

03/17/2017

Suppose $x = (1, 2, 3)$ and $y = (2, 10, 6)$

- Q1. What is Euclidean length squared of x ?
- Q2. What is the value of $x \cdot y$?
- Q3. What is the dot product between x and y (xTy)?
- Q4. What is the Euclidean length squared of $x-y$?

- A1. $1^2 + 2^2 + 3^2 = 14$
- A2. $(-1, -8, -3)$
- A3. $1 \cdot 2 + 2 \cdot 10 + 3 \cdot 6 = 40$
- A4. $x-y = (-1, -8, -3)$. $\|x\|^2 = 1^2 + 2^2 + 3^2$
 $\|x\| = \sqrt{\sum_i x_i^2}$

12/14/2016

$u = (u_1, u_2, u_3)$
 $v = (v_1, v_2, v_3)$
 $\|u - v\|^2 = \sum_i (u_i - v_i)^2$

A two dimensional matrix is a set of numbers organized in a two dimensional grid. For example M shown below is a 2×3 matrix. The first number is the number of rows and second is number of columns.

3	4	10
2	20	19

We can multiply a $n \times m$ matrix by a $m \times 1$ vector as shown below. The answer will have dimensionality $n \times 1$. Below we have a 2×3 matrix multiplying a 3×1 vector and the answer will have dimensionality 2×1 .

$$\begin{pmatrix} 3 & 4 & 10 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \cdot 2 + 4 \cdot 4 + 10 \cdot 5 \\ 2 \cdot 2 + 20 \cdot 4 + 19 \cdot 5 \end{pmatrix} = \begin{pmatrix} 72 \\ 179 \end{pmatrix}$$

The transpose of a matrix is such that each row becomes a column. So if M is $n \times m$ then M^T is of dimension $m \times n$. So the transpose of the matrix below

(3, 4, 10)

(2, 20, 19)

is given by

(3, 2)

(4, 20)

(19, 10)

The transpose of the matrix

(1, 2, 4, 1)

(1, 4, 6, 5)

(4, 5, 1, 5)

is

(1, 1, 4)

(2, 4, 5)

(4, 6, 1)

(1, 5, 5)

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$w = (1, 2)$

$x = (x_0, x_1)$

$w_0 = 4$

$w^T x = (1, 2) * (x_0) = 1 * x_0 + 2 * x_1 = x_0 + 2x_1$

(x_1)

Suppose the mean vector $m = (m_1, m_2, m_3)$

Suppose $x = (x_1, x_2, x_3)$

Then we can write the variance as $\|x - m\|^2 = \sum_i (x_i - m_i)^2$

Suppose we have four points in class 0

1 2

2 1

2 2

and four points in class 1

10 10

10 11

11 10

11 11

What are the means and variances of the two classes? For variance we need just the variance of each column.

$$m_0 = (1.5, 1.5)$$

$$m_1 = (10.5, 10.5)$$

$$v_0 = (.25, .25)$$

$$v_1 = (.25, .25)$$

These points below are in class 0.

-2 1

-1 1

0 1

1 1

2 2

And these are in class 1.

2 -2

1 -1

1 0

1 1

1 2

$$m_0 = (0, 1.2)$$

$$m_1 = (1.2, 0)$$

$$v_0 = (2, 0.16)$$

$$v1 = (0.16, 2)$$

Test data are

$$(-3, 1)$$

$$(1, 3)$$

To classify (-3, 1) calculate the weighted distance to each mean

$$\text{Dist to } m_0 = (-3-0)^2/2 + (1-1.2)^2/0.16 = 9/2 + .04/0.16 = 4.5 + 1/4$$

$$\text{Dist to } m_1 = (-3-1.2)^2/0.16 + (1-0)^2/2 = 16/0.16 + 1/2 = 100 + 1/2$$

Clearly distance to m_0 is smaller but we can also reach this conclusion by ignoring the dimension with larger variance.

To classify (1, 3) calculate the weighted distance to each mean

$$\text{Dist to } m_0 = (1-0)^2/2 + (3-1.2)^2/0.16 = 1/2 + 4/0.16 = .5 + 25$$

$$\text{Dist to } m_1 = (1-1.2)^2/0.16 + (3-0)^2/2 = .04/0.16 + 9/2 = 1/4 + 4.5$$

Since distance to m_1 is smaller we classify (1,3) as class 1. I can reach the same conclusion by ignoring the column with higher variance.