A Large Company:

Analyzing viscoelastic materials

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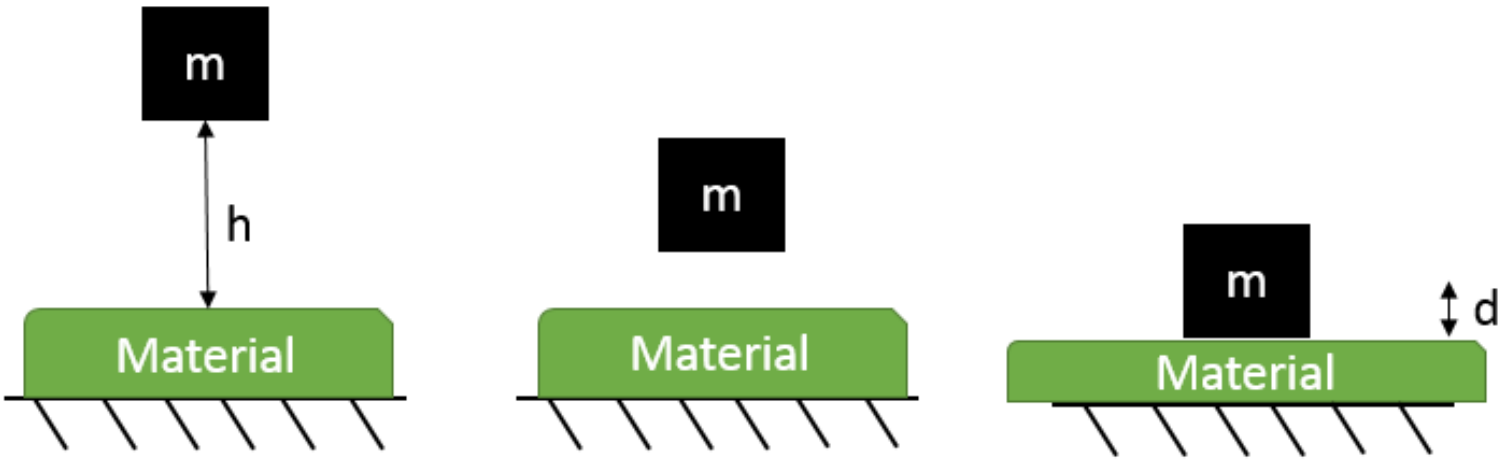
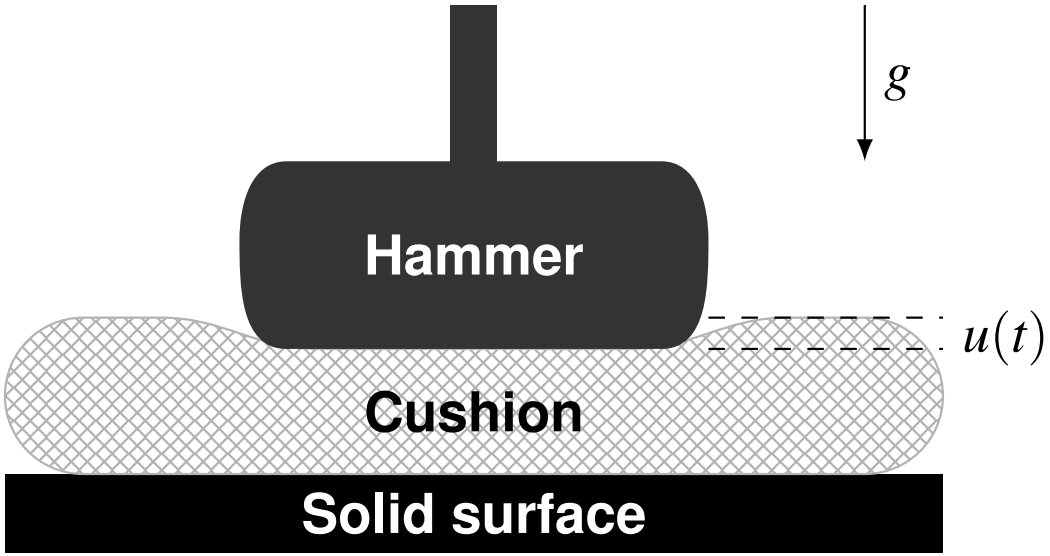
Introduction

- How to characterize cushions?
- Two approaches to find out:
  - Material Models
  - Data Analysis

Experimental Setup



Model



Newton's second law: $-F + mg = m\ddot{u}$

(F is the reaction force of the cushion on the hammer,
m is the hammer mass)

Cushion properties

What are the properties of the cushion?

