Lecture 8

Physics 106

Spring 2006

Equilibrium

http://web.njit.edu/~sirenko/

Equilibrium

Balance of Forces:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

Balance of Torques:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{P} = 0$$

Stable vs. Unstable Static Equilibrium

An equilibrium point is stable if small changes in the position lead to restoring forces back to equilibrium.

If it moves away from the equilibrium point when displaced slightly, it is unstable.

Equilibrium

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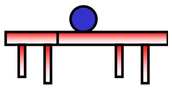
Indeterminate Equilibria

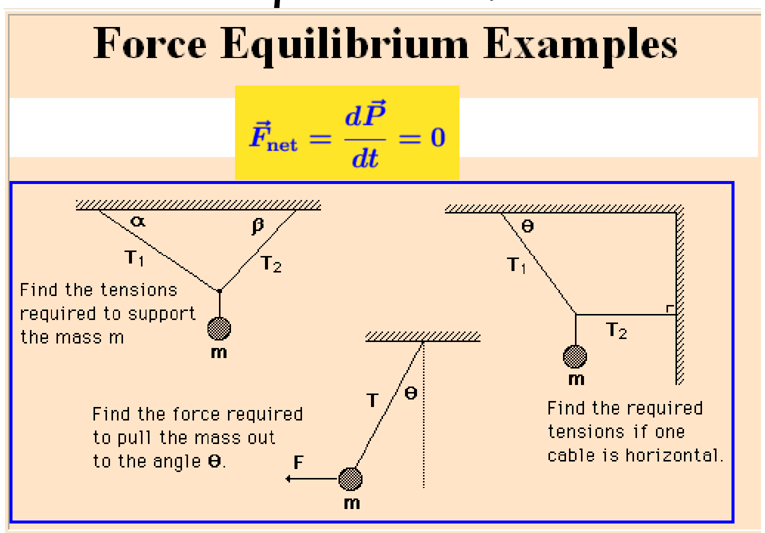
If the force and torque equations lead to more unknown forces than equations, there are an infinite number of solutions.

Examples: Four-legged table
Two axle trailer



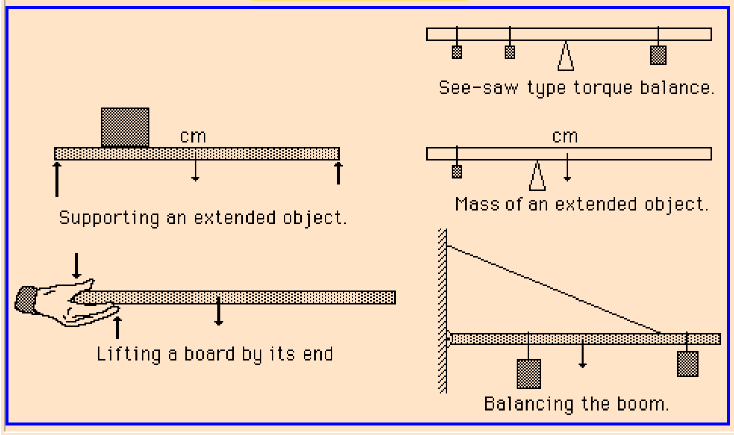
Detailed material properties and history determine the forces.

Equilibrium



Torque Equilibrium Examples

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$



Conditions for Equilibrium

An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation. The basic conditions for equilibrium are:

1. Net force = 0	x and y components of force may be separately set = 0.
$\sum_i F_i = 0$	Forces left = forces right Forces up = forces down.
2. Net torque = 0	The axis may be chosen for advantage to eliminate some unknown forces..
$\sum_i \tau_i = 0$	The sum of the clockwise torques is equal to the sum of the counterclockwise torques.

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

Equilibrium

Balance of Forces:

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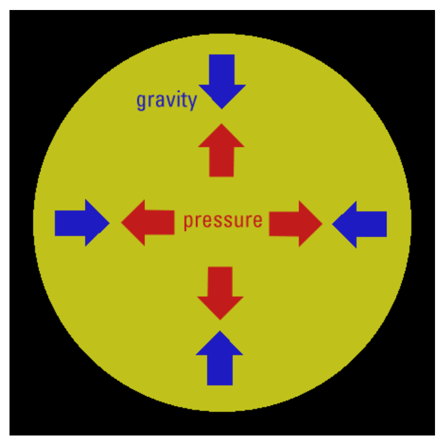
Balance of Torques:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{P} = 0$$

1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all the external torques that act on the body, measured about any possible point, must be zero.
3. The linear momentum $\underline{P}$ of the body must be zero.
4. The gravitational force $\underline{F}_g$ on a body effectively acts on a single point, called the center of gravity (cog) of the body. If g is the same for all elements of the body, then the body's cog is coincident with the body's center of mass.

