$A \xrightarrow{k_1} B$

$A+B \xrightarrow{k_2} 2B$

$B+C \xrightarrow{k_3} 2C$

$C \xrightarrow{k_4} D$

$2A+B \rightarrow D$

$$\frac{\partial A}{\partial t} = -k_1 A - k_2 AB + D_A \frac{\partial^2 A}{\partial s^2}$$

$$\frac{\partial B}{\partial t} = k_1 A + k_2 AB - k_3 BC + D_B \frac{\partial^2 B}{\partial s^2}$$

$$\frac{\partial C}{\partial t} = k_3 BC - k_4 C + D_C \frac{\partial^2 C}{\partial s^2}$$

$$\frac{\partial D}{\partial t} = k_4 C + D_D \frac{\partial^2 D}{\partial s^2}$$

uniform, steady state:

$-k_1 A - k_2 AB = 0$

$k_1 A + k_2 AB - k_3 BC = 0$

$k_3 BC - k_4 C = 0$

$k_4 C = 0$

$-k_1 A - k_2 AB = 0$

$A(k_1 + k_2 B) = 0$

$C = 0, \quad A = A_0, \quad B = B_0$

$D = D_0$

$D_B \ll D_A, D_C, D_D$

scale A w/ A_0 time with k_1^{-1}

B w/ B_0 space with $\sqrt{D_B / k_1}$

C w/ C_0

D w/ D_0

$$A_0 k_1 \frac{\partial A}{\partial \tau} = -k_1 A_0 A - k_2 A_0 B_0 AB + \frac{D_A A_0}{D_B / k_1} \frac{\partial^2 A}{\partial r^2}$$

$$B_0 k_1 \frac{\partial B}{\partial \tau} = k_1 A_0 A + k_2 A_0 B_0 AB - k_3 B_0 C_0 BC + \frac{D_B B_0}{D_B / k_1} \frac{\partial^2 B}{\partial r^2}$$

$$C_0 \cdot k_1 \frac{\partial C}{\partial \tau} = k_3 B_0 C_0 BC - k_4 C_0 C + \frac{D_C C_0}{D_B / k_1} \frac{\partial^2 C}{\partial r^2}$$

$$D_0 k_1 \frac{\partial D}{\partial t} = k_4 C_0 C + \frac{D_D D_0}{D_B / k_1} \frac{\partial^2 D}{\partial r^2}$$

$$\frac{\partial B}{\partial t} = k_1 A + k_2 AB - k_3 BC + D_B \frac{\partial^2 B}{\partial s^2}$$

$$\frac{\partial C}{\partial t} = k_3 BC - k_4 C + D_c \frac{\partial^2 C}{\partial s^2}$$

$A = A_0 = \text{const.}$

Steady uniform soln. $\Rightarrow$ $k_1 A_0 + k_2 A_0 B_0 - k_3 B_0 C_0 = 0$

$$k_3 C_0 \left(B_0 - \frac{k_4}{k_3} \right) = 0$$

$$B_0 = \frac{k_4}{k_3} \qquad \frac{k_1 A_0 + k_2 A_0 \cdot \frac{k_4}{k_3}}{k_4} = C_0$$

scale time by k_4^{-1} ~~distance~~ length by $\sqrt{D_B / k_4}$

$$\frac{\frac{k_4}{k_3} \cancel{B_0} k_4}{1} \frac{\partial B}{\partial \tau} = k_1 A_0 + k_2 A_0 \cdot \frac{k_4}{k_3} B - k_3 \cdot \frac{k_4}{k_3} \cdot \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_4} BC + \frac{D_B \, k_4 / k_3}{D_B / k_4} \frac{\partial^2 B}{\partial r^2}$$

$$\frac{\partial B}{\partial \tau} = \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0 B}{k_4^2 / k_3} - \frac{k_1 A_0 + \frac{k_2 k_4}{k_3} A_0}{k_4^2 / k_3} BC + \frac{\partial^2 B}{\partial r^2}$$

$$\alpha \equiv \frac{k_1 A_0 + \frac{k_2 k_4 A_0}{k_3}}{k_4^2 / k_3}$$

$$\beta \equiv \alpha - \frac{k_1 A_0}{k_4^2 / k_3}$$

$$\frac{\partial B}{\partial \tau} = \alpha - \beta + \beta B - \alpha BC + \frac{\partial^2 B}{\partial r^2}$$

$$\boxed{\frac{\partial B}{\partial \tau} = \alpha (1 - BC) + \beta (B - 1) + \frac{\partial^2 B}{\partial r^2}}$$

$$\frac{k_1 A_0 + \cancel{\frac{k_2 k_4}{k_3}} A_0}{1/k_4} \frac{\partial C}{\partial \tau} = \cancel{k_3} \cdot \frac{k_4}{\cancel{k_3}} \cdot \cancel{\left(k_1 A_0 + \frac{k_2 k_4}{k_3} A_0\right)} BC - k_4 \cdot \cancel{\left(k_1 A_0 + \frac{k_2 k_4}{k_3} A_0\right)} C + \frac{\left(k_1 A_0 + \cancel{\frac{k_2 k_4}{k_3}} A_0\right) D_C}{D_B / k_4} \frac{\partial^2 C}{\partial r^2}$$