Viscous — material resists deformation at a given rate

Elastic — material goes back to original shape

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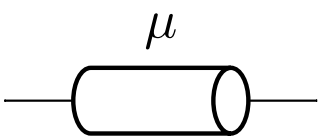
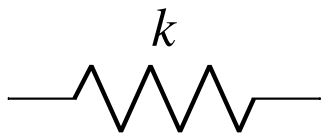


dashpot (visco)

$$F = \mu \dot{u}$$

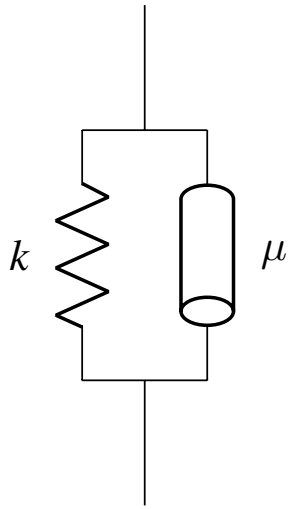


spring (elastic)

$$F = ku$$

Viscoelasticity

The simplest models that capture both of these components combine spring and dashpot in series or parallel, for example the Kelvin–Voigt model:



$$F = ku + \mu \dot{u}$$

Method

Substituting constitutive law, $F = ku + \mu\dot{u}$, into Newton's second law, $-F + mg = m\ddot{u}$, gives

