Quantum Analogs
Chapter 5
Advisor Manual

Short Description
of
Thermodynamic Experiments

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Short description of the thermodynamics experiments

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For the last five years, I have been using the spherical resonator of the Quantum Analogs apparatus in my introductory lab at the University of Kassel to make a precise measurement of the gas constant R, to measure the adiabatic exponent of nitrogen, and to measure the temperature dependence of sound velocity. Unfortunately, finishing the English manual for these experiments was delayed because so much time went into the quantum mechanics manual. The chapter in the manual on thermodynamics is on my task list for the next months. In the following, I think you will find the information you need to do the experiments. A more elegant version is in the works.

The sound velocity c in gases depends on the molar mass M, the adiabatic coefficient, γ, the temperature T, and the gas-constant R.

$$c = \sqrt{\gamma \frac{RT}{M}}$$

Any one of these quantities can be measured when all others are known. Actually, the literature value of the gas constant is measured with this type of experiment to an accuracy of 10^{-6}. (M.R.Moldover et.al. Phys. Rev. Lett. 60, 249 (1988) This paper also includes a detailed discussion of experimental errors.). In our lab, we get a relative accuracy of 10^{-4}. A typical value is R = 8.314 +/- 0.002 J/(mol K).

In our experiments, we take the molar mass of a natural argon isotope mixture as a known value and measure the temperature with a platinum thermometer (Pt-100 fixed to the aluminum of the sphere) with an accuracy of 0.1 Kelvin.

Argon can be taken as an ideal gas with known adiabatic exponent, so that we can calculate the gas constant R directly from the sound velocity c.

The sound velocity in a sphere can be measured with high accuracy in particular for the radial modes (s-wave here called n=0). For these modes, the resonance frequency f depends only on the radius r of the sphere, and the sound velocity c.

$$f_{n,s} = c\, z_{n,s} / (2\pi r) \tag{8}$$

The proportionality factor z can be calculated analytically for all resonances. The values are given in the table on the following page.

Eigenfunctions of the general Helmholtz differential equation

n	***s***	$z_{n,s}$
1	1	2.081576
2	1	3.342094
0	**2**	**4.493409**
3	1	4.514100
4	1	5.646704
1	2	5.940370
5	1	6.756456
2	2	7.289932
0	**3**	**7.725252**
6	1	7.851978
3	2	8.538755
7	1	8.934839
1	3	9.205840
4	2	9.840446
8	1	10.010371
2	3	10.613855
0	**4**	**10.904122**
5	2	11.070207
9	1	11.079418

For the precise measurement of the gas constant, we use, primarily, the $n = 0$, $s = 2$ resonance. Note that the use of n and s here differs from the labeling of stats in quantum mechanics. This resonance corresponds to the 2s-state in terms of the hydrogen eigenfunctions.

The aluminum spheres are machined to high accuracy on a computer controlled lathe so that the radius is better known than it can be measured by caliper.

Precise measurements of the gas constant:
Students fill the sphere with argon gas and measure the resonance frequencies and temperature. From that result, together with the other known quantities, students calculate the gas constant.

Adiabatic exponent:
Students fill the sphere with nitrogen gas and measure the adiabatic exponent, taking the literature value of gas constant. We find $\gamma = 1.40$, fitting well to the theoretical value of 7/5.

Temperature dependence of sound velocity:
Students fill the sphere with air and measure the sound velocity with high accuracy. Using a hair dryer, students warm the aluminum cylinder by a few degrees and measure sound velocity as function of temperature (temperature range 20°C - 40°C). Due to the high accuracy of the measurement the temperature dependence can be measured even in this small temperature interval.