$$\frac{\partial C}{\partial \tau} = BC - C + \frac{D_C}{D_B} \frac{\partial^2 C}{\partial r^2}$$

$$\boxed{\frac{\partial C}{\partial \tau} = (B-1) C + \frac{D_C}{D_B} \frac{\partial^2 C}{\partial r^2}}$$

$$\boxed{\frac{\partial C}{\partial \tau} = (B-1) C + \gamma \frac{\partial^2 C}{\partial r^2}}$$

$$B = 1 + \delta B$$
$$C = 1 + \delta C$$

$$\frac{\partial}{\partial \tau} \delta B = \alpha \cdot \left(1 - (1+\delta B)(1+\delta C)\right) + \beta\, \delta B + \frac{\partial^2 \delta B}{\partial r^2}$$

$$= \alpha \left[1 - (1 + \delta B + \delta C + \delta B \delta C)\right] + \beta\, \delta B + \frac{\partial^2 \delta B}{\partial r^2}$$

$$\boxed{\begin{aligned} \frac{\partial}{\partial \tau} \delta B &= \alpha \left[- \delta B - \delta C - \delta B \delta C\right] + \beta\, \delta B + \frac{\partial^2 \delta B}{\partial r^2} \\ \frac{\partial}{\partial \tau} \delta C &= \delta B (1 + \delta C) + \gamma \cdot \frac{\partial^2 \delta C}{\partial r^2} \end{aligned}}$$

linearized system:

$$\frac{\partial \delta B}{\partial \tau} = -\alpha (\delta B + \delta C) + \beta\, \delta B + \frac{\partial^2 \delta B}{\partial r^2}$$

$$\frac{\partial}{\partial \tau} \delta C = \delta B + \gamma \cdot \frac{\partial^2 \delta C}{\partial r^2}$$

$$\frac{\partial}{\partial \tau} = \lambda$$

$$\frac{\partial^2}{\partial r^2} = -k^2$$

$$\lambda\, \delta B = (-\alpha + \beta)\delta B - \alpha\, \delta C - k^2 \delta B$$

$$\lambda\, \delta C = \delta B - \gamma k^2 \delta C$$

$$\lambda \begin{bmatrix} \delta B \\ \delta C \end{bmatrix} = \begin{bmatrix} -\alpha+\beta-k^2 & -\alpha \\ 1 & -\gamma k^2 \end{bmatrix} \begin{pmatrix} \delta B \\ \delta C \end{pmatrix}$$

$$\det \begin{pmatrix} -\alpha+\beta-k^2-\lambda & -\alpha \\ 1 & -\gamma k^2 - \lambda \end{pmatrix} = 0$$

$$(\lambda + k^2 + \alpha - \beta)(\lambda + \gamma k^2) + \alpha = 0$$

$$\lambda^2 + (k^2 + \alpha - \beta + \gamma k^2)\lambda - \beta\gamma k^2 = 0$$

$$\lambda = \frac{-[(1+\gamma)k^2 + \alpha - \beta] \pm \sqrt{[(1+\gamma)k^2 + \alpha - \beta]^2 + 4\beta\gamma k^2}}{2}$$

$$\gamma \to 0 \qquad \lambda = 0, \qquad \beta - \alpha - (1+\gamma)k^2 = \beta - \alpha - k^2$$

$A \equiv BrO_3^-$ $X = HBrO_2$ $Y = Br^-$ red => reduced state of the catalyst

B = mixture of malonic & ~~bor~~ bromomalonic acids Z => oxidized state

$$2H^+ + A + Y \rightarrow X \quad k_1'$$

$$H^+ + X + Y \rightarrow O \quad k_2'$$

$$H^+ + A + X + 2\,\text{red} \rightarrow 2X + 2Z \quad k_3'$$

$$2X \rightarrow O \quad k_4$$

$$B + Z \rightarrow hY + \text{red} \quad k_5'$$

$$k_3' = k_3'' \cdot \frac{[\text{red}]}{[\text{red}]+c}$$

$$v_3 = k_3'' [x][A][H^+] \frac{[\text{red}]}{[\text{red}]+c}$$

if $[\text{red}] \gg c$

H^+ & $A = BrO_3^-$ remain const.

