Lecture 3

Physics 106

Fall 2006

Rotational dynamics:

- **Newton's Second Law and examples**

http://web.njit.edu/~sirenko/

Kinetic Energy of Rotation

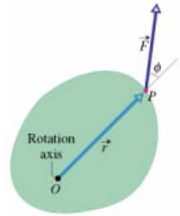
$K = \frac{1}{2} I\omega^2$ (radian measure)

$I = \sum m_i r_i^2$ (rotational inertia)

$I = I_{com} + Mh^2$ (parallel – axis theorem).

Do a calculation or see the Text Book

Torque: $\vec{\tau}$

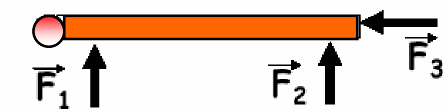


$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$

Torque: $\vec{\tau}$

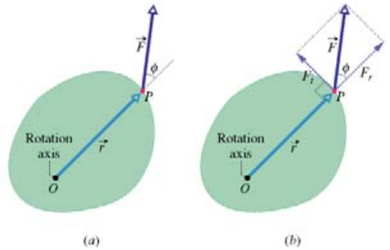
All forces are the same



Door on a hinge:
How do you slam?

Not only the force is important,
But how you apply it !

Torque: $\vec{\tau}$

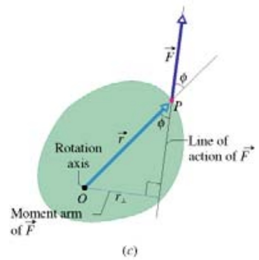


The value of *torque:*

$\tau = r \cdot F \cdot \sin\phi$

$\phi = 0 \Rightarrow \tau = 0$

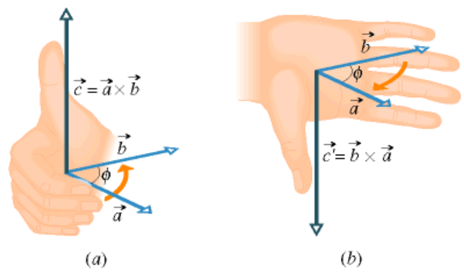
$\phi = \pi/2 \Rightarrow \tau = r \cdot F$ (max)



In vector notation form:

$\vec{\tau} = [\vec{r} \times \vec{F}]$

Vector Cross Product



The value of *cross product:*

c = a·b·sinϕ

ϕ=0 ➔ c = 0

ϕ=π/2 ➔ c = a·b (max)

Cross product is maximized when vectors are perpendicular

$$\vec{a}\times\vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

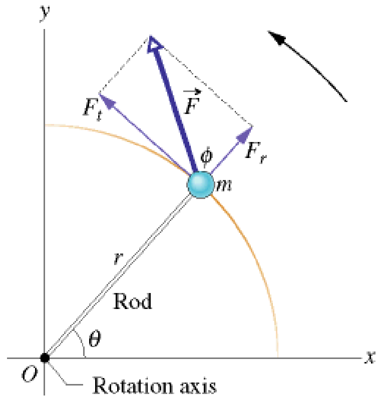
Order is important: $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

Newton's Second Law for Rotation

Torque causes the change in ω

$\tau_{net} = I \cdot \alpha$

Rotational equivalent of F = ma



$$F_t = ma_t.$$

$$\tau = F_t r = ma_t r.$$

$$\tau = m(\alpha r)r = (mr^2)\alpha.$$

Rotational Analogy to Linear Motion

	Translation	*Rotation*
position	x	θ
velocity	v = dx/dt	ω = dθ/dt
acceleration	a = dv/dt	α = dω/dt
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I \omega^2$
Force	F = ma	$\tau_{net} = I \cdot \alpha$

Work and Rotational Kinetic Energy

Work-kinetic energy theorem

$$\Delta K = K_f - K_i = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = W$$

Work, rotation about fixed axis

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$

Work, constant torque

$$W = \tau(\theta_f - \theta_i)$$

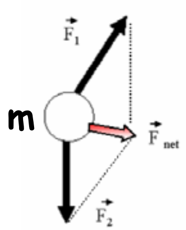
Power, rotation about fixed axis

$$P = \frac{dW}{dt} = \tau\omega$$

Newton's Second Law for Rotation

Force F = ma

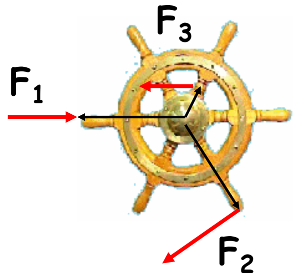
Net Force (or Total Force)



