Lecture 9(+10)

Physics 106
Spring 2006

Gravitation

HW&R

http://web.njit.edu/~sirenko/

Gravitation

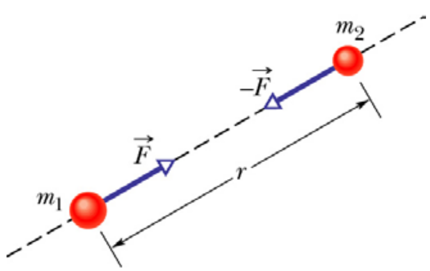
- On Earth: the Earth gravitation dominates (F =mg) $F = G\frac{m_1 m_2}{r^2}$
- In the Solar System: attraction to the Sun is the main effect
- In the Galaxy (Milky Way): Attraction to the center of the Galaxy determines everything.
- At the edge of the Universe: the conceptual problems begin … Accelerating expansion of the visible Universe is known since 1998. From that tim this problem became one of the frontiers of the modern Physics

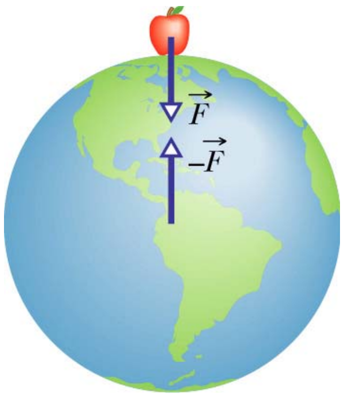
The Andromeda galaxy. Located 2.3 10^6 light-years from us

Newton's Law of Gravitation (known since 1665)

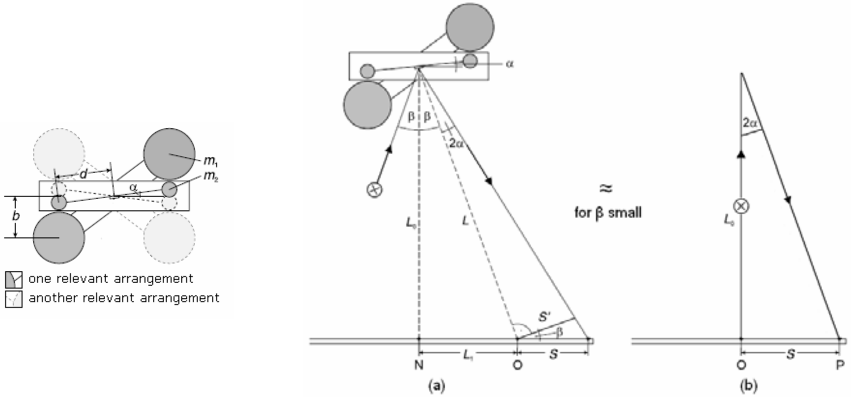
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$





Measuring the Gravitational constant G using the Cavendish method



$$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$

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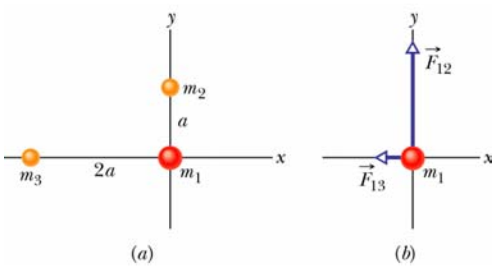


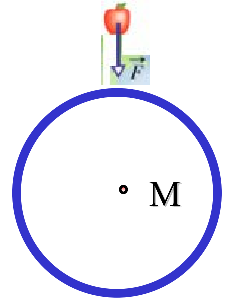
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Sample Problem from HW&R:

$m_1 = 6$ kg, $m_2 = m_3 = 4$ kg, $d_{12} = a$, and $d_{13} = 2a$. What is the net gravitational force F_1 that acts on the particle "1" due to the other particles? Use $a = 0.1$ m.

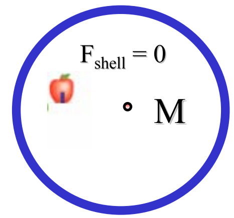
Newton's Law of Gravitation

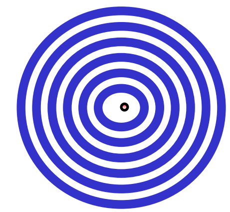


$$F = G\frac{m_1 m_2}{r^2}$$

A uniform spherical shell of matter attracts a particles that is outside the shell as if all the shell's mass is concentrated at its center!

