Lecture 2

Physics 106

Spring 2007

Rotational dynamics:

- Kinetic Energy of rotation
- Rotational inertia
- Torque
- Cross product

http://web.njit.edu/~sirenko/

Required Home Work

Specific information for the UT homework system:

UT Guest ID Registration: https://utdirect.utexas.edu/nlogon/eid_suite/essentials/create_eid.WBX?portal_role=O

UT HW Student Instructions:

https://hw.utexas.edu/bur/studentGuestEID.html

Student Login Page (Univ. of Texas):

https://utdirect.utexas.edu/security-443/UTEIDLogon.wb

UT EID Home Page (Forgotten Password): https://utdirect.utexas.edu/nlogon/eid_suite/general/

Your instructor will announce the 5 digit course number you need to use when you register for Physics 106 in the UT system.

If you already have a UT Guest login ID and password, you can continue to use it.

Fill out the following for your own future reference, and keep it someplace where you can find it:

§ Unique course number to be announced by instructors: **11787**

§ Your Login ID on the UT system (generated when you register with UT; case sensitive!):________

§ Your own password (selected upon registration with UT; confidential!): ___________

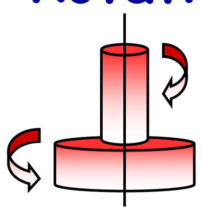
Note that NJIT instructors can not access your password.

Rotation:

Angular Displacement

Angular Velocity

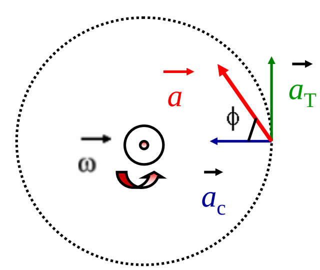
Angular Acceleration



θ, $\vec{\omega}$, $\vec{\alpha}$

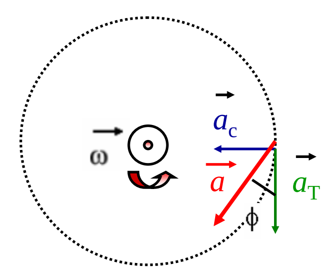
Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Acceleration in Circular Motion:



ω increases with time

$\alpha > 0$



ω decreases with time

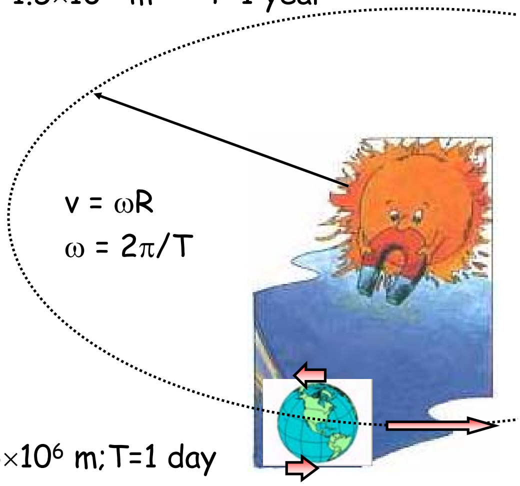
$\alpha < 0$

$a_T = r\alpha$ $\quad a_c = v^2/r = r\omega^2$

$\vec{a} = \vec{a}_c + \vec{a}_T$; $a = (a_c^2 + a_T^2)^{1/2}$, $\tan\phi = a_T / a_c$

QZ: Our linear velocity with respect to the Sun

$R_{E\text{-}S} = 1.5\times10^{11}$ m T=1 year



$v = \omega R$

$\omega = 2\pi/T$

$R=6\times10^{6}$ m; T=1 day

When do we move faster ?

(a) Day
(b) Night

What is the velocity difference between Day and Night at the Equator line ?