$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

$\vec{F}_{net} = m\vec{a}$

$\tau_{net} = I \cdot \alpha$



$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = I \cdot \vec{\alpha}$

$\vec{\tau}_{net} = [\vec{r}_1 \times \vec{F}_1] + [\vec{r}_2 \times \vec{F}_2] + [\vec{r}_3 \times \vec{F}_3] = I \cdot \vec{\alpha}$

Newton's Second Law for Rotation

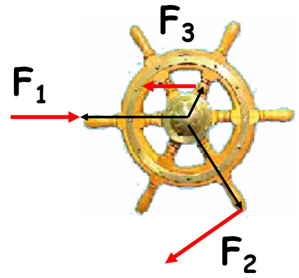
$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = I \cdot \vec{\alpha}$

$\vec{\tau}_{net} = [\vec{r}_1 \times \vec{F}_1] + [\vec{r}_2 \times \vec{F}_2] + [\vec{r}_3 \times \vec{F}_3] = I \cdot \vec{\alpha}$

When torque is positive ?

τ is positive if it rotates the body to positive direction (CCW)

"clock is negative".



$\tau_1 = 0$ $\tau_3 > 0$ $\tau_2 < 0$

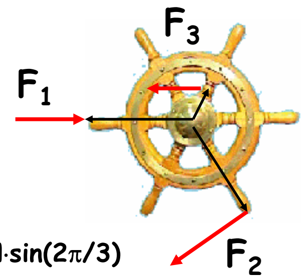
⊙ ⊗

Newton's Second Law for Rotation

$\tau_1 = 0$ $\tau_3 > 0$ $\tau_2 < 0$

⊙ ⊗

$r_1 = 1.0m$	$r_2 = 1.0m$	$r_3 = 0.5m$
$F_1 = 2.0N$	$F_2 = 3.0N$	$F_3 = 2.0N$
$\phi_1 = \pi$	$\phi_2 = -\pi/2$	$\phi_3 = 2\pi/3$
$I = 10$ kg·m²		

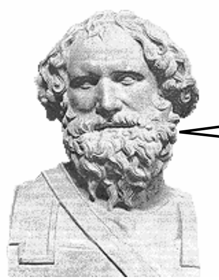


τ_{net} = 1m·2N·sin(π)+1m·3N·sin($-\pi/2$)+0.5m·2N·sin($2\pi/3$)

= 0+(-3)m·N +(-3)m·N +0.87 m·N = -2.13 m·N

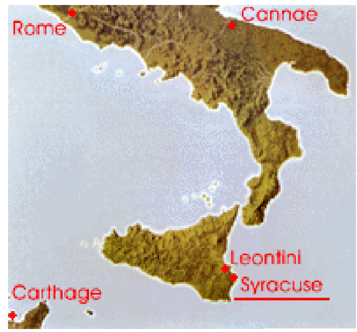
$\alpha = \tau_{net} / I$ = -2.13 m·N/ 10 kg·m² = -0.21 rad/s²

This Angular acceleration speeds up CW rotation



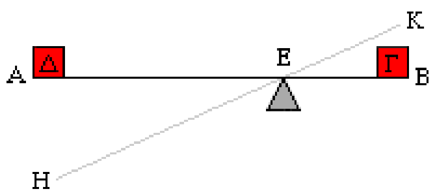
EUREKA !