Equilibrium inside a Star



Balance of Forces:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

Balance of Torques:

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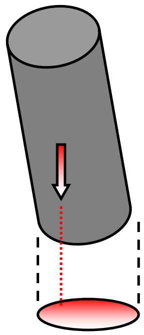
Equilibrium of the tower of Piza

Is it really stable ?



$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

Static Equilibrium

$\vec{P} = 0$

Balance of Forces:

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Balance of Torques:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

Stable vs. Unstable Static Equilibrium

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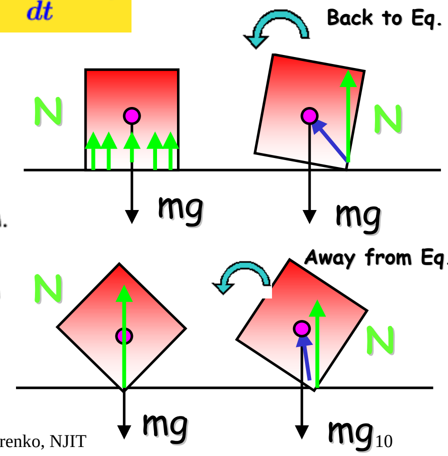
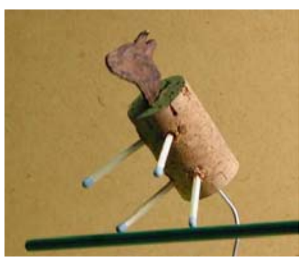


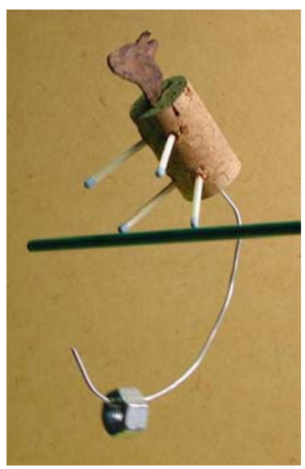
Equilibrium for fun

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$



Unstable Equil.



Stable Equil.

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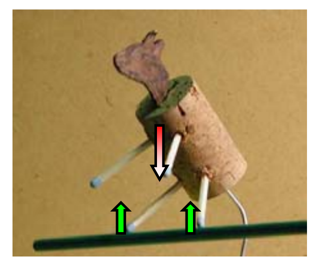
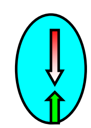
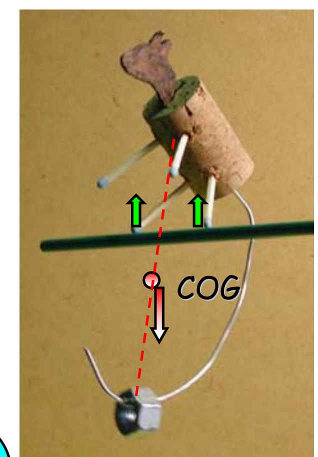
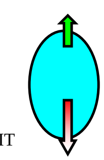


Unstable Equil.







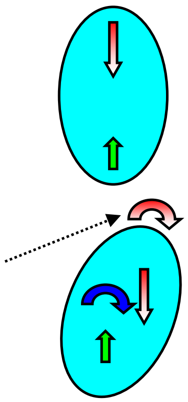
Stable Equil.

Equilibrium; Stable vs. Unstable Two Forces

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

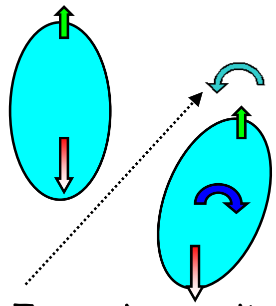
$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

Unstable Equil.



Torque is in the direction of the angular displacement

Stable Equil.



