Lab 2 : Index of Refraction, Total Internal Reflection, and Critical Angle

I. Objective :

To observe the importance of the angle of refraction and the critical angle directly in the laboratory.

II. Theory:

A. Part 1

Examine figure 1. Notice that the deviation of the beam, d, can be written in terms of the incident angle only. To do this we need to make use of Snell's Law and a simple trigonometric identity.

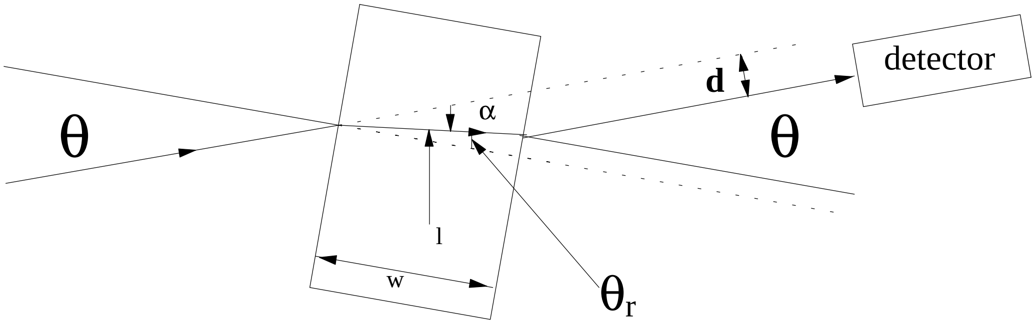


Figure 1

With the help of the following, an expression for the index of refraction as a function of beam deviation can be developed:

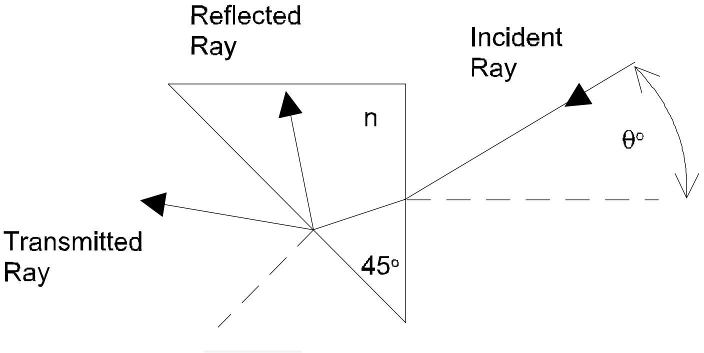
1) $\theta = \alpha + \theta_r$
2) $n(\sin(\theta_r)) = \sin(\theta)$
3) $d = l(\sin(\alpha))$
4) $w = l(\cos(\theta_r))$
5) $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

Then we get:

$$n^2 = \sin^2(\theta) + \left(\frac{\sin(\theta)\cos(\theta)}{\sin(\theta) - \frac{d}{w}} \right)^2 \quad \text{Eq. (1)}$$

B. Part 2

Total internal reflection can only occur when a beam of light is traveling from an optically denser medium to a less dense medium. In the case of the right angle prism the geometry is set up so that a beam that enters one of the short sides strikes the diagonal at 45° to its normal. In total internal reflection there is no refracted ray, so from Snell's law we may calculate the "critical" angle, $\sin(\theta_c) = \frac{n_1}{n_2}$ where n_2 is the index of refraction of the denser material. In our case the less dense material is air, so $n_1 = 1$.



For total internal reflection at the air/glass interface, (hypotenuse of the triangular prism), the transmitted of figure 2 ray vanishes. By similar geometrical arguments as in part 1 it can be shown:

$$n^2 = \sin^2(\theta_o) + (\sqrt{2} + \sin(\theta_o))^2 \qquad \text{Eq. (2)}$$

where θ_o is the critical angle defined in figure 2.

