Work and energy

1. A particle moves from a point $\vec{r}_1 = 2\hat{i} + 2\hat{j}$ (in meters) to a point $\vec{r}_2 = 6\hat{i} - 4\hat{j}$. Find the work $W$ done by a constant force $\vec{F} = 4\hat{i} + 2\hat{j}$ (in newtons).

2. Find the angle between the force and the displacement

3. If the mass of the particle in the previous problem is $m = 2\,kg$ and the initial speed at $\vec{r}_1$ is $5\,m/s$, find its speed at $\vec{r}_2$ (assume no work by other forces).

4. A $3\,kg$ block is moved along a flat horizontal surface by a constant force $F = 30\,N$ which makes $30^o$ with horizontal. The speed changes from $10\,cm/s$ to $50\,cm/s$ after a $2\,m$ displacement. Find the work done by friction.

5. A skier slides down from a hill which is $30\,m$ high and then, without losing speed, up a hill which is $10\,m$ high. What is his final speed? (a) Ignore friction. (b) Assume a small average friction force of $40\,N$ and the combined length of the slopes $L = 200\,m$. The mass of the skier is $m = 80\,kg$.

6. A mass $m = 1\,kg$ is attached to a string $L = 5\,m$ and freely revolves under the force of gravity. The speed at the top is $v = 10\,m/s$

   (a) draw clear force diagrams for the two vertical and the horizontal orientations of the string

   (b) find the speed $V$ at the lowest point

(c) find the tension $T$ ("apparent weight") at the lowest point

7. Consider a spring with $k = 100\,N/m$

   (a) find the extension length if an $m = 1.5\,kg$ mass is attached to the spring

   (b) what is the work done by the spring if it is stretched by extra $10\,cm$ starting from $x_0 = 5\,cm$.

   (c) find the elastic potential energy stored in the spring at the maximum extension

   (d) the spring with mass $m$ attached is now released and starts oscillating (ignore gravity). Find the maximum speed $V$.

   (e) $(*)$ find the speed $v$ when $x = 10\,cm$.