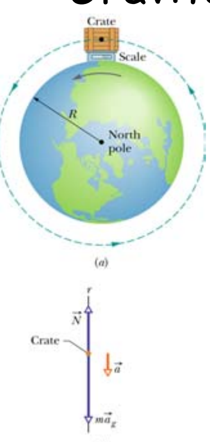
Solid sphere is a combination of spherical shells:





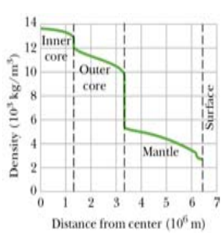
M

Gravitation Near Earth's Surface



Gravitational acceleration:

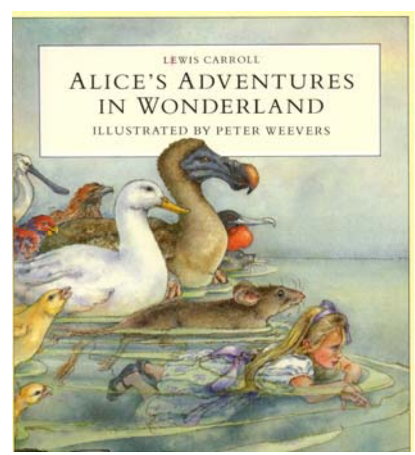
$$F = G\frac{Mm}{r^2} = ma_g \quad \Rightarrow \quad a_g = \frac{GM}{r^2}$$



1. Earth is not uniform.
2. Earth is not a sphere.
3. Earth is rotating.

Mean Earth surface (0 km, 9.83 m/s^2), Mt. Everest (8.8 km, 9.80 m/s^2), highest manned balloon (36.6 km, 9.71 m/s^2), Space Shuttle orbit (400 km, 8.70 m/s^2), and communications satellite (35,700 km, 0.225 m/s^2)

Gravitation inside the Earth "Alice in Wonderland"



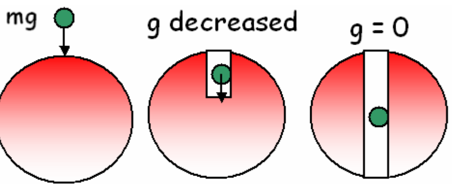
Gravitation inside the Earth
"Alice in Wonderland"

Outside the Earth
(r > R)

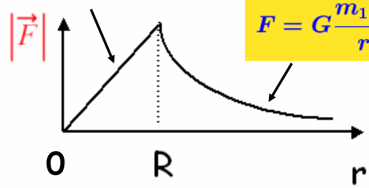
$$F = G\frac{m_1 m_2}{r^2}$$

R = 6,400 km

$a_g \approx g = 9.8\ m/s^2$

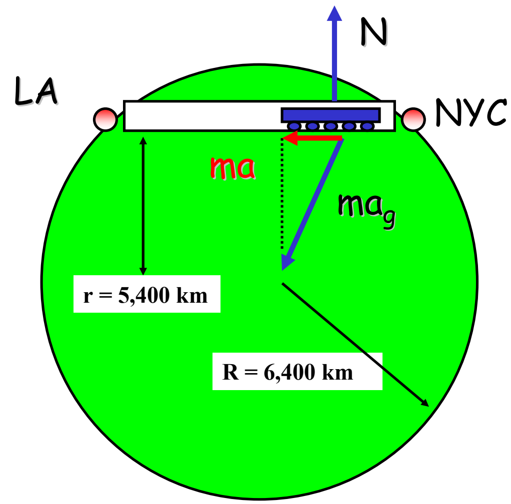


F ≈ K·r ≈ (G m_1m_2 / R^3) · r



$$F = G\frac{m_1 m_2}{r^2}$$

Gravitational Train "Alice"



(a) Will this train move at all?

(b) What is the total force on the train in the middle of the tunnel ?

(c) Where will the train stop? LA or in the middle of the tunnel

(c) What is the speed of the train in the middle of the tunnel ?

Answers:

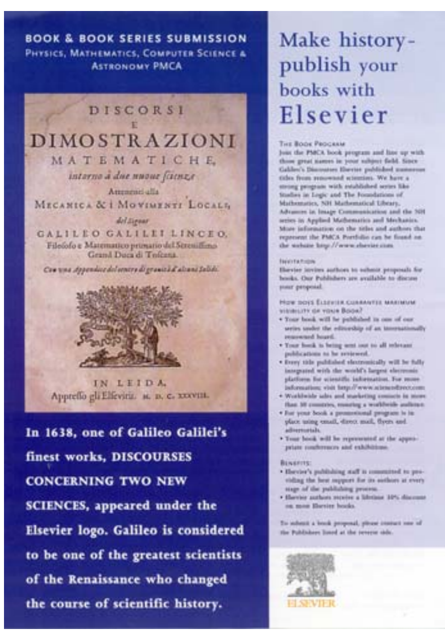
(a) v ≈ 6000 m/s

(b) v ≈ 600 m/s

(c) v ≈ 60 m/s

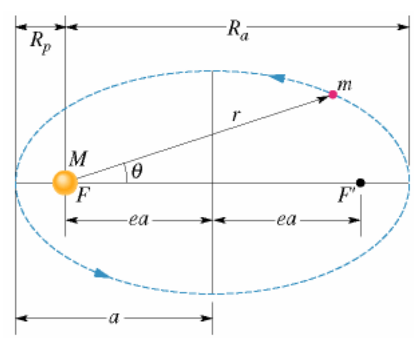
(d) v ≈ 6 m/s

(e) v ≈ 0



Planets and Satellites: Kepler's Laws

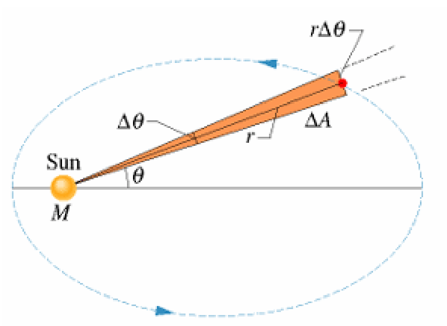
THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.



A planet of mass *m* moving in an elliptical orbit around the Sun. The Sun, of mass *M*, is at one focus *F* of the ellipse. The other focus is *F*′, which is located in empty space. Each focus is a distance *ea* from the ellipse's center, with *e* being the eccentricity of the ellipse. The semimajor axis *a* of the ellipse, the **perihelion** (nearest the Sun) distance R_p, and the **aphelion** (farthest from the Sun) distance R_a are also shown.

Planets and Satellites: Kepler's Laws

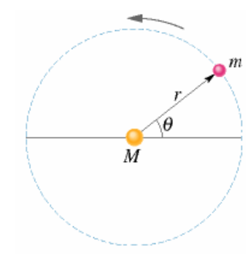
THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal times; that is, the rate dA/dt at which it sweeps out area A is constant.



In time Δt, the line r connecting the planet to the Sun (of mass M) sweeps through an angle $\Delta\theta$, sweeping out an area ΔA (shaded).

Planets and Satellites: Kepler's Laws

THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.



$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

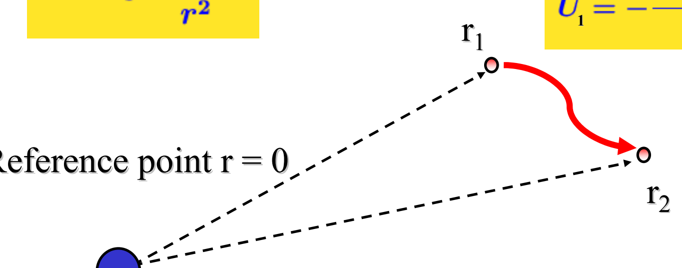
Potential Energy :

ΔU between r_1 and r_2 is the work done by the Gravitation Force during the move from r_1 to r_2 :

$$F = G\frac{m_1 m_2}{r^2}$$

$$U_1 = -\frac{GMm}{r_1}$$

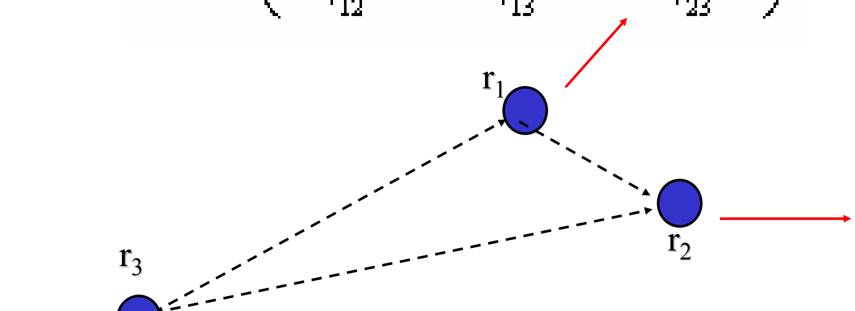
$$U_2 = -\frac{GMm}{r_2}$$



U = 0; at infinity ! (far away)

Potential Energy of a System :

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$



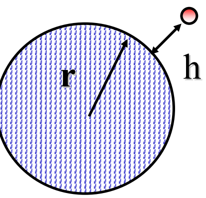
U = 0; at infinity ! (far away)

Potential Energy :

Is it $\Delta U = mgh$ or $U = -\frac{GMm}{r}$, anyway ?