$$[\text{red}] + [Z] \equiv C_0 = \text{const.}$$

if $[Z] \sim C_0$

$[\text{red}] \ll c$

$$BrO_2^* + \text{red} \rightarrow HBrO_2 + Z$$

$$BrO_2^* + BrO_2^* \rightarrow BrO_3^- + HBrO_2$$

$$\frac{\partial x}{\partial t} = k_1' \cdot H^2 A Y - k_2' H x Y + k_3' H A R^2 X - k_4 X^2 + D_x \nabla^2 X$$

$$\frac{\partial Z}{\partial t} = k_3' H A X R^2 - k_5' B Z + D_z \nabla^2 Z$$

$$k_3' X \cdot A H^+ \equiv k_3 X$$

$$k_3' A H^+ = k_3$$

$$k_3 = k_3'' \cdot \frac{[\text{red}]}{[\text{red}]+c} \cdot A H^+ = k_3$$

$k_3' \, HA \, [\mathrm{red}]^2$

$\Rightarrow k_3' \, [H^+][A][\mathrm{red}]^2 = \dfrac{k_3'' \, [H^+][A][\mathrm{red}]}{[\mathrm{red}] + c}$

$k_3'' \, [H^+][A] = k_3$

$$\frac{\partial Z}{\partial t} = k_3 \cdot X \cdot \frac{(C_0 - Z)}{C_0 - Z + c} - k_5 Z + D_Z \nabla^2 Z$$

$Z \to \dfrac{k_3^2}{k_4 k_5} z$

$t \to \tau / k_5$

$X \to \dfrac{k_3}{k_4} x$

$$\frac{\frac{k_3^2}{k_4 k_5} \partial z}{\frac{1}{k_5} \partial \tau} = k_3 \cdot \frac{k_3}{k_4} x \, \frac{C_0 - \frac{k_3^2}{k_4 k_5} z}{C_0 - \frac{k_3^2}{k_4 k_5} z + C} - k_5 \cdot \frac{k_3^2}{k_4 k_5} z + D_z \frac{k_3^2}{k_4 k_5} \nabla^2 z$$

$$\boxed{\frac{\partial z}{\partial \tau} = x \cdot \frac{(1 - m z)}{(1 - m z + \varepsilon_1)} - z + D_z' \nabla^2 z}$$

Similarly

$$\boxed{\frac{\partial x}{\partial \tau} = \frac{1}{\varepsilon} \left[\frac{(q - x) f z}{q + x} + \frac{x \, (1 - m z)}{(1 - m z + \varepsilon_1)} - x^2 \right] + D_x' \nabla^2 x}$$

$$\frac{\partial x}{\partial \tau} = \frac{1}{\varepsilon}\left[\frac{(q-x) f z}{q+x} + \frac{x(1-mz)}{\varepsilon_1 + 1 - mz} - x^2 \right] + D_x \Delta x$$

$$\frac{\partial z}{\partial \tau} = \frac{x \cdot (1-mz)}{\varepsilon_1 + 1 - mz} - z + D_z \Delta z$$

x_0 & z_0 s.t. $\begin{cases} z_0 \dfrac{q-x_0}{q+x_0} \cdot f + \dfrac{x_0(1-mz_0)}{\varepsilon_1+1-mz_0} - x_0^2 = 0 \\[2ex] \dfrac{x_0(1-mz_0)}{\varepsilon_1+1-mz_0} - z_0 = 0 \end{cases}$

$x \to x + \delta x$

$z \to z + \delta z$

$$\frac{\partial \delta x}{\partial \tau} = \frac{1}{\varepsilon}\left[-\frac{f z_0}{q+x_0} - \frac{q-x_0}{(q+x_0)^2} f \cdot z_0 + \frac{1-mz_0}{\varepsilon_1+1-mz_0} - 2x_0 \right] \delta x$$
$$+ \frac{1}{\varepsilon}\left[\frac{q-x_0}{q+x_0} \cdot f - \frac{m x_0}{\varepsilon_1+1-mz_0} - \frac{x_0(1-mz_0)(-m)}{(\varepsilon_1+1-mz_0)^2} \right] \delta z + D_x \Delta \delta x$$

$$\frac{\partial \delta z}{\partial \tau} = \frac{1-mz_0}{\varepsilon_1+1-mz_0} \cdot \delta x + \left[\frac{x_0(-m)}{\varepsilon_1+1-mz_0} - \frac{x_0(1-mz_0)(-m)}{(\varepsilon_1+1-mz_0)^2} - 1 \right] \delta z + D_z \Delta \delta z$$

$$\lambda \begin{pmatrix} \delta x \\ \delta z \end{pmatrix} = \begin{bmatrix} \frac{1}{\varepsilon}\left[-\frac{f z_0}{q+x_0} - \frac{q-x_0}{(q+x_0)^2} f z_0 + \frac{1-mz_0}{\varepsilon_1+1-mz_0} - 2x_0 \right] - D_x k^2 & \frac{1}{\varepsilon}\left[\frac{q-x_0}{q+x_0} f - \frac{m x_0}{\varepsilon_1+1-mz_0} - \frac{x_0(1-mz_0)(-m)}{(\varepsilon_1+1-mz_0)^2} \right] \\ \frac{1-mz_0}{\varepsilon_1+1-mz_0} & \left[\frac{x_0(-m)}{\varepsilon_1+1-mz_0} - \frac{x_0(1-mz_0)(-m)}{(\varepsilon_1+1-mz_0)^2} \right] - D_z k^2 \end{bmatrix}$$