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Name (Print): $\qquad$
$\qquad$ Section: $\qquad$

## I nstructions:

- Questions 1 through 10 are multiple-choice questions worth 5 points each. Answer each of them on the Scantron sheet using \#2 pencil. Answer all of them as there is no penalty for guessing. You will need to do calculations on the exam papers for most of the questions and you may use backs of the exam papers for extra space.
- Questions A and B are workout problems worth 25 points each. Answer them on the exam booklet and show ALL work, otherwise there is no way to give partial credit.
- Be sure your name and section number are on both the Scantron form and the exam booklet. Also write your name, id, and section at the top of each page with long answer questions $A$ and $B$ on them.
- You may bring and use your own formula sheet, using one side of an $8.5 \times 11$ sheet or two sides of a $5 \times 8$ card. A default formula sheet is also provided (see final page of this booklet).
- Make sure to bring your own calculator: sharing of calculators is not permitted and will need it.
- As you know, NJIT has a zero-tolerance policy for ethics code violations. Students are not to communicate with each other once the exam has started. All cell phones, pagers, or similar electronic devices should be turned off.
- If you have questions or need something call your proctor or instructor.

1. The drawing shows the blade of a chain saw. The rotating sprocket tip at the end of the guide bar has a radius of 25 cm . The angular speed of the sprocket tip is $12 \mathrm{rad} / \mathrm{s}$. Find the linear speed of a chain link at point A .
A) $4 \mathrm{~m} / \mathrm{s}$
B) $5 \mathrm{~m} / \mathrm{s}$
C) $1 \mathrm{~m} / \mathrm{s}$
D) $2 \mathrm{~m} / \mathrm{s}$

E) $3 \mathrm{~m} / \mathrm{s}$
2. A disk with radius $R=3.0 \mathrm{~m}$. is spinning about its center. Initially the disc has an angular velocity of $150 \mathrm{rev} / \mathrm{min}$, and is slowing down uniformly at a rate of $6.0 \mathrm{rad} / \mathrm{s}^{2}$. By the time it stops spinning, the total number of revolutions the disk will make is:
A) 7.1
B) 9.3
C) 1.1
D) $\mathbf{3 . 3}$
E) 5.5
3. Two identical wheels are spinning, but wheel $B$ is spinning with twice the angular velocity of wheel $A$ and its radius is also twice as large as the radius of wheel $A$. The ratio of the radial acceleration of a point on the rim of $B$ to the radial acceleration of a point on the rim of $A$ is:
A) $1 / 8$
B) $1 / 4$
C) 1
D) 2
E) 8
4. Five equal $2.0-\mathrm{kg}$ point masses are arranged in the $x-y$ plane as shown. They are connected by massless sticks to form a rigid body. The distance $a$ is 4 m . Find the rotational inertia in $\mathrm{kg} \cdot \mathrm{m}^{2}$ about an axis pqarallel to the z - axis through point P. The result is:
A) 128
B) 512
C) $\mathbf{2 8 8}$
D) 256
E) 192

5. A uniform rod of mass $M=4 \mathrm{~kg}$ and length $L=3.0 \mathrm{~m}$. is pivoted about an axis perpendicular to the rod and 50 cm . from it's left end. Find the rotational inertia about this axis (in $\mathrm{kg} . \mathrm{m}^{2}$ )
A) $\mathbf{7 . 0}$
B) 3.0
C) $\quad 36.0$
D) 5.0
E) $\quad 11.0$
6. A force $F$ is applied to the edge of the wheel as shown at point $P$. The torque that it produces about an axis of rotation through the center of the wheel is:
A) FR
B) $\operatorname{FRsin}(\phi)$
C) $F \sin (\phi)$
D) $R \sin (\phi)$
E) $F R \cos (\phi)$

7. The dumbell in the figure consists of a uniform rod fastened to two. masses attached to each end. The rotational inertia of the rod about an axis perpendicular to the plane of the figure through point " $O$ " is $I_{\text {ROD }}=8.0 \mathrm{~kg} . \mathrm{m}^{2}$. Each mass is 2.0 kg located 2.0 meters from the center point " 0 " of the rod. What force F must be applied to one end (perpendicular to the rod) to give the system an instantaneous angular acceleration of $4.0 \mathrm{rad} / \mathrm{s}^{2}$ about the center?

