

Lab 10 : Fraunhofer Diffraction

I. Objective :

To observe the diffraction pattern that arises from plane waves incident upon a single slit, and to obtain a measurement of the slitwidth based upon data collected.

II. Theory :

The simplest treatment of Fraunhofer diffraction appeals to Huygens' principle for explanation. A plane wave is incident upon a long, narrow slit and the instant the wave occupies the plane of the slit secondary wavelengths are emitted. These secondary waves are spherical. There are effectively an infinite number of sources across the aperture. For a particular observation point, each source has a different optical path which introduces a phase relationship between the waves that are emitted from the aperture. The resultant sum becomes an integral over the plane of the aperture and a simple relationship between the "angle of diffraction" and the intensity of the light in the observation plane can be developed. The details are worked out in many texts, most notably Hecht chapter 10.

In the observation plane we may write:

$$I = I_0 \frac{\sin^2(b)}{b^2} \quad \text{Eq.(1)}$$

where $b = \frac{2p}{l} \frac{a}{2} \sin(q)$, a is the slitwidth and θ is the angle between the normal to the aperture and the distance to the observation point. When $\beta = n\pi$ where n is an integer, minima occur. Then

$\sin(q) = \frac{l}{a}$ is the condition for the first minima to occur. We can exploit this relationship in order to obtain an expression for the slitwidth if we know the width of the central maximum. First, equating the sine to its argument and calling Δx_0 the width of the central maximum (see Figure 2), we have:

$$\Delta x_0 = \frac{2lz}{a} \quad \text{Eq. (2)}$$

This can be solved for the width of the slit and compared to a value obtained by other means.

III. Procedure :

- 1) Set up the equipment as shown in Figure 1. Use two translation stages (motion parallel to each other) to increase the range of motion from 1 to 2 inches. The top stage should be computer-controllable. Make sure to leave about 1.5 meters in between the slit and the photodetector. Once again, make sure the beam is parallel to the tabletop. As a final check make sure the beam enters the photodetector at the same height it leaves the second mirror.
- 2) Make sure the detector is linear over the range that you plan to use it. Appropriate choice of a neutral density filter will insure this.
- 3) Before measurements are taken try several different slitwidths. Using a piece of paper observe the central maximum and several orders. Make sure to select a pattern whose minima are not too small with respect to the size of the detector (a good pattern would be PASCO 9165A Pattern C).

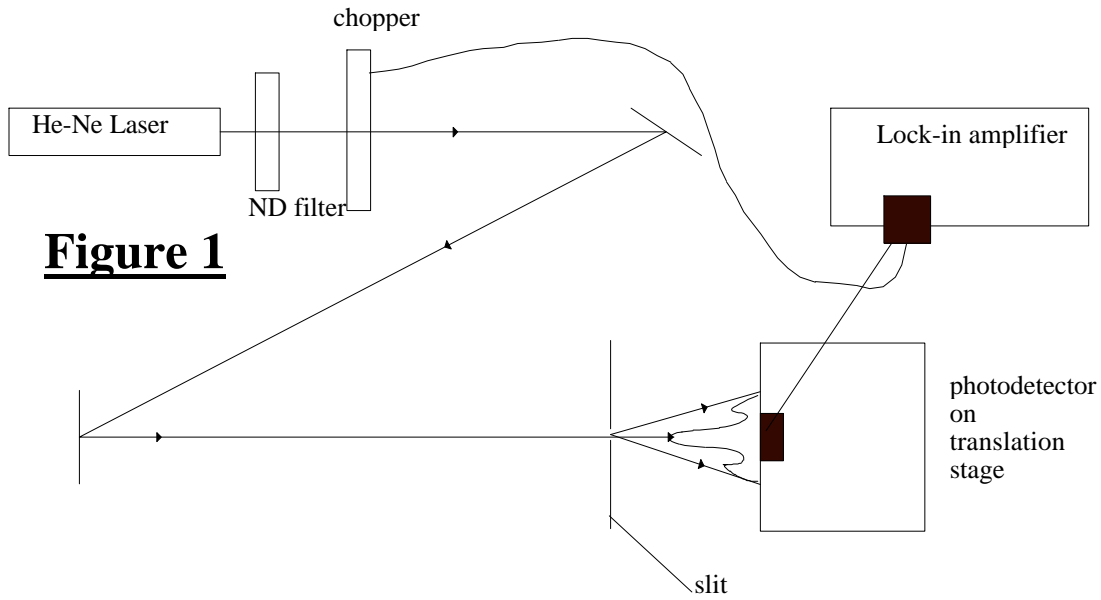
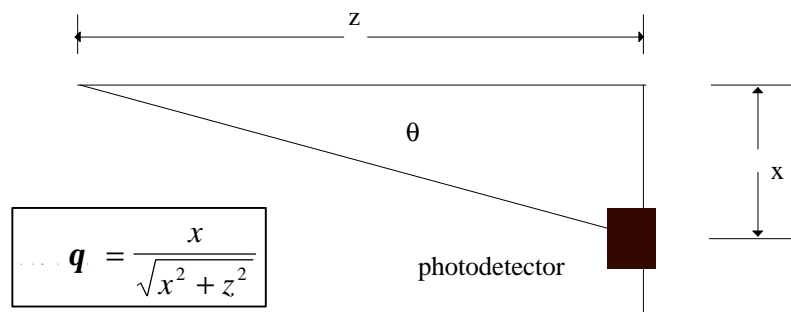


