

$$\frac{\partial A}{\partial t} = -k_1 A - k_2 AB + D_A \frac{\partial^2 A}{\partial s^2}$$

$$\frac{\partial B}{\partial t} = k_1 A + k_2 AB - k_3 BC + D_B \frac{\partial^2 B}{\partial s^2}$$

$$\frac{\partial C}{\partial t} = k_3 BC - k_4 C + D_C \frac{\partial^2 C}{\partial s^2}$$

$$\frac{\partial D}{\partial t} = k_4 C + D_D \frac{\partial^2 D}{\partial s^2}$$

uniform, steady state:

$$-k_1 A - k_2 AB = 0$$

$$k_1 A + k_2 AB - k_3 BC = 0$$

$$k_3 BC - k_4 C = 0$$

$$k_4 C = 0$$

$$-k_1 A - k_2 AB = 0$$

$$A(k_1 + k_2 B) = 0$$

$$C = 0, \quad A = A_0, \quad B = B_0$$

$$D = D_0$$

$$D_0 \ll D_A, D_C, D_D$$

scale A w/ A_0 time with k_1^{-1}
 B w/ B_0 space with $\sqrt{D_B/k_1}$
 C w/ C_0
 D w/ D_0

$$A_0 k_1 \frac{\partial A}{\partial t} = -k_1 A_0 A - k_2 A_0 B_0 AB + \frac{D_A A_0}{D_B^0/k_1} \frac{\partial^2 A}{\partial r^2}$$

$$B_0 k_1 \frac{\partial B}{\partial t} = k_1 A_0 A + k_2 A_0 B_0 AB - k_3 B_0 C_0 BC + \frac{D_B B_0}{D_B^0/k_1} \frac{\partial^2 B}{\partial r^2}$$

$$C_0 \cdot k_1 \frac{\partial C}{\partial t} = k_3 B_0 C_0 BC - k_4 C_0 C + \frac{D_C C_0}{D_B^0/k_1} \frac{\partial^2 C}{\partial r^2}$$

$$D_0 \cdot k_1 \frac{\partial D}{\partial t} = k_4 C_0 C + \frac{D_D D_0}{D_B^0/k_1} \frac{\partial^2 D}{\partial r^2}$$

$$\frac{\partial B}{\partial t} = k_1 A + k_2 AB - k_3 BC + D_B \frac{\partial^2 B}{\partial s^2}$$

$$\frac{\partial C}{\partial t} = k_3 BC - k_4 C + D_C \frac{\partial^2 C}{\partial s^2}$$

$$A = A_0 = \text{const.}$$

$$\text{Steady uniform soln.} \Rightarrow k_1 A_0 + k_2 A_0 B_0 - k_3 B_0 C_0 = 0$$

$$k_3 C_0 \left(B_0 - \frac{k_4}{k_3} \right) = 0$$

$$B_0 = \frac{k_4}{k_3} \quad \frac{k_1 A_0 + k_2 A_0 \frac{k_4}{k_3}}{k_4} = C_0$$

scale time by $\frac{1}{k_4}$ length distance by $\sqrt{D_B/k_4}$

$$\frac{k_4}{k_3} \frac{B_0 k_4}{1} \frac{\partial B}{\partial \tau} = k_1 A_0 + k_2 A_0 \frac{k_4}{k_3} B - k_3 \frac{k_4}{k_3} \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_4} BC + \frac{D_B k_4/k_3}{D_B/k_4} \frac{\partial^2 B}{\partial r^2}$$

$$\frac{\partial B}{\partial \tau} = \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0 B}{k_4^2/k_3} - \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_4^2/k_3} BC + \frac{\partial^2 B}{\partial r^2}$$

$$\alpha \equiv \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_4^2/k_3}$$

$$\beta \equiv \alpha - \frac{k_1 A_0}{k_4^2/k_3}$$

$$\frac{\partial B}{\partial \tau} = \alpha - \beta + \beta B - \alpha BC + \frac{\partial^2 B}{\partial r^2}$$

$$\frac{\partial B}{\partial \tau} = \alpha (1 - BC) + \beta (B - 1) + \frac{\partial^2 B}{\partial r^2}$$

$$\frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{1/k_4} \frac{\partial C}{\partial t} = k_3 \cdot \frac{k_4}{k_3} \cdot \left(\frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_3} \right) B C - k_4 \cdot \left(\frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_3} \right) C$$

$$+ \frac{\left(k_1 A_0 + \frac{k_2 k_4}{k_3} A_0 \right) D_c}{D_B / k_4} \frac{\partial^2 C}{\partial r^2}$$

$$\frac{\partial C}{\partial t} = B C - C + \frac{D_c}{D_B} \frac{\partial^2 C}{\partial r^2}$$

$$\boxed{\frac{\partial C}{\partial t} = (B-1)C + \frac{D_c}{D_B} \frac{\partial^2 C}{\partial r^2}}$$