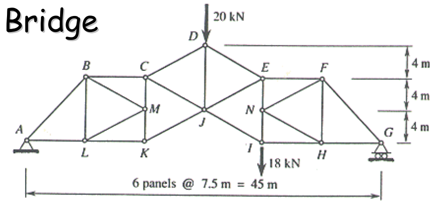
Torque is opposite to the direction of the angular displacement

Equilibrium of Mechanical Constructions

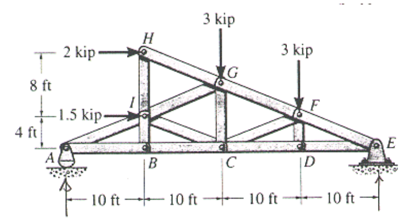
$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

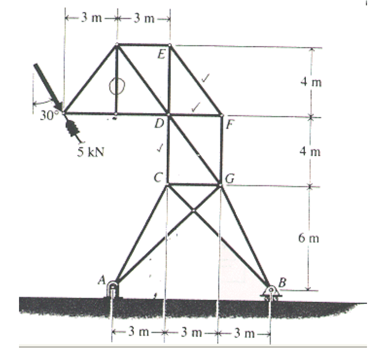
Bridge



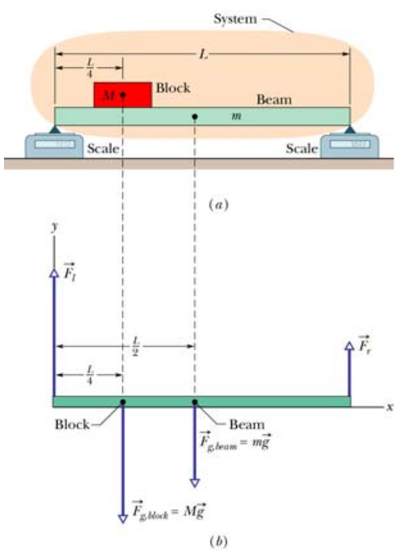
Roof



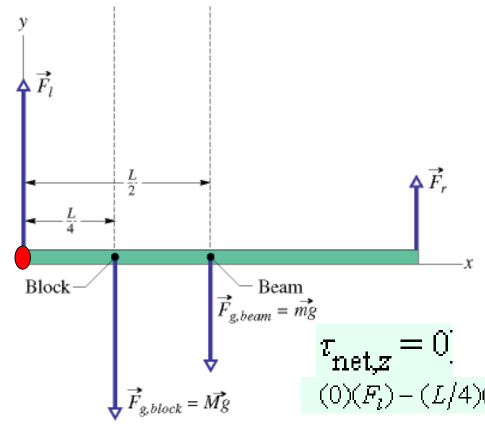
Tower:



Sample Problem XIII – 1



A uniform beam of length L and mass m = 1.8 kg is at rest with its ends on two scales. A uniform block with mass M = 2.7 kg is at rest on the beam, with its center a distance $L/4$ from the beam's left end. What do the scales read?



From the force balance we have Two unknowns

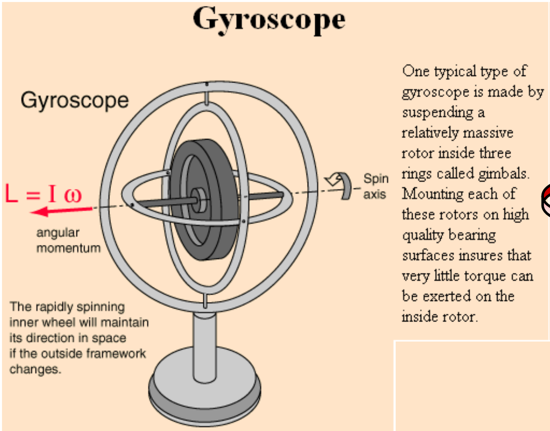
$$F_l + F_r - Mg - mg = 0$$

$$\tau_{net,z} = 0$$

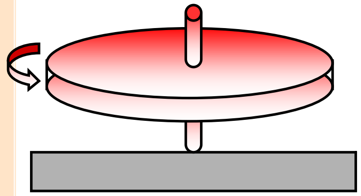
$$(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0$$

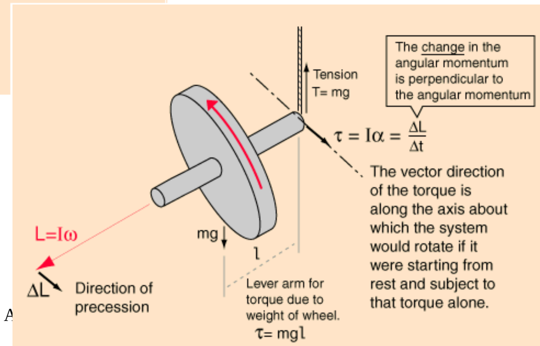
$$F_r = \tfrac{1}{4}Mg + \tfrac{1}{2}mg$$
$$= \tfrac{1}{4}(2.7\text{ kg})(9.8\text{ m/s}^2) + \tfrac{1}{2}(1.8\text{ kg})(9.8\text{ m/s}^2)$$
$$= 15.44\text{ N} \approx 15\text{ N}.$$

$$F_l = (M + m)g - F_r$$
$$= (2.7\text{ kg} + 1.8\text{ kg})(9.8\text{ m/s}^2) - 15.44\text{ N}$$
$$= 28.66\text{ N} \approx 29\text{ N}.$$