A. 32 N
B. 48 N
C. 64 N
D. 27 N
E. $\quad 18 \mathrm{~N}$

## KEY

8. A disk drive with rotational inertia $4.0 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}$ is to accelerate uniformly from rest to an angular velocity of 7200 $\mathrm{rev} / \mathrm{min}$ in 10 sec . The torque that its motor must provide to cause this acceleration is:
A) $\quad 0.30 \mathrm{~N} . \mathrm{m}$
B) $\quad 0.72 \mathrm{~N} . \mathrm{m}$
C) $\quad 3.30 \mathrm{~N} . \mathrm{m}$
D) $\quad 180 \mathrm{~N} . \mathrm{m}$
E) $\quad 2.9 \mathrm{~N} . \mathrm{m}$
9. What value of the angle $\phi$ would cause the angular acceleration to be zero in the diagram below. The forces $\mathbf{F}_{\mathbf{1}}$ and $\mathbf{F}_{\mathbf{2}}$ act on a thin rigid rod pivoted at the rotation axis shown. The rotation axis is perpendicular to the page.
A. 1.0 rad
B. 0.0 rad
C. $90^{\circ}$
D. $45^{\circ}$
E. none of these
$45^{\circ}$
none

10. A 12 kg block hangs on a cord that is wrapped around the rim of a flywheel of radius 0.25 m . The rotational inertia of the flywheel about a horizontal axis is $0.60 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. When the block is released, the cord unwinds with no slipping, the system accelerates, and the tension in the cord is no longer equal to the weight of the block hanging from it. Find the acceleration of the block:
A) $\quad 49 \mathrm{~m} / \mathrm{s}^{2}$
B) $\quad 9.8 \mathrm{~m} / \mathrm{s}^{2}$
C) $\quad 5.4 \mathrm{~m} / \mathrm{s}^{2}$
D) $\quad 4.9 \mathrm{~m} / \mathrm{s}^{2}$
E) $\quad 0.02 \mathrm{~m} / \mathrm{s}^{2}$

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## Show all work for the following problem

A. ( 25 points) A small ball of mass 2.5 kg is attached to one end of a $4.0-\mathrm{m}$-long massless rod and the other end of the rod is attached to a pivot as shown. The bar is released from rest when it is horizontal at $t=0$, after which it swings down due to gravity. The sketch shows the bar at a time when it makes an angle $\theta$ with the vertical and also when $t=0$.
a) Calculate the net torque and angular acceleration of the bar at the instant after it is released.
b) Find the net torque on the bar and its angular acceleration when $\theta=\pi / 6$. Show the moment arm you are using on the sketch.
c) Find the angular velocity $\omega$ when $\theta=0$; that is, when the bar has swung so that it is momentarily vertical. There is no friction in the pivot. Note that the angular acceleration is not constant in this problem.
d) Find the angular acceleration when $\theta=0$. Describe what happens to the torque and angular acceleration as the bar swings past this point.

## Answers:

a) $\tau=-98 \mathrm{~nm}, \alpha=-2.5 \mathrm{rad} / \mathrm{s}^{2}$
b) $\tau=-49 \mathrm{~nm}, \alpha=-1.2 \mathrm{rad} / \mathrm{s}^{2}$
c) $\omega=-2.2 \mathrm{rad} / \mathrm{s}$
d) $\alpha=0$ since the moment arm is proportional to $\sin (\theta)$, which vanishes for $\theta=\mathbf{0}$. Afterwards, both $\alpha$ and $\tau$ reverse direction and become counter-clockwise. The angular velocity remains negative (CW) until the bar has swung to its most leftmost position.

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## Show all work for the following problem:

B. (25 points) Two masses are attached by a cord to a pulley as shown. The cord does not stretch and it can not slip where it touches the pulley, but there is no other frictional force. The rotational inertia of the pulley is $5.0 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
The radius $R=.75 \mathrm{~m}$ as shown.
a) Draw free-body diagrams and use Newton's second law to get the equations that describe the motion of mass $A$ and mass B.
b) Draw the free-body diagram and get the rotational second law equation for the pulley. Include the forces and torques acting on it.
c) You need 2 more equations to solve for the 5 unknowns in your equations above. What are they and why are they justified?
d) Calculate the angular acceleration of the pulley. Ans - $\mathbf{1 . 1} \mathbf{r a d} / \mathbf{s}^{\mathbf{2}} \mathbf{C W}$
e) Calculate the acceleration of mass $B$. Ans: - $\mathbf{0 . 8 6} \mathbf{~ m} / \mathbf{s}^{\mathbf{2}}$ (down)