$$\boxed{\frac{\partial C}{\partial t} = (B-1)C + \gamma \frac{\partial^2 C}{\partial r^2}}$$

$$B = 1 + \delta B$$

$$C = 1 + \delta C$$

$$\frac{\partial}{\partial t} \delta B = \alpha \cdot (1 - (1 + \delta B)(1 + \delta C)) + \beta \delta B + \frac{\partial^2 \delta B}{\partial r^2}$$

$$= \alpha [1 - (1 + \delta B + \delta C + \delta B \delta C)] + \beta \delta B + \frac{\partial^2 \delta B}{\partial r^2}$$

$$\boxed{\frac{\partial}{\partial t} \delta B = \alpha [-\delta B - \delta C - \delta B \delta C] + \beta \delta B + \frac{\partial^2 \delta B}{\partial r^2}}$$

$$\frac{\partial}{\partial t} \delta C = \delta B (1 + \delta C) + \gamma \cdot \frac{\partial^2 \delta C}{\partial r^2}$$

Linearized system: $\frac{\partial \delta B}{\partial t} = -\alpha (\delta B + \delta C) + \beta \delta B + \frac{\partial^2 \delta B}{\partial r^2}$

$$\frac{\partial \delta C}{\partial t} = \delta B + \gamma \cdot \frac{\partial^2 \delta C}{\partial r^2}$$

$$\frac{\partial}{\partial t} = \lambda$$

$$\frac{\partial^2}{\partial r^2} = -k^2$$

$$\lambda sB = (-\alpha + \beta) sB - \alpha sC - k^2 sB$$

$$\lambda sC = sB - \gamma k^2 sC$$

$$\lambda \begin{bmatrix} sB \\ sC \end{bmatrix} = \begin{bmatrix} -\alpha + \beta - k^2 & -\alpha \\ 1 & -\gamma k^2 \end{bmatrix} \begin{bmatrix} sB \\ sC \end{bmatrix}$$

$$\det \begin{pmatrix} -\alpha + \beta - k^2 - \lambda & -\alpha \\ 1 & -\gamma k^2 - \lambda \end{pmatrix} = 0$$

$$(\lambda + k^2 + \alpha - \beta)(\lambda + \gamma k^2) + \alpha = 0$$

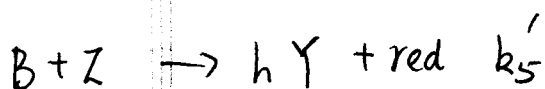
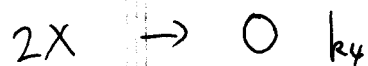
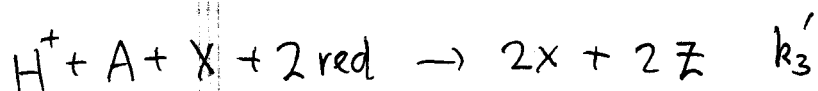
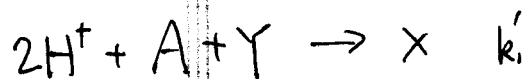
$$\lambda^2 + (k^2 + \alpha - \beta + \gamma k^2)\lambda - \beta \gamma k^2 = 0$$

$$\lambda = \frac{-[(1+\gamma)k^2 + \alpha - \beta] \pm \sqrt{[(1+\gamma)k^2 + \alpha - \beta]^2 + 4\beta\gamma k^2}}{2}$$

$$\gamma \rightarrow 0 \quad \lambda = 0, \quad \beta - \alpha - (1+\gamma)k^2 = \beta - \alpha - k^2$$

$A \equiv \text{BrO}_3^-$ $X = \text{HBrO}_2$ $Y = \text{Br}^-$
 $B = \text{mixture of malonic \& bromomalonic acids}$

red \Rightarrow reduced state
of the catalyst
 $Z \Rightarrow$ oxidized state



$$k_3' = k_3'' \cdot \frac{[\text{red}]}{[\text{red}] + c}$$

$$v_3 = k_3'' \frac{[X][A][\text{H}^+][\text{red}]}{[\text{red}] + c}$$

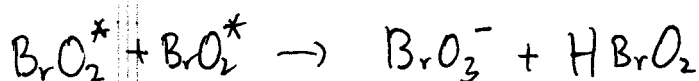
if $[\text{red}] \gg c$

H^+ & $A = \text{BrO}_3^-$ remain const.

