F-K Equation:

$$\frac{\partial u}{\partial t} = u(1-u) + \frac{\partial^2 u}{\partial x^2}$$

$$U(z) = u(x,t), \quad z \equiv x - ct$$

$$U'' + cU' + U(1-U) = 0$$

boundary condition $\quad U(-\infty) = 1$, $\quad U(\infty) = 0$

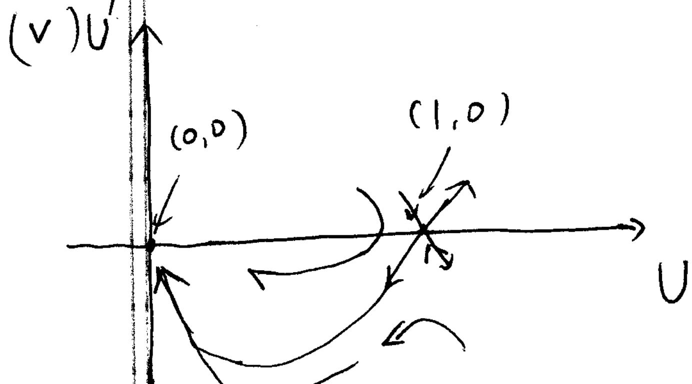
$$U' = V$$
$$V' = -cV - U(1-U)$$

$$\frac{dV}{dU} = \frac{-cV - U(1-U)}{V}$$

$(U,V) = (0,0)$ $\qquad (1,0) \rightarrow \lambda_\pm = \frac{1}{2}\left[-c \pm (c^2+4)^{1/2}\right]$ saddle

$$\lambda_\pm = \frac{1}{2}\left[-c \pm (c^2-4)^{1/2}\right]$$

saddle node if $c^2 > 4$
stable spiral if $c^2 < 4$



$$U(z) = g(\xi) \qquad \xi = \frac{z}{c} = \varepsilon^{1/2} \cdot z$$

$$\frac{dU}{dz} = \frac{dg}{d\xi}\frac{d\xi}{dz} = \varepsilon^{1/2}\frac{dg}{d\xi}$$

$$\varepsilon g'' + c \cdot \varepsilon^{1/2} g' + g(1-g) = 0$$

$$c = \frac{1}{\varepsilon^{1/2}} \qquad \boxed{\varepsilon g'' + g' + g(1-g) = 0}$$

multiplying $e^{-\ln g_0'}$

$$\left(\frac{dg_1}{d\xi} - \frac{g_0''}{g_0'}\cdot g_1\right) e^{-\ln g_0'} = -g_0'' e^{-\ln g_0'}$$

$$g_1' \cdot e^{-\ln g_0'} - \frac{g_0''}{g_0'} e^{-\ln g_0'} g_1 = \left(g_1 \cdot e^{-\ln g_0'}\right)' = -g_0'' \cdot e^{-\ln g_0'}$$

$$= -\frac{g_0''}{g_0'}$$

$$g_1 \cdot e^{-\ln g_0'} \Big|_{-\infty}^{\xi} = -\ln g_0' \Big|_{-\infty}^{\xi}$$

$$g_1(\xi)\cdot e^{-\ln g_0'(\xi)} - 0 = -\ln g_0'(\xi) + \ln g_0'(-\infty)$$

$$g_1(\xi) = \left[-\ln g_0'(\xi) + \ln g_0'(-\infty)\right] e^{\ln g_0'(\xi)}$$

$$= -g_0' \cdot \ln\left(4\,|g_0'|\right)$$

$$U(z;\varepsilon) = \left(1+e^{z/c}\right)^{-1} + \frac{1}{c^2}\cdot e^{z/c}\left(1+e^{z/c}\right)^{-2} \ln\frac{4e^{z/c}}{\left(1+e^{z/c}\right)^2}$$

for $c \geq c_{min} = 2$

inflection pt. where $U'' = 0$

$$g_0''(\xi) + \varepsilon \cdot g_1''(\xi) + O(\varepsilon^2) = 0$$

for the soln. of g_0 & g_1

$$\xi = 0$$

$$\varepsilon \frac{d^2 g}{d\xi^2} + \frac{dg}{d\xi} + g(1-g) = 0$$

$$g(-\infty) = 1 \qquad g(\infty) = 0 \qquad 0 < \varepsilon \le \frac{1}{C_{min}^2} = 0.25$$