$|(V_{day} - V_{night})|/V_{average}$

(a) 0.00008
(b) 0.015
(c) 0.03
(d) 0.3
(e) 100 %

Kinetic Energy of Rotation

$K = \frac{1}{2} mv^2$ Point mass (no rotation); v of the COM

$$K = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \cdots$$
$$= \sum \tfrac{1}{2} m_i v_i^2 ,$$

System of particles or an object

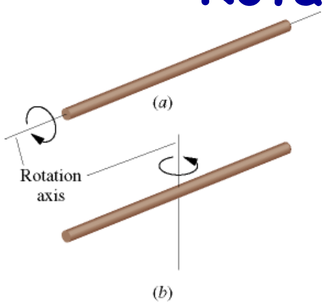
$v = \omega r$

$$K = \sum \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$I = \sum m_i r_i^2$ (rotational inertia)

$K = \frac{1}{2} I\omega^2$ (radian measure)

Rotational Inertia



$K = \frac{1}{2} I\omega^2$ (radian measure)

$I^{(a)} \neq I^{(b)}$

I – rotational equivalent of mass m

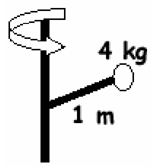
Main difference between m and I:
Rotational Inertia depends on the direction of rotation !

For a rigid body, I depends on how the mass is distributed in an object relative to the axis of rotation

Rotational Inertia Of Point Mass

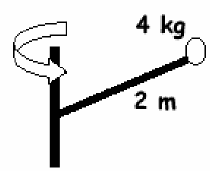
For a single particle I = mr²
(all mass at same r)

A single particle



$I = (4\ \text{kg})(1\ \text{m})^2$
$= 4\ \text{kg m}^2$

The same particle farther out



$I = (4\ \text{kg})(2\ \text{m})^2$
$= 16\ \text{kg m}^2$

Four times the rotational "mass"

Rotational Inertia

Checkpoint 1

The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.

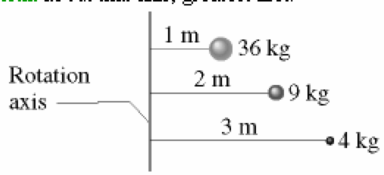
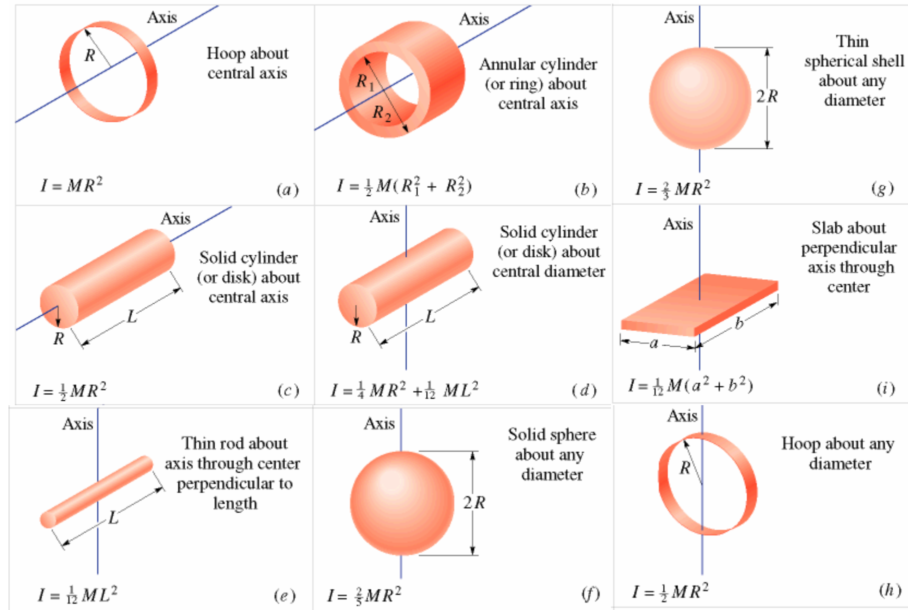
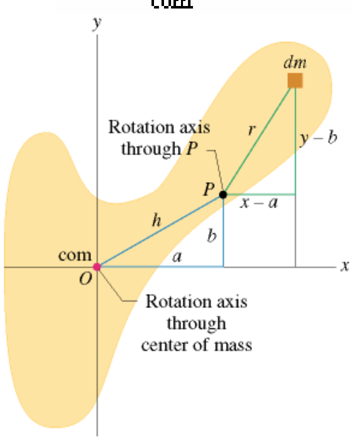