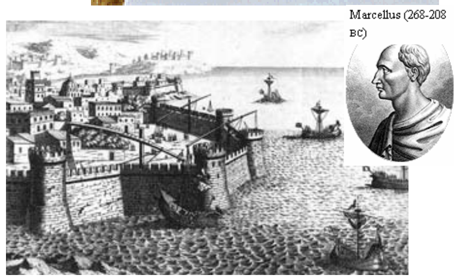
π = 3.1415926…



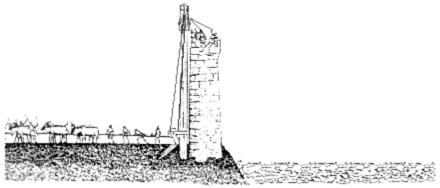
Archimedes

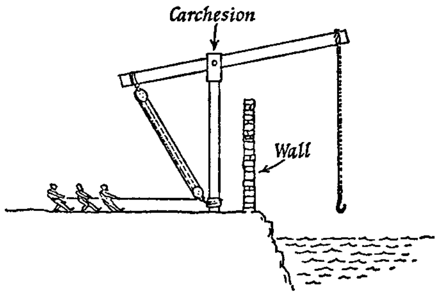
(287 BC - 211 BC)





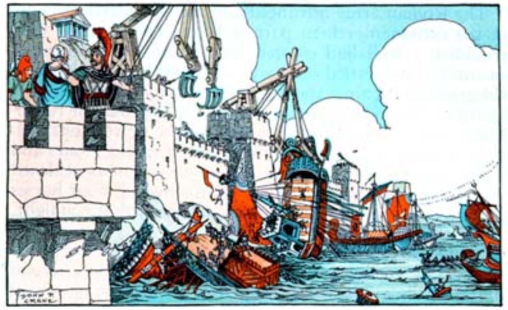
Archimedes' Claw

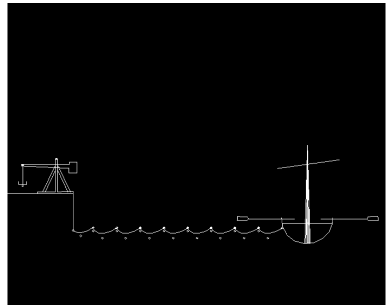




http://www.mcs.drexel.edu/~crorres/Archimedes/Claw/illustrations.html

Archimedes' Claw





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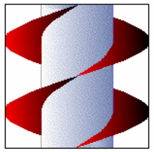
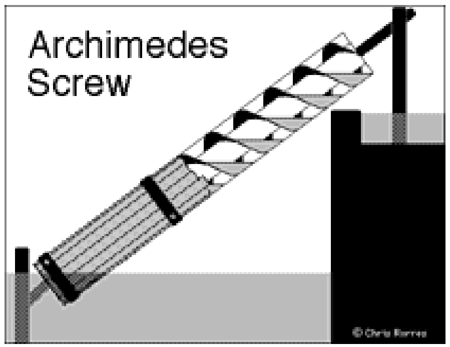
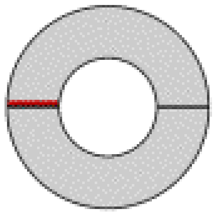


Archimedes Screw





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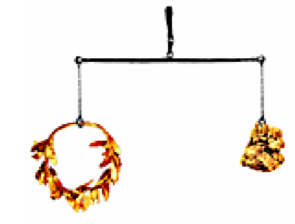
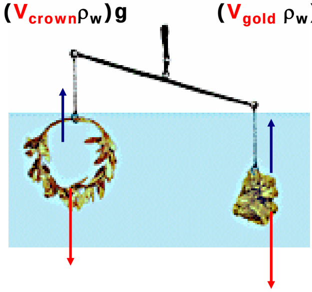
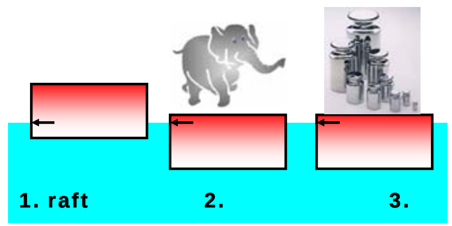
Hiero: "Is it really 100% gold ?"

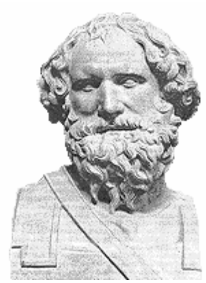


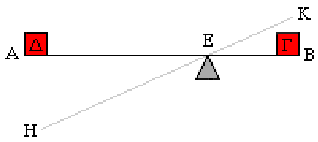
$(V_{crown}\rho_w)g$ $(V_{gold}\ \rho_w)g$



mg mg







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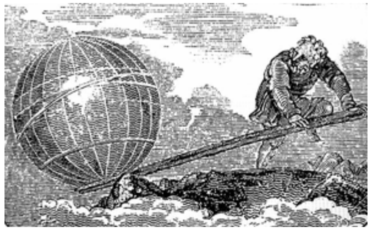
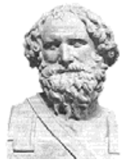


