Positive Solutions for Classes of $n \times n$ Nonlinear Positone Elliptic Systems

Ratnasingham Shivaji

Department of Mathematics and Statistics
Center for Computational Sciences
Mississippi State University

Abstract

We study the existence and multiplicity of positive solutions to $n \times n$ systems of the form

$$\begin{aligned} -\Delta u_1 &= \lambda f_1(u_2) && \text{in } \Omega \\ -\Delta u_2 &= \lambda f_2(u_3) && \text{in } \Omega \\ \vdots &= \vdots \\ -\Delta u_{n-1} &= \lambda f_{n-1}(u_n) && \text{in } \Omega \\ -\Delta u_n &= \lambda f_n(u_1) && \text{in } \Omega \\ u_1 = u_2 &= \cdots = u_n = 0 && \text{on } \partial\Omega. \end{aligned}$$

Here Δ is the Laplacian operator, λ is a non-negative parameter and Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. The nonlinearities $f_i \in C^1([0,\infty)),\ i \in \{1,2,\cdots,n\}$ are strictly increasing functions such that $f_i(0) \geq 0,\ i \in \{1,\cdots,l-1,l+1,\cdots,n\}$ and $f_l(0) > 0$ for some $l \in \{1,\cdots,n\}$ (positone systems), and satisfy a combined sublinear condition at ∞. We establish our results by the method of sub and supersolutions. We also discuss our results in the case when one of the nonlinearities, say f_k, is given by $f_k(z) = e^{\frac{\alpha z}{\alpha+z}};\ \alpha > 0$ which arises in the theory of combustion.