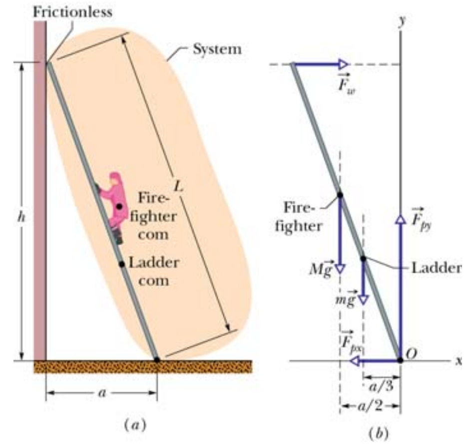
Is this rotating wheel stable ?



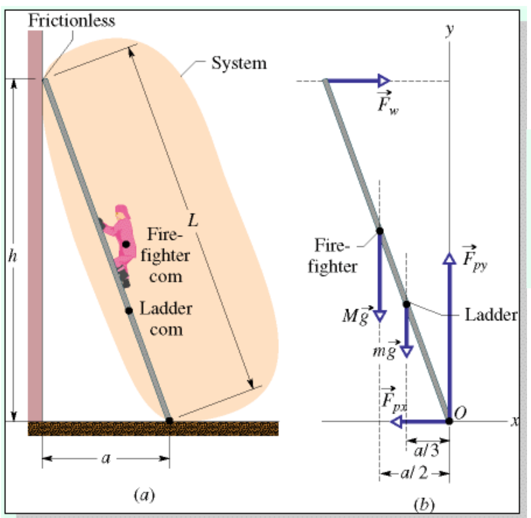


Dynamic and Static Equilibriums are different !

Sample Problem XIII – 2



A ladder of length L = 12 m and mass m = 45 kg leans against a slick (frictionless) wall. Its upper end is at height h = 9.3 m above the pavement on which the lower end rests (the pavement is not frictionless). The ladder's center of mass is $L/3$ from the lower end. A firefighter of mass M = 72 kg climbs the ladder until her center of mass is $L/2$ from the lower end. What the are the magnitudes of the forces of the ladder from the wall and the pavement?



$$a = \sqrt{L^2 - h^2} = 7.58\ \text{m}.$$

$$F_w = \frac{ga(M/2 + m/3)}{h} = \frac{(9.8\ \text{m/s}^2)(7.58\ \text{m})(72/2\ \text{kg} + 45/3\ \text{kg})}{9.3\ \text{m}} = 407\ \text{N} \approx 410\ \text{N}.$$

$$F_w - F_{px} = 0$$

$$F_{px} = F_w = 410\ \text{N}.$$

$$F_{net,y} = 0$$

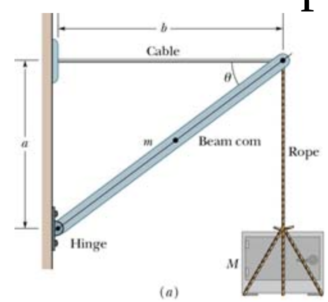
$$F_{py} - Mg - mg = 0,$$

$$F_{py} = (M + m)g = (72\ \text{kg} + 45\ \text{kg})(9.8\ \text{m/s}^2) = 1146.6\ \text{N} \approx 1100\ \text{N}.$$