$$m\ddot{u} + \mu\dot{u} + ku = mg$$

with initial conditions given from experimental set-up

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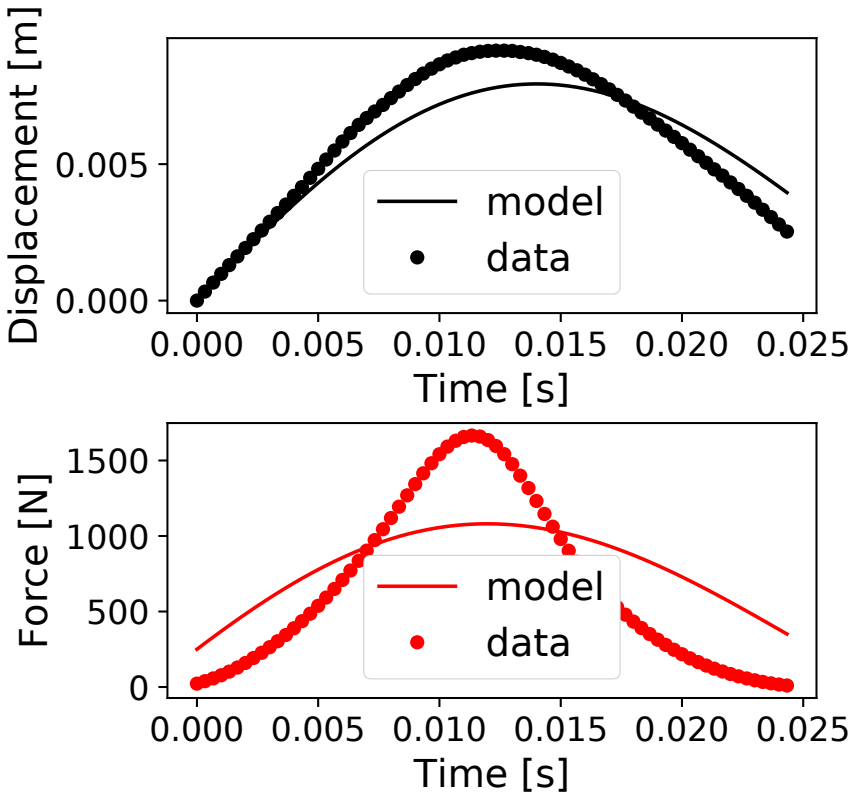
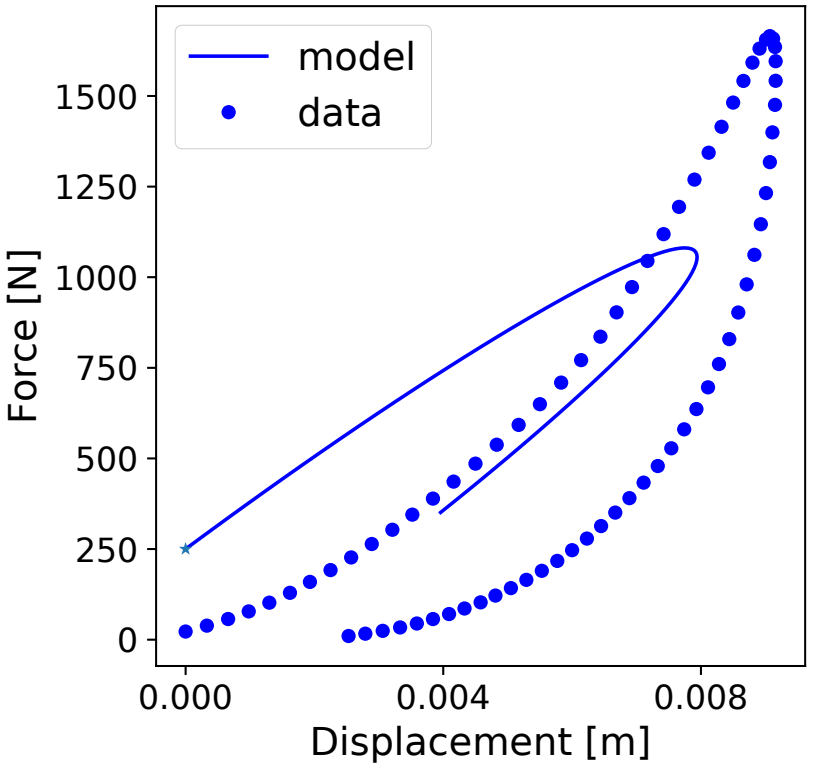
Now fit the data using the free model parameters:

1. Guess parameters μ and k
2. Simulate the forward model to give force and displacement
3. Compare to data: Good enough? Quit, else refine parameters and return to 2.

Results


Displacement [m]
0.005
0.000
model
data
0.000 0.005 0.010 0.015 0.020 0.025
Time [s]
Force [N]
1500
1000
500
0
model
data
0.000 0.005 0.010 0.015 0.020 0.025
Time [s]


Force [N]
1500
1250
1000
750
500
250
0
model
data
0.000
0.004
0.008
Displacement [m]

Nonlinear viscoelasticity

Nonlinear spring $F = k(u)$ for a **function** k. We take a cubic form $k(u) = k_1 u + k_2 u^2 + k_3 u^3$