$$[\text{red}] + [Z] \equiv C_0 = \text{const.}$$

if $[Z] \sim C_0$

$[\text{red}] \ll c$



$$\frac{\partial X}{\partial t} = k_1' \text{H}^2 A Y - k_2' \text{H} X Y + k_3' \text{H} A \text{R}^2 X - k_4 X^2 + D_x \nabla^2 X$$

$$\frac{\partial Z}{\partial t} = k_3' \text{H} A X \text{R}^2 - k_5' B Z + D_z \nabla^2 Z$$

$$k_3' X \cdot A \text{H}^+ \equiv k_3 X$$

$$k_3' A \text{H}^+ = k_3$$

$$k_3 = k_3'' \cdot \frac{[\text{red}]}{[\text{red}] + c} \cdot A \text{H}^+ = k_3$$

$$k_3' HA [red]^2$$

$$\Rightarrow k_3' [H^+][A][red]^2 = \frac{k_3'' [H^+][A][red]}{[red] + c}$$

$$k_3'' [H^+][A] = k_3$$

$$\frac{\partial Z}{\partial t} = k_3 \cdot x \cdot \frac{(C_0 - Z)}{C_0 - Z + c} - k_5 Z + D_z \nabla^2 Z$$

$$Z \rightarrow \frac{k_3^2}{k_4 k_5} \zeta \quad \frac{\frac{k_3^2}{k_4 k_5} \partial \zeta}{\frac{1}{k_5} \partial \tau} = k_3 \cdot \frac{k_3}{k_4} x \cdot \frac{C_0 - \frac{k_3^2}{k_4 k_5} \zeta}{C_0 - \frac{k_3^2}{k_4 k_5} \zeta + c} - k_5 \cdot \frac{k_3^2}{k_4 k_5} \zeta$$

$$t \rightarrow \tau / k_5$$

$$x \rightarrow \frac{k_3}{k_4} x \quad + D_z \frac{k_3^2}{k_4 k_5} \nabla^2 \zeta$$

$$\frac{\partial \zeta}{\partial \tau} = x \cdot \frac{(1 - m \zeta)}{(1 - m \zeta + \epsilon_1)} - \zeta + D_z' \nabla^2 \zeta$$

Similarly

$$\frac{\partial x}{\partial \tau} = \frac{1}{\epsilon} \left[\frac{(\zeta - x) \zeta}{\zeta + x} + \frac{x (1 - m \zeta)}{(1 - m \zeta + \epsilon_1)} - x^2 \right] + D_x' \nabla^2 x$$

$$\frac{\partial X}{\partial \tau} = \frac{1}{\varepsilon} \left[\frac{(g-x) f z}{g+x} + \frac{x(1-mz)}{\varepsilon_1 + 1 - mz} - x^2 \right] + D_x \Delta X$$

$$\frac{\partial z}{\partial \tau} = \frac{x \cdot (1-mz)}{\varepsilon_1 + 1 - mz} - z + D_z \Delta z$$

$$X_0 \text{ \& } z_0 \text{ s.t. } \begin{cases} z_0 \frac{g-X_0}{g+X_0} \cdot f + \frac{X_0(1-mz_0)}{\varepsilon_1 + 1 - mz_0} - X_0^2 = 0 \\ \frac{X_0(1-mz_0)}{\varepsilon_1 + 1 - mz_0} - z_0 = 0 \end{cases}$$

$$X \rightarrow X + \delta X$$

$$z \rightarrow z + \delta z$$

$$\frac{\partial \delta X}{\partial \tau} = \frac{1}{\varepsilon} \left[-\frac{f z_0}{g+X_0} - \frac{g-X_0}{(g+X_0)^2} f \cdot z_0 + \frac{1-mz_0}{\varepsilon_1 + 1 - mz_0} - 2X_0 \right] \delta X$$

$$+ \frac{1}{\varepsilon} \left[\frac{g-X_0}{g+X_0} \cdot f - \frac{mX_0}{\varepsilon_1 + 1 - mz_0} - \frac{X_0(1-mz_0)(-m)}{(\varepsilon_1 + 1 - mz_0)^2} \right] \delta z + D_x \Delta \delta X$$

$$\frac{\partial \delta z}{\partial \tau} = \frac{1-mz_0}{\varepsilon_1 + 1 - mz_0} \cdot \delta X + \left[\frac{X_0(-m)}{\varepsilon_1 + 1 - mz_0} - \frac{X_0(1-mz_0)(-m)}{(\varepsilon_1 + 1 - mz_0)^2} - z_0 \right] \delta z + D_z \Delta \delta z$$

$$a \begin{pmatrix} \delta X \\ \delta z \end{pmatrix} = \begin{pmatrix} \frac{1}{\varepsilon} \left[-\frac{f z_0}{g+X_0} - \frac{g-X_0}{(g+X_0)^2} f \cdot z_0 + \frac{1-mz_0}{\varepsilon_1 + 1 - mz_0} - 2X_0 \right] \delta X + \frac{1}{\varepsilon} \left[\frac{g-X_0}{g+X_0} \cdot f - \frac{mX_0}{\varepsilon_1 + 1 - mz_0} - \frac{X_0(1-mz_0)(-m)}{(\varepsilon_1 + 1 - mz_0)^2} \right] \delta z \\ \frac{1-mz_0}{\varepsilon_1 + 1 - mz_0} \delta X + \left[\frac{X_0(-m)}{\varepsilon_1 + 1 - mz_0} - \frac{X_0(1-mz_0)(-m)}{(\varepsilon_1 + 1 - mz_0)^2} - z_0 \right] \delta z \end{pmatrix}$$