It is the same thing, just different zero levels.

$U = -\frac{GMm}{r}$ is more universal (always correct)



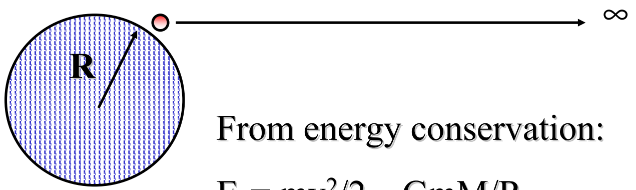
$\Delta U = mgh$ works for $h \ll r$,
zero at the Earth surface

$U = -\frac{GMm}{r}$ always works, zero at ∞

$\Delta U = GMm/r - GMm/(r+h) = GMm(r+h-r)/(r\cdot(r+h))$
$= mh\cdot[GM/(r\cdot(r+h))] \approx mgh$

Escape Speed:

$F = G\frac{m_1 m_2}{r^2}$



From energy conservation:

$E_1 = mv^2/2 - GmM/R$

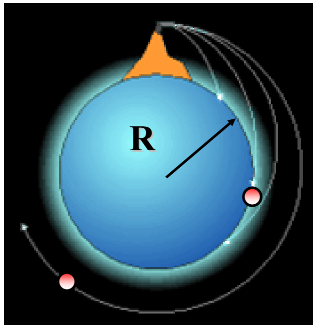
$E_2 = 0$ (velocity is small)

$v^2 = 2GM/R = 2gR$

$v = (2GM/R)^{1/2} \approx 11{,}200$ m/s

First Satellite Speed:

$F = G\frac{m_1 m_2}{r^2}$



"Newton's cannon"

in 1687 in "*Principia Mathematica*"

$v_{satellite} \approx (gR)^{1/2}$

$v_{satellite} \approx 8{,}000$ m/s

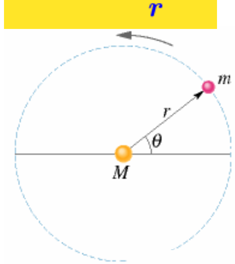
$g \approx 8.70$ m/s^2

An object in orbit is weightless not because 'it is beyond the earth's gravity' but because it is in 'free-fall' - just like a skydiver.

Satellites: Orbits and Energy

Potential Energy

$U = -\frac{GMm}{r}$

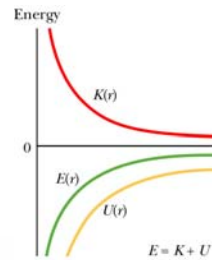


$\frac{GMm}{r^2} = m\frac{v^2}{r}$

Kinetic Energy for the orbital motion

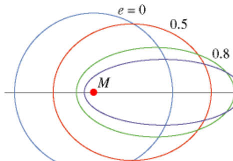
$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

$K = -\frac{U}{2}$ (circular orbit)



Total Energy: $E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ (circular orbi

$E = -\frac{GMm}{2a}$ (elliptical orbit)
semimajor axis a



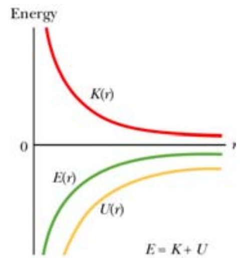
Potential and Kinetic Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

Kinetic Energy for the orbital motion

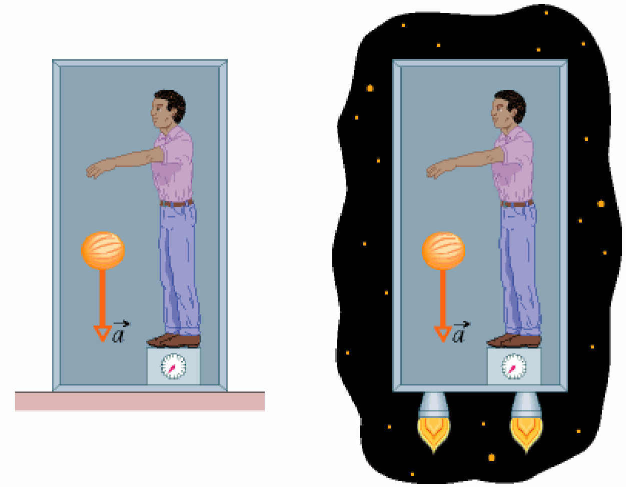
$$F = G\frac{Mm}{r^2} = m\frac{v^2}{r} \quad \Rightarrow \quad K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$



Total Energy

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Einstein and Gravitation



(a) (b)