downarrow$

$$K = \sum \tfrac{1}{2}m_i(\omega r_i)^2 = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$I = \sum m_i r_i^2$ (rotational inertia)

$K = \frac{1}{2}I\omega^2$ (radian measure)

Rotational Inertia



$K = \frac{1}{2}I\omega^2$ (radian measure)

$I^{(a)} \neq I^{(b)}$

I - rotational equivalent of mass m

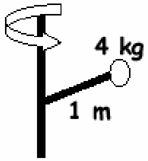
Main difference between m and I: *Rotational Inertia* depends on the direction of rotation !

For a rigid body, I depends on how the mass is distributed in an object relative to the axis of rotation

Rotational Inertia Of Point Mass

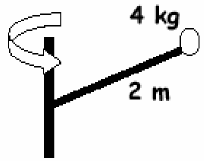
For a single particle I = mr^2 (all mass at same r)

A single particle



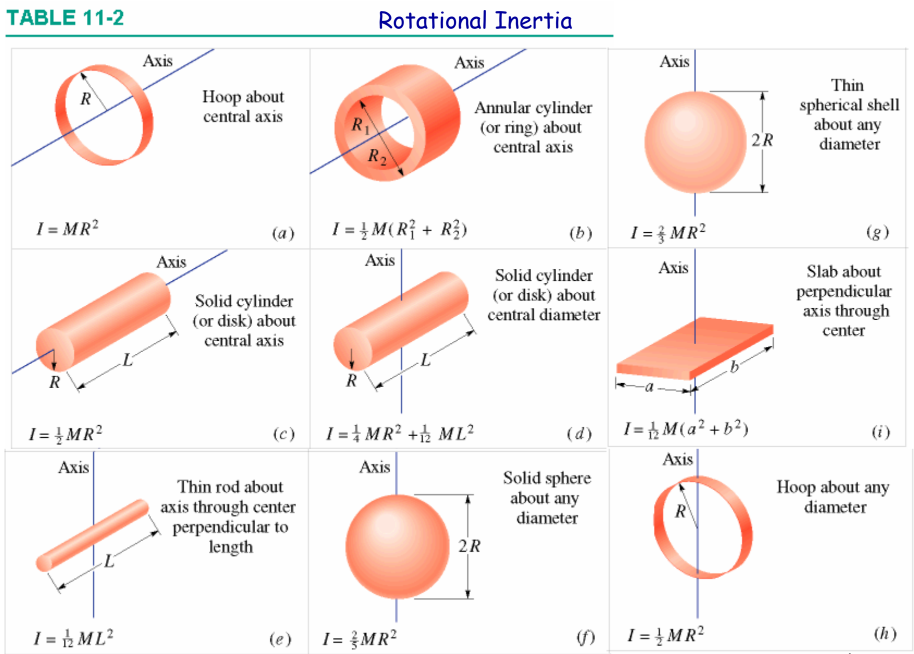
I = (4 kg)(1 m)^2 = 4 kg m^2

The same particle farther out



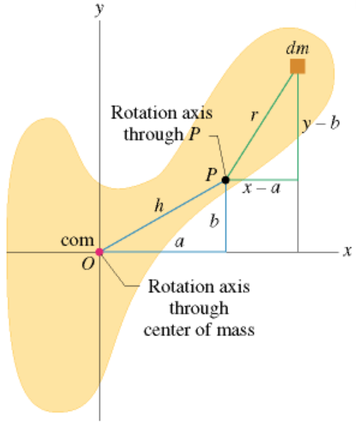
I = (4 kg)(2 m)^2 = 16 kg m^2

Four times the rotational "mass"



Parallel-Axis Theorem for Rotational Inertia

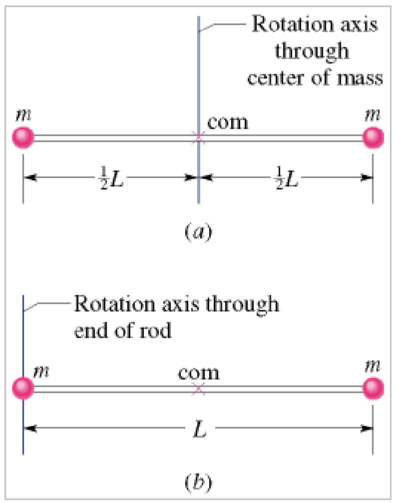
$$I = I_{com} + Mh^2 \quad \text{(parallel-axis theorem).}$$



