# SUPPLEMENT TO "FURTHER RESULTS ON CONTROLLING THE FALSE DISCOVERY PROPORTION" 

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S.1. Additional Figures. Due to space constraints, Figures S.1-S.7 we discussed in the simulation section of the paper are appended here.
S.2. Findings of Additional Simulations on $\gamma$-kFDP. Here we present the findings of simulation studies examining the effect of $k$ on a $\gamma$-kFDP controlling procedure and providing an insight into the choice of $k$ under different types and strengths of dependence.

We generated dependent normal random variables $N\left(\mu_{i}, 1\right), i=1, \ldots, n$, with correlation matrix $\Sigma$, where $\pi_{0} n$ of the $\mu_{i}$ 's equal to 0 and the rest equal to 2 . The following three types of dependence structure were considered: (1) $\Sigma=(1-\rho) I_{n}+\rho 1_{n} 1_{n}^{\prime}$, in case of equi-correlated dependence, (ii) $\Sigma=$ $\left(\rho^{|i-j|}\right)$, with $i, j=1, \ldots, n$, in case of $\operatorname{AR}(1)$ dependence, and (iii) $\Sigma=$ $I_{\frac{n}{s}} \otimes\left[(1-\rho) I_{s}+\rho 1_{s} 1_{s}^{\prime}\right]$, with $s$ satisfying $s \times g=n$ for some positive integer $g$ less than or equal to $n$, in case of block dependence. In each scenario, we fixed $n=5000$ and $\pi_{0}=0.5$ and chose each of the values in $\{0,0.3,0.6,0.9\}$ for $\rho$ before applying the $\gamma$ - kFDP controlling procedure in Theorem 3.3 at level $\alpha=0.05$ with $\gamma=0.1$ to test $\mu_{i}=0$ vs. $\mu_{i}>0$, simultaneously for $i=$ $1, \ldots, n$, and determining the average power of the procedure for each of the $k / n$ values in $\{0.0004,0.0008,0.0012, \ldots, 0.05\}$. Specifically, when applying the procedure in Theorem 3.3, we chose the $\alpha_{i}^{\prime}$ in that Theorem to be $\alpha_{i}^{\prime}=$ $\{\lfloor\gamma i\rfloor+1\} \alpha /\{n+\lfloor\gamma i\rfloor+1-i\}$.

Our findings are presented in Figure S. 8 and Table S.1. The figure seems to support our intuition (mentioned in Remark 2.1), although more clearly under independence, that the difference between $\gamma$-kFDP and $\gamma$-FDP and the stipulated power gain in using a $\gamma$-kFDP procedure over the corresponding $\gamma$-FDP procedure are not realized until $k / n$ reaches a certain critical


Fig S.1. Simulated values of $\gamma-F D P$ and average powers of the original Lehmann-Romano stepdown procedure ( $L R S D$ ) and its stepup analogue ( $L R S U$ ) under block dependence for $n=100, s=20$ and $\alpha=0.05$.


Fig S.2. As in Figure S.1, simulated values of $\gamma-F D P$ and average powers of the $L R S D$ and LR SU procedures under block dependence for $s=50$.


Fig S.3. As in Figure S.1, simulated values of $\gamma-F D P$ and average powers of the $L R S D$ and $L R S U$ procedures under $A R(1)$ dependence.


Fig S.4. Simulated values of $\gamma-k F D P$ and average powers of BH-type stepdown and stepup (BH SD and BH SU) and GBS-type stepdown and stepup (GBS SD and GBS SU) in Theorem 3.2 and 3.3, all developed under common correlation, for $n=100$ and $\alpha=0.05$.


Fig S.5. As in Figure S.4, simulated values of $\gamma-k F D P$ and average powers of the BH SD and $S U$, and $G B S S D$ and $S U$ procedures under block dependence for $s=20$.


Fig S.6. As in Figure S.4, simulated values of $\gamma-k F D P$ and average powers of the BH SD and $S U$, and $G B S S D$ and $S U$ procedures under block dependence for $s=50$.


Fig S.7. As in Figure S.4, simulated values of $\gamma-k F D P$ and average powers of the $B H S D$ and $S U$, and $G B S S D$ and $S U$ procedures under $A R(1)$ dependence.

Table S. 1
The smallest $k$ at which the power starts increasing

| $\rho$ | 0 | 0.3 | 0.6 | 0.9 |
| :---: | :---: | :---: | :---: | :---: |
| Equicorrelated | 70 | 10 | 10 | 10 |
| AR(1) | 48 | 48 | 48 | 10 |
| Block | 48 | 10 | 6 | 6 |

value, with the power gain steadily increasing with $k / n$ beyond that point. This critical value is seen to decrease with increasing dependence, which in a way justifies the rationale behind our proposal of using $\gamma$-kFDP instead of $\gamma$-FDP, for some judiciously chosen $k / n$, under high dependence.

Table S. 1 provides a more precise idea about this critical value of $k / n$ under different types and varying strengths of dependence. It is interesting to note from this table that the $\gamma$-kFDP control seems most effective under block dependence (often referred to as clumpy dependence and assumed in many scientific investigations such as in microarray experiments, which we mentioned in the introduction), among the different types of dependence considered.


Fig S.8. Average power of the stepup procedure in Theorem 3.3 with increasing $k / n$ values

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