## Signature

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Name (Print): $\qquad$ 4 Digit ID: $\qquad$ Section: $\qquad$

## I nstructions:

- Answer all questions. Questions 1 through 10 are multiple choice questions worth 5 points each. You may need to do some calculation. Answer each of these on the Scantron sheet using \#2 pencil. You may use backs of the question papers for extra space. Questions A and B are work-out problems worth 25 points each. Answer them on the exam booklet and show ALL work, otherwise there is no way to give partial credit.
- Be sure your name and section number are on both the Scantron form and the exam booklet. Also write your name, Id, and section at the top of each page with long answer questions $A$ and $B$ on them. You may lose points if your name and section number does not appear on each page. Be sure you are in the right room for your section.
- You may use one side of an $8.5 \times 11$ sheet or two sides of a $5 \times 8$ card as a formula sheet. A default formula sheet is also provided (see page 6 of this booklet). Make sure to bring your own calculator.
- All cell phones, pagers or similar electronic devices should be turned off. Sharing of calculators is not permitted. Students are not to communicate with each other once the test has started. If you have questions or need something call your proctor or instructor.
- As you know, NJIT has a zero tolerance policy for ethics code violations during and also after an exam.

1. When the speed of a front wheel drive car is increasing on a straight horizontal road, which way do the frictional forces on the front and rear tires point? Hint: does the car accelerate if the wheels are on ice? Which way does friction point in order to avoid slipping?
A) forward for all tires
B) backward for all tires
C) forward for the front tires and backward for the rear tires
D) forward for the rear tires and backward for the front tires
E) zero
2. A bicycle wheel (a hoop with $I_{\text {hoop }}=M R^{2}$ ) rolls without slipping along the floor. The ratio of its translational kinetic energy to its rotational kinetic energy (about an axis through its center of mass) is:
A) 1
B) 2
C) 3
D) $1 / 2$
E) $1 / 3$
3. A solid ball starts from rest and rolls without slipping down the ramp as shown in the sketch. The height of the ramp is $h$, the ball's mass is $M$ and its radius is $R$. What is the ball's final mass center velocity?
(Hint: $\mathbf{I}_{\text {sphere }}=\mathbf{2 / 5} \mathbf{M R}^{\mathbf{2}}$ )
A) Can not be determined from the information given
B) $v_{\text {com }}=(2 g h)^{1 / 2}$
C) $v_{\text {com }}=(\mathrm{gh})^{1 / 2}$
D) $\mathbf{v}_{\text {com }}=(10 \mathrm{gh} / 7)^{1 / 2}$
E) $v_{\text {com }}=5 \mathrm{Mgh} / 2$
4. A single particle is located somewhere on the positive $\mathbf{x}$ axis. A net force acting on this particle points in the negative $\mathbf{y}$ direction. The vector of resulting torque points in the:
A) positive $\mathbf{x}$ direction
B) positive $\mathbf{y}$ direction
C) positive $\mathbf{z}$ direction
D) negative $\mathbf{y}$ direction
E) negative $\mathbf{z}$ direction

A
5. The cross product $\mathbf{A} \times \mathbf{B}$ of two vectors $\mathbf{A}=2 \mathbf{i}+3 \mathbf{j}$ and $\mathbf{B}=-4 \mathbf{i}+6 \mathbf{j}$ is equal to
A) $15 \mathbf{j}$
B) 10
C) $5 \mathbf{i}$
D) $24 k$
E) $-5 \mathbf{k}$
6. A $\mathbf{1 0 . 0} \mathbf{- k g}$ block travels around a $\mathbf{2 . 0} \mathbf{- m}$ radius circle with an angular velocity of $\mathbf{2 0} \mathbf{r a d} / \mathbf{s}$. Its angular momentum $L$ about the center of the circle is:
A) $800 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
B) $400 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
C) $200 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
D) $80 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
E) $8 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
7. A figure skater goes into a spin, keeping her arms as shown. When she extends her arms horizontally:
A) her angular velocity increases.
B) her angular velocity remains the same.
C) her rotational inertia decreases.
D) her rotational kinetic energy increases.
E) her angular momentum remains the same.