III. Procedure:

A. Part 1

1) Set up the experiment as shown in Figure 4. Make sure the laser beam is parallel to the tabletop and along one of the lines of screw holes after the second mirror (m2). To ensure this, measure the height of the beam after the second steering (m2) mirror making sure that the beam enters the photodetector at this height.
2) Make sure that the detector is linear. To do this, place a neutral density filter in front of the beam before it enters the chopper. Then take a 0.3 neutral density filter and place it in front of the detector. If the detector is linear, the lock-in's reading will be exactly ½ of what it was. Try different neutral density filters until you feel that you have found the best combination.
3) Find the position of the undeflected beam by adjusting the position of the detector with the x-y-z translation stage and locating the peak reading on the lock-in. Now place the glass sample on the rotation stage. Make sure to retroreflect the beam back upon the mirror to ensure normal incidence.
4) Record the deflection of the beam as a function of the angle of incidence. Make sure that you are recording the total deviation, i.e. the reading - the zero point. Obtain about 10 different data points between 0° and 90°.
5) Calculate the index of refraction for each data point. How does these compare with the value for crown glass, n = 1.56?

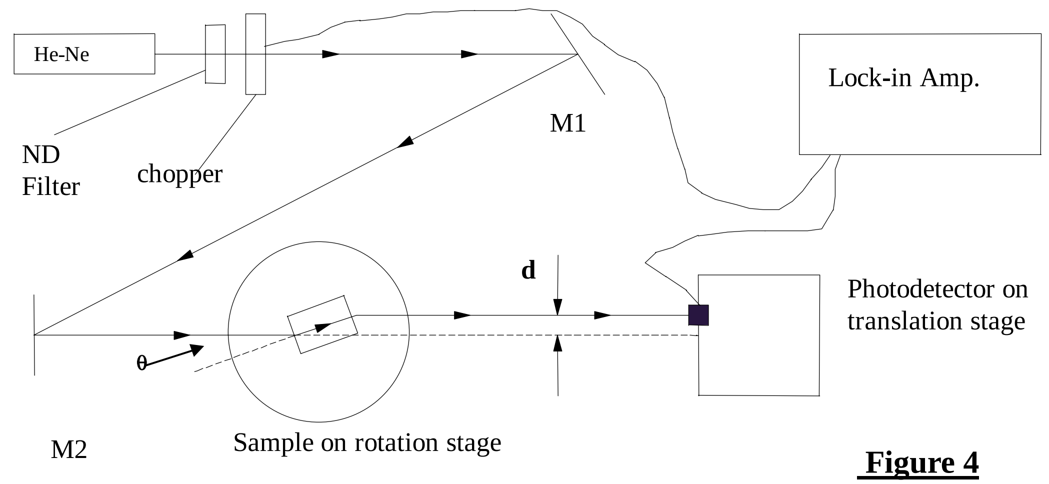


Figure 4

B Part 2

1) Leave everything in place except the piece of glass. Mount a right angle prism on the rotation stage as illustrated in Fig. 2. Be careful with the reflected beam, block it to avoid hitting a classmate.
2) Adjust the angle of incidence so that the transmitted ray of Fig. 2 just disappears. This angle is the critical angle. Use this value and Equation 2 to calculate the index of refraction. Compare this with the value for crown glass.

IV Discussion :

1) Derive Equation 1, the index of refraction squared as a function of incident angle.
2) Using Equation 1, obtain an expression for d as a function of angle of incidence. Plot it (with n =1.56 and d = thickness of your plate from $\theta = 0$ to 90^o) and describe it's features. Also plot on the same graph your experimental data points. How sensitive is d to the angle of incidence? How does the physical size of the photo detector element compare with d? Over what range of angles would you expect to get the best experimental values for n? Why? Which of your experimental values from Part 1-5 is most accurate?
3) Derive Equation 2.
4) Describe a more accurate way to do Part 2(B) in order to determine the index of refraction. Think of how the angle may be determined by measuring distances.