Figure 1

- 4) Use PASCO 9165A Pattern C. Align the detector to the left of the central maximum. Make sure that as you scan across the diffraction pattern you will include three orders of diffraction on at least one side. Take readings for the intensity as a function of distance (x in Figure 2). A reasonable number of points is roughly 1 every 250-500 μm . This will yield about 100-50 points per 25mm. Use the LABVIEW program provided in order to take the data. Note that 36 steps of the stepping motor correspond to a linear translation of 50 μm . If one micrometer driven translation stage does not have enough range to measure across 3 orders of diffraction, translate the second stage to continue the scan. Make sure to overlap the two ranges so you can merge the data set! If the signal on the lock-in becomes too small for the higher order diffraction peaks, make sure to change to a more sensitive scale.
- 5) Repeat the experiment over the same range of translation for pattern B and D.
- 6) Switch the pattern to PASCO 9165C Pattern D and repeat the experiment over the same range of translation. For this pattern, the number of slits is 5.



$$q = \frac{x}{\sqrt{x^2 + z^2}}$$

Figure 2

IV. Discussion :

- 1) Derive Equation 2.

- 1) Use equation 2 to calculate a value for the slitwidth of pattern PASCO 9165A Pattern C. Compare to the value provided by the lab instructor.
- 2) What error is introduced into the calculation of the slitwidth by the small angle approximation? ($\sin(\theta) = \theta$) Using an exact expression for the sine, recalculate the slitwidth. Is the error significant? Why or Why not?
- 3) Plot your data and the theoretical curve for each pattern of PASCO 9165A on the same plot (normalize curves). The y-axis should be normalized intensity. The x-axis should be distance. For the theoretical curve, adjust the theoretical width of the slit until you get a good fit between theory and experiment. Compare your data to the theoretical expression for the intensity as a function of position x . Explain any discrepancies between theory and experiment.
- 4) Based on your results for Discussion Question 3, make a table of the experimentally inferred slitwidths and their given value for each of the three patterns.
- 5) Plot your normalize experimental curves for pattern B, C, and D on the same plot. What happens to the diffraction pattern if a smaller slit is used? A larger slit?
- 6) Does the measured intensity go to zero at the diffraction minimum? Why or Why not? Theoretically, do you expect the value to be zero? HINT: Does the size of the detector matter?
- 7) Plot normalized experimental data for PASCO 9165A pattern B and PASCO 9165C pattern D on the same plot. The difference between these slit patterns is the number of slits (slit width is the same). Compare the two curves. Qualitatively, what effect does increasing the number of slits have on the diffraction pattern?