8. A horizontal disc of rotational inertia $\mathbf{I}=\mathbf{0 . 0 1} \mathbf{~ k g} \cdot \mathbf{m}^{\mathbf{2}}$ and radius $\mathbf{0 0} \mathbf{~ c m}$ is rotating about a vertical axis through its center with an angular speed of $\mathbf{3 . 5} \mathbf{r a d} / \mathbf{s}$. A wad of wet putty of mass $\mathbf{1 0 0}$ grams drops vertically onto the disc from above and sticks to the edge of the disk. What is the angular speed of the disk right after the putty sticks to it?
A) $1.3 \mathrm{rad} / \mathrm{s}$
B) $\mathbf{2 . 5} \mathbf{r a d} / \mathrm{s}$
C) $3.3 \mathrm{rad} / \mathrm{s}$
D) $4.0 \mathrm{rad} / \mathrm{s}$
E) $5.0 \mathrm{rad} / \mathrm{s}$

9. Two disks are mounted on frictionless bearings on a common shaft. Disc 1 has rotational inertia $I_{1}$ and is spinning with angular velocity $\omega_{2}$ Disc 2 has rotational inertia $I_{2}=3 . I_{1}$ and is spinning in the same direction as Disc 1 with angular velocity $\omega_{\mathbf{2}}=\mathbf{2} \cdot \omega_{\text {? }}$, as shown. The two disks are slowly forced toward each other along the shaft until they stick together and rotate with a common final angular velocity of:
A) $\mathbf{7} \cdot \omega_{1} / 4$
B) $\omega_{1}$
C) $3 \cdot \omega_{1}$
D) $\omega_{1} / 3$
E) $\quad 1.8 \cdot \omega_{1}$


Disk 1
Disk 2
10. If the angular momentum of a system of particles is constant, which of the following statements must be true?
A) No torques are acting on any part of the system.
B) A constant torque acts on the system.
C) The net torque is zero on each part of the system.
D) The net torque is zero for the whole system.
E) The net external torque on the system is constant.

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## Show all equations and work for the following problem

A). (25 points) A 50.0-kg wheel, which can be viewed as a hoop with the diameter of $\mathbf{1 . 0} \mathbf{~ m}$ is initially at rest. It must be brought smoothly (at constant angular acceleration) to rotate at $\mathbf{3 6 0} \mathbf{r e v} / \mathbf{m i n}$ in $\mathbf{2 0 . 0}$ seconds. Neglect friction and calculate the following quantities: $\quad$ Hint: $I_{\text {hoop }}=M R^{2}$
a) The constant torque of the external force on the wheel:

Ans: $\mathbf{2 3 . 6}$ N.m

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\begin{aligned}
& \mathrm{I}=\mathrm{M} \cdot \mathrm{D}^{2} / 4=50 \mathrm{~kg} \cdot(0.5 \mathrm{~m})^{2}=12.5 \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& \omega_{\mathrm{f}}=(360 \mathrm{rev} / \mathrm{min}) \cdot 2 \pi /(60 \mathrm{~s})=37.7 \mathrm{rad} / \mathrm{s} \\
& \alpha=\omega_{\mathrm{f}} / \mathrm{t}=37.7 \mathrm{rad} / \mathrm{s}^{2} / 20 \mathrm{~s}=1.88 \mathrm{rad} / \mathrm{s}^{2} \\
& \tau=\alpha \cdot \mathrm{I}=\left(12.5 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \cdot\left(1.88 \mathrm{rad} / \mathrm{s}^{2}\right)=23.6 \mathrm{~N} \cdot \mathrm{~m}
\end{aligned}
$$

b) The work done by external force to accelerate the wheel:
$\quad$ Ans: $\quad 8.9 \times 10^{\mathbf{3}}$ Joules
$\theta=\alpha \cdot t^{2} / 2=0.5 \cdot\left(1.88 \mathrm{rad} / \mathrm{s}^{2}\right) \cdot(20 \mathrm{~s})^{2}=376 \mathrm{rad}$
Work $=\tau \cdot \theta=(23.6 \mathrm{~N} \cdot \mathrm{~m}) \cdot 376 \mathrm{rad}=8883 \mathrm{~J}=8.9 \times 10^{3} \mathrm{~J}$ oules

You can get the same result using Work-Kinetic Energy theorem: W $=\Delta \mathrm{K}=\mathrm{K}_{\mathrm{f}}-\mathrm{K}_{\mathrm{i}}$
Since $K_{i}=0$, then Work $=1 / 2 \mid \omega f^{2}=0.5 \cdot\left(12.5 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \cdot(37.7 \mathrm{rad} / \mathrm{s})^{2}=8883 \mathrm{~J}=8.9 \times 10^{3} \mathrm{~J}$ oules
c) The average power of the external force:

Ans: 444 Watts

Power $=$ Work $/ \mathrm{t}=8883 \mathrm{~J} / 20 \mathrm{~s}=444$ Watts
You can also use:
Power $=\tau \cdot \omega$, but you should input "average" vangular velocity, which is equal to $\omega_{\mathrm{f}} / 2=(37.7 \mathrm{rad} / \mathrm{s}) / 2=18.9 \mathrm{rad} / \mathrm{s}$. Then you will get
Power $=(23.6 \mathrm{~N} \cdot \mathrm{~m}) \cdot(18.9 \mathrm{rad} / \mathrm{s})=444 \mathrm{Watts}$

## Show all equations and work for the following problem:

B) (25 points) A uniform thin rod of length $\mathbf{L}=\mathbf{2 . 0} \mathbf{m}$ and mass $\mathbf{M}$ $=\mathbf{5 . 0} \mathbf{~ k g}$ can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a $\mathbf{1 0} \mathbf{g}$ bullet traveling in the horizontal plane of the rod is fired into one end of the rod. As viewed from above, the direction of the bullet's velocity makes an angle of $\pi / \mathbf{2}$ with the rod (see the figure). The bullet lodges in the rod and the angular velocity of the rod+bullet is $\mathbf{2} \mathbf{r a d} / \mathbf{s}$ immediately after the collision.
Hint: $I_{\text {rod (center) }}=1 / 12 \mathrm{ML}^{2}$, where L is the length of the rod.

a) What is the bullet's speed just before the impact?

Ans: $335 \mathrm{~m} / \mathrm{s}$

Initial angular momentum ( $\mathrm{L}_{\mathrm{i}}$ ) is equal to the angular momentum of the bullet:
$L_{i}=L_{\text {bullet }}=m v L / 2$. Hence, the Velocity of the bullet $V_{\text {bullet }}=2 L_{i} / m L$,
where $L$ is the length of the rod.
Rotational Inertia after the impact: $\mid=I_{\text {rod (center) }}+I_{\text {bullet }}$
$L_{\text {final }}=\omega I=(2 \mathrm{rad} / \mathrm{s}) \cdot\left[(1 / 12) \cdot(5 \mathrm{~kg}) \cdot(2 \mathrm{~m})^{2}+(0.01 \mathrm{~kg}) \cdot(1 \mathrm{~m})^{2}\right]=3.35 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
Assuming that $\mathrm{L}_{\mathrm{i}}=\mathrm{L}_{\text {final }}$, we calculate the velocity of the bullet
$v_{\text {bullet }}=2 \cdot\left(3.35 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}\right) /(0.01 \mathrm{~kg} \cdot 2 \mathrm{~m})=335 \mathrm{~m} / \mathrm{s}$
b) What is the ratio of the kinetic energy of the entire system after the impact to that of the bullet just before the impact?

Ans: $\mathrm{K}_{\mathrm{f}} / \mathrm{K}_{\mathrm{i}}=0.006$

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\begin{aligned}
& K_{i}=1 / 2 m v^{2}=1 / 2^{*}(0.01 \mathrm{~kg}) \cdot(335 \mathrm{~m} / \mathrm{s})^{2}=561 \mathrm{~J} \\
& \mathrm{~K}_{\mathrm{f}}=1 / 2 \mathrm{I} \omega^{2}=1 / 2(2 \mathrm{rad} / \mathrm{s})^{2} \cdot\left[(1 / 12) \cdot(5 \mathrm{~kg}) \cdot(2 \mathrm{~m})^{2}+(0.01 \mathrm{~kg}) \cdot(1 \mathrm{~m})^{2}\right]=3.35 \mathrm{~J} \\
& K_{f} / K_{i}=0.006
\end{aligned}
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c) Through what angle will the bullet-thin rod system rotate in 10 s after the impact?

Ans: 20 rad
$\theta=\omega t=2 \mathrm{rad} / \mathrm{s} \cdot 10 \mathrm{~s}=20 \mathrm{rad}$
note that after the impact the rod+bullet rotate at constant angular velocity (zero angular acceleration). If you were trying to use
$\theta=1 / 2\left(\omega_{\mathrm{f}}+\omega_{0}\right) \cdot \mathrm{t}$
then you should get the same result, since $\omega_{\mathrm{f}}=\omega_{0}$