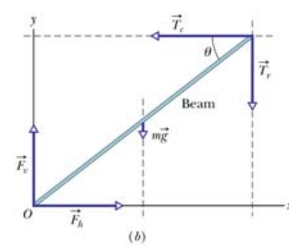
Torque balance for the point O

$$-(h)(F_w) + (a/2)(Mg) + (a/3)(mg) + (0)(F_{px}) + (0)(F_{py}) = 0.$$

Sample Problem XIII – 3



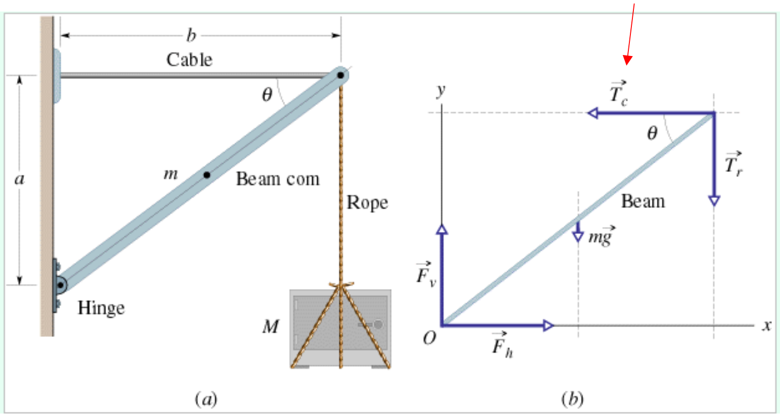
A safe of mass M = 430 kg is hanging by a rope from a boom with dimensions a = 1.9 m and b = 2.5 m. The boom consists of a hinged beam and a horizontal cable that connects the beam to a wall. The uniform beam has a mass m = 85 kg. The masses of the cable and the rope are negligible.



a) What are the tension T_c in the cable? In other words, what is the magnitude of the force $\vec{T}_c$ on the beam from the cable?

b) Find the magnitude F of the net force on the beam from the hinge.

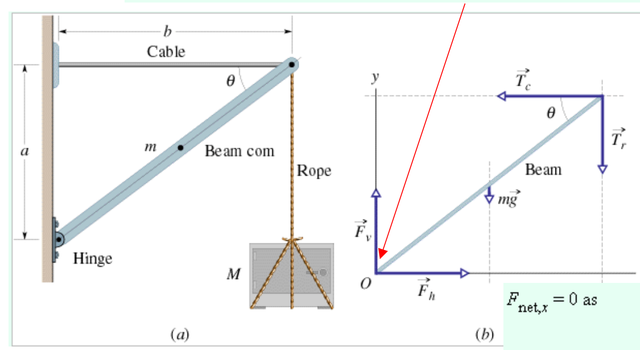
What is the tension T_c in the cable?



$$(a)(T_c) - (b)(T_r) - (\tfrac{1}{2}b)(mg) = 0.$$

$$T_c = \frac{gb(M + \frac{1}{2}m)}{a} = \frac{(9.8\ \text{m/s}^2)(2.5\ \text{m})(430\ \text{kg} + 85/2\ \text{kg})}{1.9\ \text{m}} = 6093\ \text{N} \approx 6100\ \text{N}.$$

Find the magnitude F of the net force on the beam from the hinge.



$F_{\text{net},x} = 0$ as

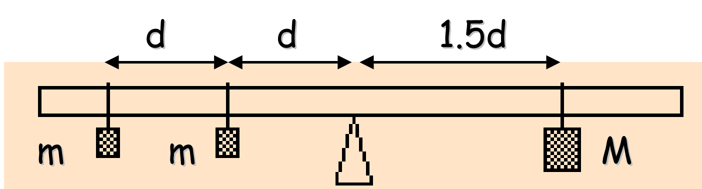
$$F_h - T_c = 0.$$

$$F_h = T_c = 6093\ \text{N}.$$

$$F_v - mg - T_r = 0.$$

$$F_v = (m + M)g = (85\ \text{kg} + 430\ \text{kg})(9.8\ \text{m/s}^2) = 5047\ \text{N}.$$

$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{(6093\ \text{N})^2 + (5047\ \text{N})^2} \approx 7900\ \text{N}.$$

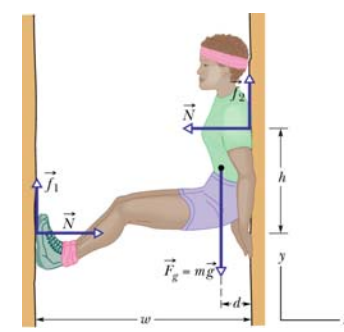


QZ#8 m = 5 kg d = 2 m

- Show all forces acting on the beam. Mass of the beam is zero
- Write the force and torque balances
- Calculate M to keep the whole system in equilibrium

Sample Problem XIII – 4

A rock climber with mass m = 55 kg rests during a “chimney climb”, pressing only with her shoulders and feet against the walls of a fissure of width w = 1.0 m. Her center of mass is is a horizontal distance d = 0.2 m from the wall against which her shoulders are pressed. The coefficient of static friction between her shoes and the wall is μ_1 = 1.1, and between her shoulders and the wall it is μ_2 = 0.7. To rest, the climber wants to minimize her horizontal push on the walls. The minimum occurs when her feet and her shoulders are on the verge of sliding.



a) What is the minimum horizontal push on the walls?

b) For that push, what must be the vertical distance h between her feet and her shoulders if she is to be stable?