Calculate I_{com} for the axis going through the COM

Use *Parallel-Axis Theorem* to calculate I

Example:Rotational Inertia



$$I = \sum m_i r_i^2 = (m)(\tfrac{1}{2}L)^2 + (m)(\tfrac{1}{2}L)^2 = \tfrac{1}{2}mL^2.$$

$$I = m(0)^2 + mL^2 = mL^2$$

$$I = I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2 = mL^2.$$

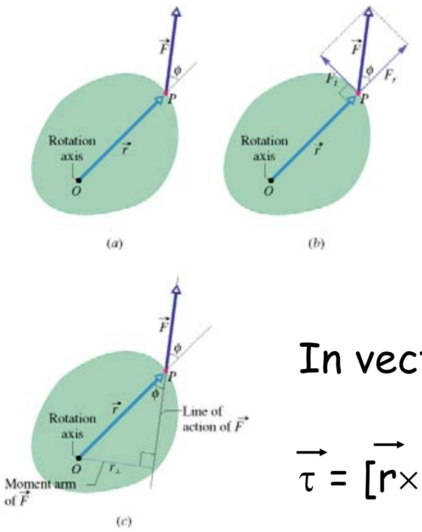
Torque: $\vec{\tau}$

All forces are the same

Door on a hinge: How do you slam?

Not only the force is important,
But how you apply it !

Torque: $\vec{\tau}$



The value of *torque:*

$\tau = r \cdot F \cdot \sin\phi$

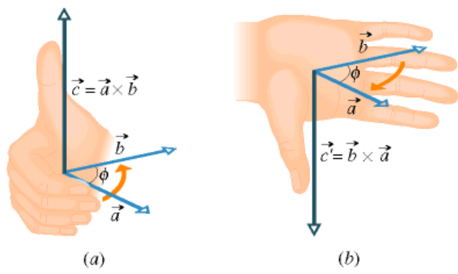
$\phi=0 \Rightarrow \tau=0$

$\phi=\pi/2 \Rightarrow \tau= r \cdot F$ (max)

In vector notation form:

$\vec{\tau} = [\vec{r} \times \vec{F}]$

Vector Cross Product



The value of *cross product:*

$c = a \cdot b \cdot \sin\phi$

$\phi=0 \Rightarrow c = 0$

$\phi=\pi/2 \Rightarrow c = a \cdot b$ (max)

Cross product is maximized when vectors are perpendicular

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

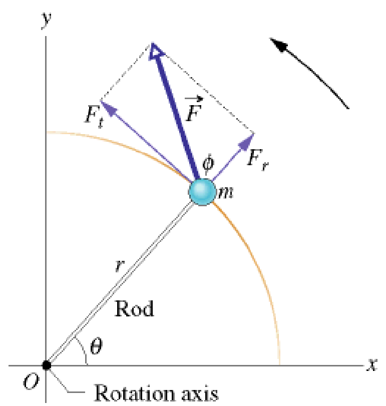
Order is important: $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Newton's Second Law for Rotation

Torque causes the change in ω

$\tau_{net} = I \cdot \alpha$

Rotational equivalent of F = ma



$$F_t = m a_t.$$

$$\tau = F_t r = m a_t r.$$

$$\tau = m(\alpha r) r = (m r^2)\alpha.$$

Rotational Analogy to Linear Motion

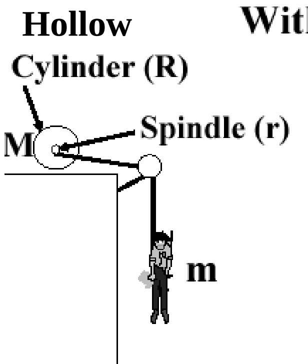
	Translation	*Rotation*
position	x	θ
velocity	v = dx/dt	ω = dθ/dt
acceleration	a = dv/dt	α = dω/dt
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2} m v^2$	$K = \frac{1}{2} I \omega^2$
Force	F = ma	$\tau_{net} = I \cdot \alpha$

A Falling Stuntman

Stuntmen sometimes need to fall large distances

Without getting hurt!

But it has to look like they fall



$$I_{cyl} = MR^2 \qquad \tau = r\,T$$

$$\alpha = \frac{r\,T}{MR^2} \qquad a = \alpha r$$

The rope is around the spindle

$ma = mg - T$

$a = g/(1+MR^2/mr^2)$

For $m = 70$ kg, $M = 10$ kg,
$r = 0.1$ m and $R = 0.5$ m

$a = 9.8/(1+25/7) \approx 2\,m/s^2$

Homework

See the Physics 106 Course Syllabus

U of Texas HW is required

http://web.njit.edu/~sirenko/