$$g(0) = 1/2$$

$$g(\xi;\varepsilon) = g_0(\xi) + \varepsilon g_1(\xi) + \cdots$$

$g(0;\varepsilon) = \frac{1}{2}$ for all ε

$$g_0(-\infty) = 1 \qquad g_0(\infty) = 0 \qquad g_0(0) = \frac{1}{2}$$

$g_i(\pm\infty) = 0 \qquad g_i(0) = 0 \qquad$ for $i = 1, 2, 3, \cdots$

at leading order: $\frac{dg_0}{d\xi} = -g_0(1-g_0) \qquad g_0 = \frac{1}{1+e^{\xi}}$

$O(1)$: $\frac{d^2 g_0}{d\xi^2} + \frac{dg_1}{d\xi} + g_1(1-2g_0) = 0$

$$\frac{d^2 g_0}{d\xi^2} = -\frac{dg_0}{d\xi}(1-g_0) - g_0\left(-\frac{dg_0}{d\xi}\right)$$

$$= -\frac{dg_0}{d\xi}(1-2g_0)$$

$$\frac{dg_1}{d\xi} = -g_1(1-2g_0) + \frac{dg_0}{d\xi}(1-2g_0)$$

$$= (1-2g_0)\left(-g_1 + \frac{dg_0}{d\xi}\right)$$

$$\boxed{\frac{dg_1}{d\xi} - \left(\frac{g_0''}{g_0'}\right) g_1 = -g_0''}$$

$$U'(0) = \frac{-\frac{1}{c}}{(1+e^{0/c})^2} + \frac{1}{c^3} e^{\frac{0}{c}} (1+e^{0/c})^{-2} \cdot \ln \frac{4}{1}$$
$$+ \frac{1}{c^2} \cdot e^{\frac{0}{c}} \frac{-2/c}{(1+e^{0/c})^3} \ln \frac{4}{1}$$
$$+ \frac{1}{c^2} \cdot e^{\frac{0}{c}} \cdot (1+e^{0/c})^{-2} \cdot \left[\frac{1}{c} - \frac{2/c}{1+e^{0/c}} \right]$$
$$= -\frac{1}{4c}$$

$$-U'(0) \equiv s = \frac{1}{4c}$$

the faster the wave moves
the less steep is the wave front

stability $\begin{cases} z = x - ct \\ t = t \end{cases}$

$$u_t = u(1-u) + u_{xx} \rightarrow u_t = u(1-u) + c u_z + u_{zz}$$

U_c is the traveling wave soln. s.t.

$$U_c(1-U_c) + c \cdot U_{cz} + U_{czz} = 0$$

$$U = U_c + \omega \cdot v(z,t)$$

$$v_t = (1-2U_c) v + c v_z + v_{zz}$$

U_c is stable if $\lim_{t\to\infty} v(z,t) = 0$

$\lim_{t\to\infty} v(z,t) = \frac{dU_c}{dz}$

$$\boxed{v(z,t) = g(z) \cdot e^{-\lambda t}}$$

$g'' + cg' + (\lambda + 1 - 2U_c)g = 0$

if $\lambda = 0$, $g(z) = \frac{dU_c}{dz}$ is a soln. of this eq.

$\Rightarrow$ the traveling wave is invariant under translation along the z-axis.

write $g(z) = h(z)e^{-cz/2}$

$h'' + \left[\lambda - \left\{2U_c(z) + \frac{c^2}{4} - 1\right\}\right]h = 0$

$\downarrow$ $\quad 2U_c + \frac{c^2}{4} - 1 \geq 2U_c(z) > 0 \quad$ for $c \geq 2$ and $U_c > 0$

all eigenvalues must be real & positive

$\Rightarrow V(z,t) \to 0$ as $t \to \infty$

$\Rightarrow U_c$ is stable to all small finite domain perturbations effect of convection & nonlinear convection

$$u_t + k u u_x = u(1-u) + u_{xx}$$

$$u(x,t) = U(z), \quad z = x - ct$$

$$U'' + c(1-kU)U' + U(1-U) = 0$$

$$U(-\infty) = 1 \qquad U(\infty) = 0$$

$$V \equiv U'$$

$$\frac{dV}{dU} = \frac{-(c-kU)V - U(1-U)}{V}$$