TABLE 11-2 Rotational Inertia



Parallel-Axis Theorem for Rotational Inertia

$$I = I_{com} + Mh^2 \qquad \text{(parallel-axis theorem).}$$



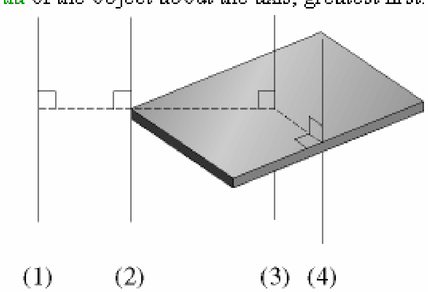
Calculate I_{com} for the axis going through the COM

Use *Parallel-Axis Theorem* to calculate I

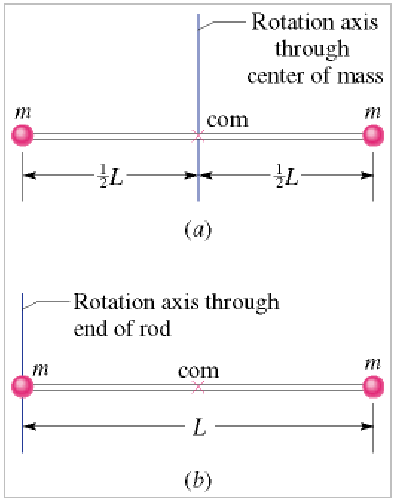
Rotational Inertia

Checkpoint 2

The figure shows a booklike object (one side is longer than the other) and four choices of rotation axes, all perpendicular to the face of the object. Rank the choices according to the rotational inertia of the object about the axis, greatest first.



Example: Rotational Inertia



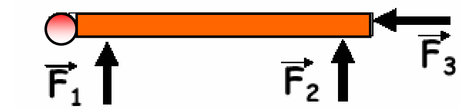
$$I = \sum m_i r_i^2 = (m)(\tfrac{1}{2}L)^2 + (m)(\tfrac{1}{2}L)^2 = \tfrac{1}{2}mL^2.$$

$$I = m(0)^2 + mL^2 = mL^2$$

$$I = I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2 = mL^2.$$

Torque: $\vec{\tau}$

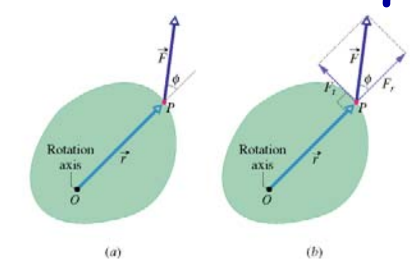
All forces are the same

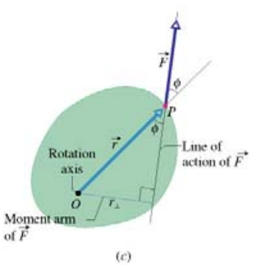


Door on a hinge: How do you slam?

Not only the force is important,
But how you apply it !