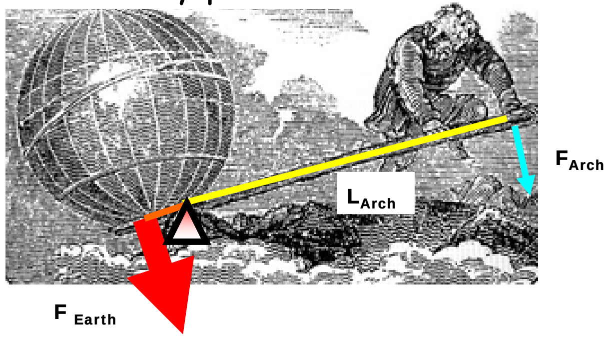
GIVE ME A PLACE TO STAND AND I WILL MOVE THE EARTH

Give me a lever long enough and a place to stand, and I will move the world

Is it really possible ???

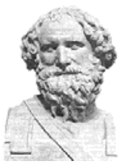


Is it really possible ???

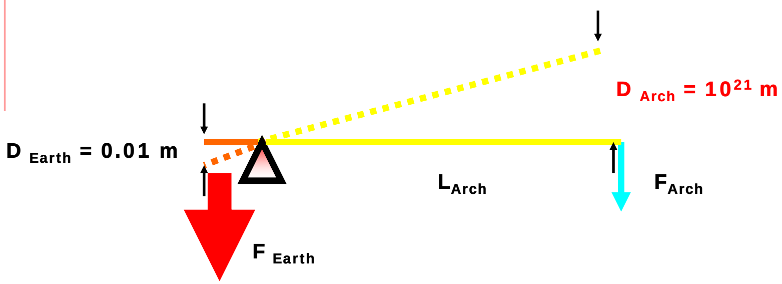


$F_{Earth} = 6\times10^{25}$ N **$F_{Arch} = 600$ N**

($M_{Earth} = 6\times10^{24}$ kg) **(60 kg)**



Is it really possible ???



$F_{Arch} = 600$ N (60 kg)

$F_{Earth} = 6\times10^{25}$ N ($M_{Earth} = 6\times10^{24}$ kg)

$L_{Arch} / L_{Earth} = 1\times10^{23}$; ➔ $D_{Arch} / D_{Earth} = 1\times10^{23}$

$D_{Arch} = 10^{21}$ m With the power of P=600N·m/s , t = 10^{21}s ≅ 5×10^{14} years

(people do not live that long)

If Archimedes moves his arm with the speed of light, then

$D_{Arch} = 50$ m during 5×10^{9} years (life time of the Earth)

QZ Problem

A rigid sculpture consists of a thin hoop (of mass $m=1$ kg and radius $R = 1$ m) and a thin radial rod (of mass $M=2$ kg and length $L = 2$ m). The sculpture can pivot around a horizontal axis in the plane of the hoop, passing through its center.

Rod: $I_{com} = (1/12)\, ML^2$

Hoop: $I_{com} = (1/2)\, mR^2$

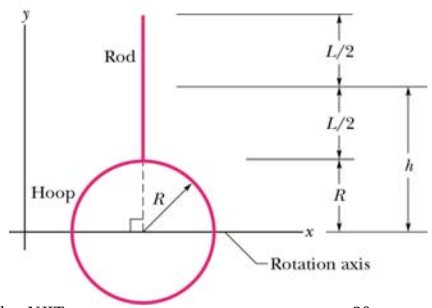
$I = I_{com} + Mh^2$; $I_{tot} = I_1 + I_2$; $K = ½ I\omega^2$

a) What is the sculpture's rotational inertia ***I*** about the rotation axis?

b) Starting from rest, the sculpture rotates around the rotation axis from the initial upright position. What is the change of the sculpture's Potential Energy ΔU when it is inverted?

c) What is the Kinetic Energy of rotation when it is inverted?

d) What is the angular speed ω around the horizontal axis ?



Homework

See the Physics 106 Course Syllabus

U of Texas HW is required

http://web.njit.edu/~sirenko/