$$\frac{\partial u}{\partial t} = u(1-u^q) + \frac{\partial^2 u}{\partial x^2} \qquad q > 0$$

$u = 0$
$u = 1$

$$u(x,t) = U(z), \quad z = x - ct, \qquad \begin{aligned} U(-\infty) &= 1 \\ U(\infty) &= 0 \end{aligned}$$

$$U'' + cU' + U(1-U^q) = 0 \equiv L(U)$$

$$U(z) = \frac{1}{(1+\cancel{ak}ae^{bz})^s}$$

$a,\ b,\ s$ positive constants

at $z = \infty \qquad U(z) \to 0$
$\quad z = -\infty \qquad U(z) \to 1$

$$U' = -\frac{s \cdot a \cdot b e^{bz}}{(1+ae^{bz})^{s+1}}$$

$$U'' = -\frac{s \cdot ab^2 e^{bz}}{(1+ae^{bz})^{s+1}} + \frac{s(s+1)\, ab^3 e^{2bz}}{(1+ae^{bz})^{s+2}}$$

$$L(U) = \frac{1}{(1+ae^{bz})^{s+2}} \Big\{ \big[s(s+1)b^2 - sb(b+c) + 1 \big] a^2 e^{2bz}$$

$$+ [2 - sb(b+c)]\, a e^{bz} + 1 - [1 + a e^{bz}]^{2-sq} \}$$

$L(U) = 0$ for all z

$$\Rightarrow \quad 2 - sq = 0,\ 1,\ 2$$

$$sq = 0,\ 1,\ 2$$

$$s = 0,\ \frac{1}{q},\ \frac{2}{q}$$

$s = 0$ is impossible

$s = \frac{1}{q}$: $e^{bz} \to \quad 2 - sb(b+c)^{-1} = 0 \qquad sb(b+c) = 1$ (?)

$e^{2bz} \to \quad s(s+1)b^2 - sb(b+c) + 1 = 0 \qquad \Rightarrow s(s+1)b^2 = 0$

$b = 0$

because $s > 0$

impossible scenario

$s = \frac{2}{q}$: e^{bz} : $sb(b+c) = 2$

e^{2bz} : $s(s+1)b^2 - sb(b+c) + 1 = 0 \quad \Rightarrow \quad s(s+1)b^2 = 1$

$$\therefore \quad s = \frac{2}{q}, \qquad b = \frac{1}{[s(s+1)]^{1/2}}, \qquad c = \frac{2}{sb} - b$$

$$\Rightarrow \quad s = \frac{2}{q}, \qquad b = \frac{q}{[2(q+2)]^{1/2}}, \qquad c = \frac{q+4}{[2(q+2)]^{1/2}}$$

point of inflection $z_i = -b^{-1} \ln(as)$

$$s = \frac{b}{\left(1+\frac{1}{s}\right)^{s+1}} = \frac{\frac{1}{2}q}{\left(1+\frac{q}{2}\right)^{3/2+2/q}}$$

$q = 1 \Rightarrow$ Fisher-Kolmogoroff Eq.

$$s = 2, \quad b = \frac{1}{\sqrt{6}}, \quad c = \frac{5}{\sqrt{6}} \approx 2.04 > 2$$

$$\frac{\partial u}{\partial t} = u^{q+1}(1-u^q) + \frac{\partial^2 u}{\partial x^2} \Rightarrow U'' + cU' + U^{q+1}(1-U^q) = 0$$

$$U(z) = \frac{1}{(1+ae^{bz})^s}$$

$$\Rightarrow s = \frac{1}{q}, \quad b = \frac{q}{(q+1)^{1/2}}, \quad c = \frac{1}{(q+1)^{1/2}}$$

$$\frac{\partial u}{\partial t} = u(1-u) + \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right)$$

$$u = U(z) = U(x-ct)$$

$$(UU')' + cU' + U(1-U) = 0$$

$$U(-\infty) = 1$$

$$U(\infty) = 0$$

$$U' = V$$

$$UV' = -cV - V^2 - U(1-U)$$

$$U\frac{d}{d\xi} = \frac{d}{d\zeta}$$

$$\begin{cases} \dfrac{dU}{d\zeta} = UV \\ \dfrac{dV}{d\zeta} = -cV - V^2 - U(1-U) \end{cases}$$

critical points in the (U, V) phase plane are

$$(U,V) = (0,0), \quad (1,0), \quad (0,-c)$$

$(1,0)$, $(0,-c)$ $\Rightarrow$ saddle points

$(0,c)$ $\Rightarrow$ stable nonlinear node