Torque: $\vec{\tau}$





The value of *torque:*

$\tau = r \cdot F \cdot \sin\phi$

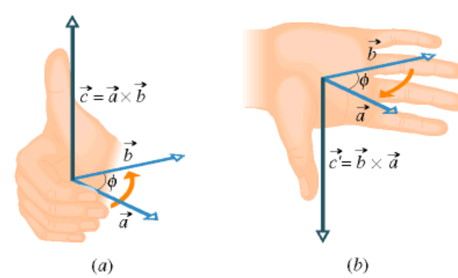
$\phi = 0 \rightarrow \tau = 0$

$\phi = \pi/2 \rightarrow \tau = r \cdot F$ (max)

In vector notation form:

$\vec{\tau} = [\vec{r} \times \vec{F}]$

Vector Cross Product



The value of *cross product:*

$c = a \cdot b \cdot \sin\phi$

$\phi = 0 \rightarrow c = 0$

$\phi = \pi/2 \rightarrow c = a \cdot b$ (max)

Cross product is maximized when vectors are perpendicular

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

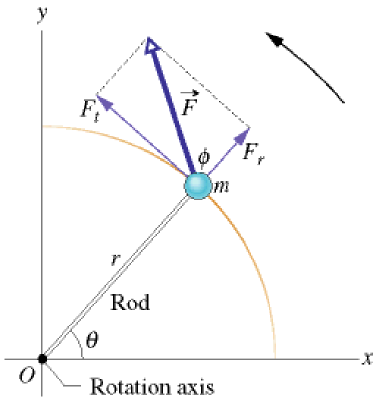
Order is important: $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Newton's Second Law for Rotation

Torque causes the change in ω

$$\tau_{net} = I \cdot \alpha$$

Rotational equivalent of F = ma



$$F_t = m a_t.$$

$$\tau = F_t r = m a_t r.$$

$$\tau = m(\alpha r)r = (m r^2)\alpha.$$

Rotational Analogy to Linear Motion

	Translation	*Rotation*
position	x	θ
velocity	$v = dx/dt$	$\omega = d\theta/dt$
acceleration	$a = dv/dt$	$\alpha = d\omega/dt$
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I\ \omega^2$
Force	$F = ma$	$\tau_{net} = I \cdot \alpha$

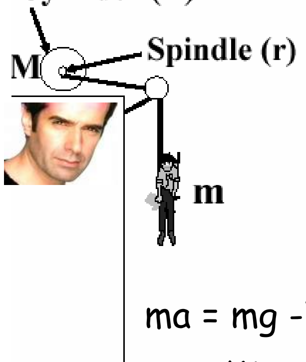
A Falling Stuntman

Stuntmen sometimes need to fall large distances

Without getting hurt!

But it has to look like they fall

Hollow Cylinder (R)



$$I_{cyl} = MR^2 \qquad \tau = r\ T$$

$$\alpha = \frac{r\ T}{MR^2} \qquad a = \alpha r$$

The rope is around the spindle

$ma = mg - T$

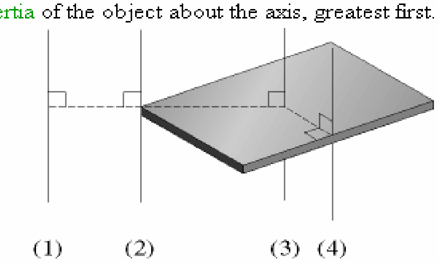
$a = g/(1+MR^2/mr^2)$

For m = 70 kg, M = 10 kg, r = 0.1 m and R = 0.5 m

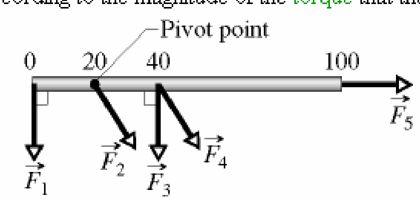
$a = 9.8/(1+25/7) \approx 2 m/s^2$

QZ#2 Parallel axis Theorem and Torque

The figure shows a booklike object (one side is longer than the other) and four choices of rotation axes, all perpendicular to the face of the object. Rank the choices according to the rotational inertia of the object about the axis, greatest first.



The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five horizontal forces on the stick have the same magnitude. Rank those forces according to the magnitude of the torque that they produce, greatest first.



Homework

See the Physics 106 Course Syllabus

U of Texas HW is required

http://web.njit.edu/~sirenko/