INFINITE SYMMETRIES AND INFINITE CONSERVATION LAWS

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In the talk we discuss differential systems possessing infinite symmetries. Specifically, we consider partial differential equations of a variational problem whose symmetry group generators contain arbitrary function(s). We study the relations between infinite symmetries and local conservation laws and introduce *essential conservation laws*. Essential (or integral) conservation laws are defined as continuity equations that lead to non-vanishing conserved densities. We classify different cases of infinite symmetries and analyze corresponding essential conservation laws. The case of arbitrary functions of all independent variables of the problem naturally leads to the Second Noether Theorem.

We will show that in R^n for a general situation of an arbitrary function of $k < n$ independent variables, infinite symmetries may lead to a *finite* number of essential conservation laws. We study the role of boundary conditions for generation of conservation laws and show that each of essential conservation laws is determined by an appropriate boundary condition. We will demonstrate that class of equations that admit infinite symmetries with arbitrary functions of independent variables includes a number of interesting systems, such as the equation of transonic gas flows, the short waves equation, Navier-Stokes equations, Zabolotskaya-Khokhlov equation, Kadomtsev-Petviashvili equation, and Davey-Stewartson equations. We will describe a general method of how to construct a finite set of essential conservation laws corresponding to infinite symmetries and show its application.

We will demonstrate that the case of arbitrary functions of dependent variables is radically different, and leads to an infinite set of essential conservation laws. Two known examples of this situation are equations of Liouville type, that can be integrated by the Darboux method, and systems of hydrodynamic type. We will show how to generate (Lagrangian) equations of this class and give some interesting examples.

It order to compare systems with infinite symmetries and known integrable models we analyze boundary conditions for infinite conserved densities of some soliton equations, and compare them with boundary conditions giving rise to essential conservation laws related to infinite symmetries with arbitrary functions of independent and dependent variables.