## PHYSI CS I FORMULAS - Common Exam 1

## Physics 106:

360 degrees $=2 \pi$ radians $=1$ revolution. $s=r \theta \quad v_{t}=r \omega \quad a_{t}=r \alpha \quad a_{c}=a_{r}=v_{t}^{2} / r=\omega^{2} r \quad a_{\text {tot }}^{2}=a_{r}^{2}+a_{t}^{2}$ $\omega=\omega_{0}+\alpha t \quad \theta_{f}-\theta_{0}=\omega_{0} t+1 / 2 \alpha t^{2} \quad \omega_{\mathrm{f}}{ }^{2}-\omega_{0}^{2}=2 \alpha\left(\theta-\theta_{0}\right) \quad \theta-\theta_{0}=1 / 2\left(\omega+\omega_{0}\right) t \quad K_{\text {rot }}=1 / 2 l \omega^{2} \quad \mathrm{I}=\Sigma \mathrm{m}_{\mathrm{i}}{ }^{2}$ $I_{\text {point }}=m r^{2} \quad I_{\text {noop }}=M R^{2} \quad I_{\text {disk }}=1 / 2 M R^{2} \quad I_{\text {sphere }}=2 / 5 M R^{2} \quad I_{\text {shell }}=2 / 3 M R^{2} \quad I_{\text {rod (center) }}=1 / 12 M L^{2}$ $I_{\text {rod (end) }}=1 / 3 \mathrm{ML}^{2} \quad \mathrm{I}_{\mathrm{p}}=\mathrm{I}_{\mathrm{cm}}+\mathrm{Mh}^{2}$
$\tau=$ forcex moment arm $=\operatorname{Frsin}(\phi)=\mathbf{r} \times \mathbf{F} \quad \tau_{\text {net }}=\Sigma \tau=\mathrm{I} \alpha \quad \mathbf{F}_{\text {net }}=\Sigma \mathbf{F}=\mathrm{m} \mathbf{a}$
$\mathrm{W}_{\text {tot }}=\Delta \mathrm{K}=\mathrm{K}_{\mathrm{f}}-\mathrm{K}_{\mathrm{I}} \quad \mathrm{W}=\tau_{\text {net }} \Delta \theta \quad \mathrm{P}=\tau . \omega(\tau$ constant $) \quad \mathrm{K}=\mathrm{K}_{\text {rot }}+\mathrm{K}_{\mathrm{cm}} \quad \mathrm{E}_{\text {mech }}=\mathrm{K}+\mathrm{U}$
$\Delta \mathrm{E}_{\text {mech }}=0$ (isolated system) $\quad \mathrm{V}_{\mathrm{cm}}=\omega r$ (rolling, no slipping)
$\mathbf{I}=\mathbf{r} \mathbf{x p} \quad \mathbf{p}=\mathrm{mv} \quad \mathbf{L}=\Sigma \mathbf{I}_{\mathbf{i}} \quad \tau_{\text {net }}=\mathrm{dL} / \mathrm{dt} \quad \mathrm{L}=\mathrm{I} \omega \quad \mathrm{I}_{\text {point mass }}=\operatorname{mrvsin}(\phi)$
For isolated systems: $\tau_{\text {net }}=0 \quad \mathbf{L}$ is constant $\quad \Delta \mathbf{L}=0 \quad L_{0}=\Sigma l_{0} \omega_{0}=L_{f}=\Sigma \mid \omega_{t}$
Equilibrium: $\Sigma$ forces $=0$ and $\Sigma$ torques $=0$, If net force on a system is zero, then the net torque is the same for any chosen rotation axis. CG definition: point about which torques due to gravity alone add to zero.

