

Lecture 4

Physics 106

Fall 2006

Review for Rotational dynamics

Q&A for the First Exam

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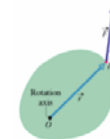
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Rotational Analogy to Linear Motion

	Translation	Rotation
position	x	θ
velocity	$v = dx/dt$	$\omega = d\theta/dt$
acceleration	$a = dv/dt$	$\alpha = d\omega/dt$

mass	m	$I = \sum m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I \omega^2$
Force	$F = ma$	$\tau_{\text{net}} = I \alpha$



$$\vec{\tau} = [\vec{r} \times \vec{F}]$$

$$\tau = r \cdot F \cdot \sin\phi$$

Angular Displacement
Angular Velocity
Angular Acceleration

$\theta, \quad \omega, \quad \alpha$



Linear Equation	Missing Variable	Angular Equation
$v = v_0 + at$	$x - x_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2} at^2$	v	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2} at^2$	v_0	$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$

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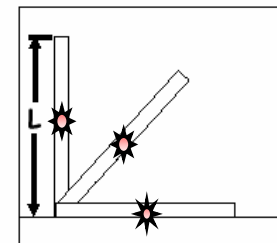
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TABLE 11-2

Rotational Inertia

<p>Hoop about central axis</p> <p>$I = MR^2$</p> <p>(a)</p>	<p>Annular cylinder (or ring) about central axis</p> <p>$I = \frac{1}{2} M(R_1^2 + R_2^2)$</p> <p>(b)</p>	<p>Thin spherical shell about any diameter</p> <p>$I = \frac{2}{3} MR^2$</p> <p>(g)</p>
<p>Solid cylinder (or disk) about central axis</p> <p>$I = \frac{1}{2} MR^2$</p> <p>(c)</p>	<p>Solid cylinder (or disk) about central axis</p> <p>$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$</p> <p>(d)</p>	<p>Slab about perpendicular axis through center</p> <p>$I = \frac{1}{12} M(a^2 + b^2)$</p> <p>(i)</p>
<p>Thin rod about axis through center perpendicular to length</p> <p>$I = \frac{1}{12} ML^2$</p> <p>(e)</p>	<p>Solid sphere about any diameter</p> <p>$I = \frac{2}{5} MR^2$</p> <p>(f)</p>	<p>Hoop about any diameter</p> <p>$I = \frac{1}{2} MR^2$</p> <p>(h)</p>

Faster than a Falling Rock



The rod starts at rest and falls over

How fast is it going when it hits the table?

Conservation of Energy: $K_i + U_i = K_f + U_f$

Initial: $U = mg(L/2)$ and $K = 0$

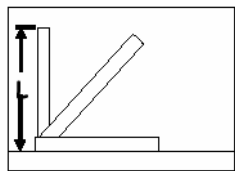
Final: $U = 0$ and $K = \frac{1}{2} I \omega^2$

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Faster than a Falling Rock



Set $E_i = E_f$ $\frac{1}{2}I\omega^2 = mgL/2$

$I = ml^2/3$ for a bar rotating around its end

$$\omega = \sqrt{\frac{3g}{L}}$$

Center of mass $v = r\omega = \frac{L}{2}\omega = \frac{1}{2}\sqrt{3gL}$

End $v = r\omega = L\omega = \sqrt{3gL}$

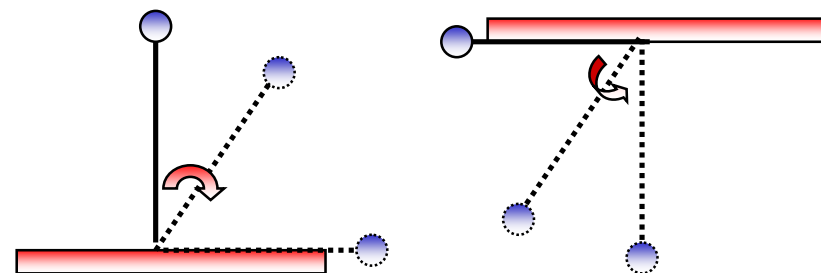
Dropped Rock

$$v = \sqrt{2gL}$$

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$L = 1 \text{ m}; M = 1 \text{ kg}; g = 9.8 \text{ m/s}^2$

v and ω at the lowest point ???

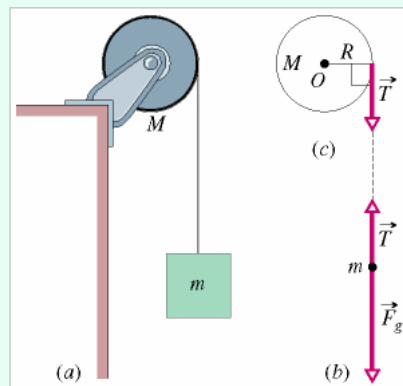
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Sample Problem 11-7

Figure 11-17a shows a uniform disk, with mass $M = 2.5 \text{ kg}$ and radius $R = 20 \text{ cm}$, mounted on a fixed horizontal axle. A block with mass $m = 1.2 \text{ kg}$ hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



I of the disk is $\frac{1}{2}MR^2$.

Fig. 11-17 Sample Problems 11-7 and 11-9. (a) The falling block causes the disk to rotate. (b) A free-body diagram for the block. (c) An incomplete free-body diagram for the disk.

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Chapter 11 Rotation

PROBLEM 55

In Fig. 11-42, one block has mass $M = 500 \text{ g}$, the other has mass $m = 460 \text{ g}$, and the pulley, which is mounted in horizontal frictionless bearings, has a radius of 5.00 cm . When released from rest, the heavier block falls 75.0 cm in 5.00 s (without the cord slipping on the pulley). (a) What is the magnitude of the blocks' acceleration? What is the tension in the part of the cord that supports (b) the heavier block and (c) the lighter block? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

I of the disk is $\frac{1}{2}MR^2$.

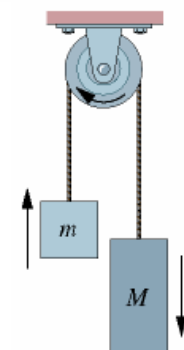


Fig. 11-42 Problem 55.

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