## Physics 105:

$\mathrm{F}_{\mathrm{g}}=\mathrm{mg} \quad \mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2} \quad 1 \mathrm{~m}=100 \mathrm{~cm}=1000 \mathrm{~mm}, 1 \mathrm{~kg}=1000 \mathrm{~g}$
$v=v_{0}+a t \quad x-x_{0}=v_{0} t+1 / 2 a t^{2} \quad v^{2}-v_{0}{ }^{2}=2 a\left(x-x_{0}\right) \quad x-x_{0}=1 / 2\left(v+v_{0}\right) t$
$F_{\text {net }}=m a \quad \Sigma F=m a \quad F_{s, \max }=\mu_{s} N \quad F_{k}=\mu_{k} N \quad$ incline: $W_{m g x}=m g s i n \theta \quad W_{\text {mgy }}=m g \cos \theta$
$F_{r}=m a_{r}=m v^{2} / r \quad a_{r}=v^{2} / r \quad f=1 / T \quad T=(2 \pi r / v)$ Impulse: $\quad F_{a v r} \Delta t=m v_{f}-m v_{1}$
Momentum conserved if net impulse $=0$. Then $(\Sigma m v)_{\text {initial }}=(\Sigma \mathrm{mv})_{\text {final }}$
Work: $W=F d(\cos \theta), \quad W_{\text {grav }}=-m g\left(y-y_{0}\right), \quad W_{\text {spring }}=-1 / 2 k\left(x^{2}-x_{0}^{2}\right), \quad W_{\text {trict }}=-F_{k} d, \quad W_{\text {tot }}=K_{f}-K_{i}$
$U_{g}=m g\left(y-y_{0}\right), \quad$ spring: $F=-k x, \quad U_{s}=1 / 2 k x^{2}, \quad K E=1 / 2 m v^{2}$
$W_{n c}=K_{f}-\mathrm{K}_{\mathrm{l}}+\mathrm{U}_{\mathrm{gf}}-\mathrm{U}_{\mathrm{gi}}+\mathrm{U}_{\mathrm{sf}}-\mathrm{U}_{\mathrm{si}} \quad$ or $\quad \mathrm{K}_{\mathrm{l}}+\mathrm{U}_{\mathrm{gi}}+\mathrm{U}_{\mathrm{si}}+\mathrm{W}_{\mathrm{nc}}=\mathrm{K}_{\mathrm{f}}+\mathrm{U}_{\mathrm{gf}}+\mathrm{U}_{\mathrm{sf}}$
Mass center: $\mathrm{X}_{\mathrm{cm}}=\Sigma\left(\mathrm{x}_{\mathrm{i}} \mathrm{m}_{\mathrm{i}}\right) / \Sigma \mathrm{m}_{\mathrm{i}}$, similarly for $\mathrm{Y}_{\mathrm{cm}}, \mathrm{Z}_{\mathrm{cm}}$

## Vectors:

Components: $a_{x}=\operatorname{a\cdot cos}(\theta) \quad a_{y}=a \cdot \sin (\theta) \quad a=a_{x} \mathbf{i}+a_{y} \mathbf{j} \quad a=\operatorname{sqr}\left[a_{x}{ }^{2}+a_{y}{ }^{2}\right] \quad \theta=\tan ^{-1}\left(a_{y} / a_{x}\right)$
Addition: $\mathbf{a}+\mathbf{b}=\mathbf{c}$ implies $\mathrm{c}_{x}=\mathrm{a}_{\mathrm{x}}+\mathrm{b}_{\mathrm{x}}, \mathrm{c}_{\mathrm{y}}=\mathrm{a}_{\mathrm{y}}+\mathrm{b}_{\mathrm{y}}$
Dot product: $a_{0} \mathbf{b}=a \cdot b \cdot \cos (\phi)=a_{x} b_{x}+a_{y} b_{y}+a_{z} b_{z}$ unit vectors: $\mathbf{i}_{0} \mathbf{i}=\mathbf{j}_{\mathbf{0}} \mathbf{j}=\mathbf{k}_{\mathbf{0}} \mathbf{k}=1, \mathbf{i}_{0} \mathbf{j}=\mathbf{i}_{\mathbf{0}} \mathbf{k}=\mathbf{j}_{0} \mathbf{k}=\mathbf{0}$
Cross product: $|\mathbf{a x b}|=a \cdot b \cdot \sin (\phi) \mathbf{a x} \mathbf{b}=-\mathbf{b} \mathbf{x} \mathbf{a}, \mathbf{a} \mathbf{x} \mathbf{a}=\mathbf{0}$ always. $\mathbf{c}=\mathbf{a} \mathbf{x} \mathbf{b}$ is perpendicular to $\mathbf{a}-\mathbf{b}$ plane $\mathbf{i} \mathbf{x} \mathbf{i}=\mathbf{j} \mathbf{x} \mathbf{j}=\mathbf{k} \mathbf{x} \mathbf{k}=0, \quad \mathbf{i x j}=\mathbf{k} \quad \mathbf{j} \mathbf{x} \mathbf{k}=\mathbf{i} \quad \mathbf{k} \mathbf{x} \mathbf{i}=\mathbf{j} \quad$ etc.