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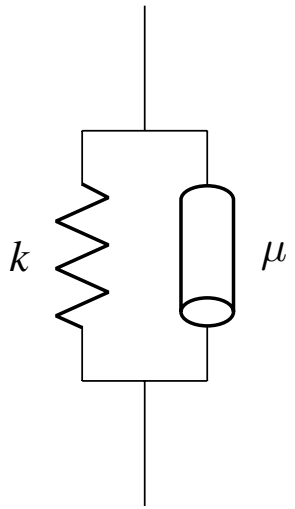
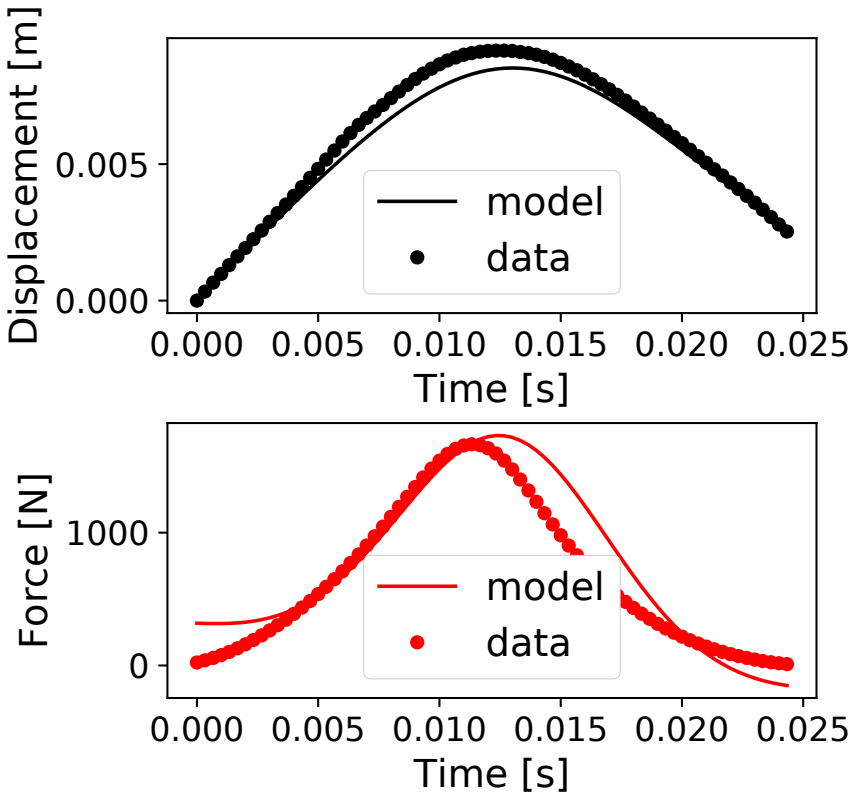
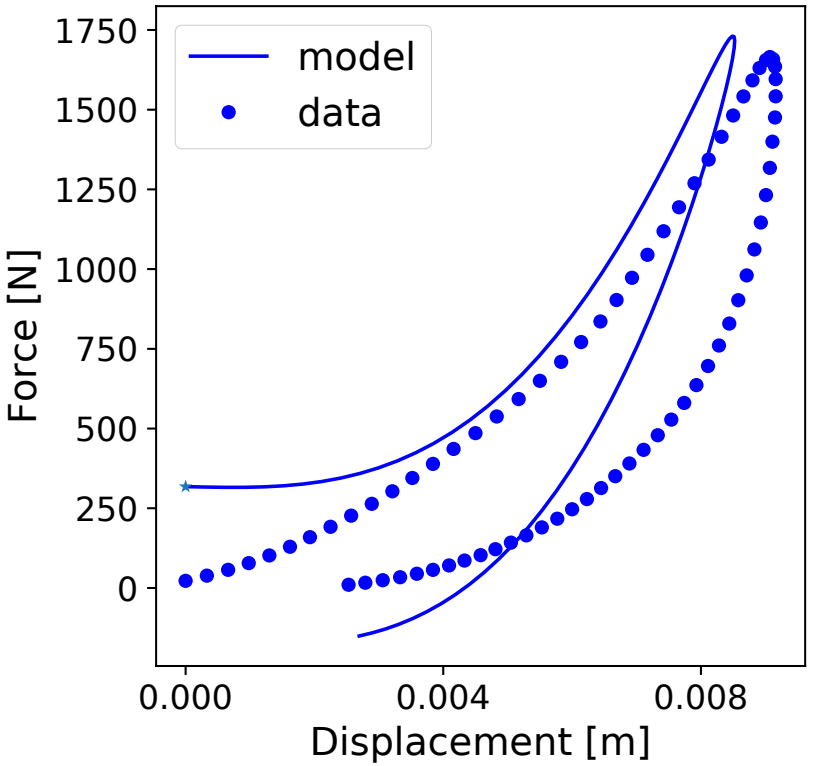


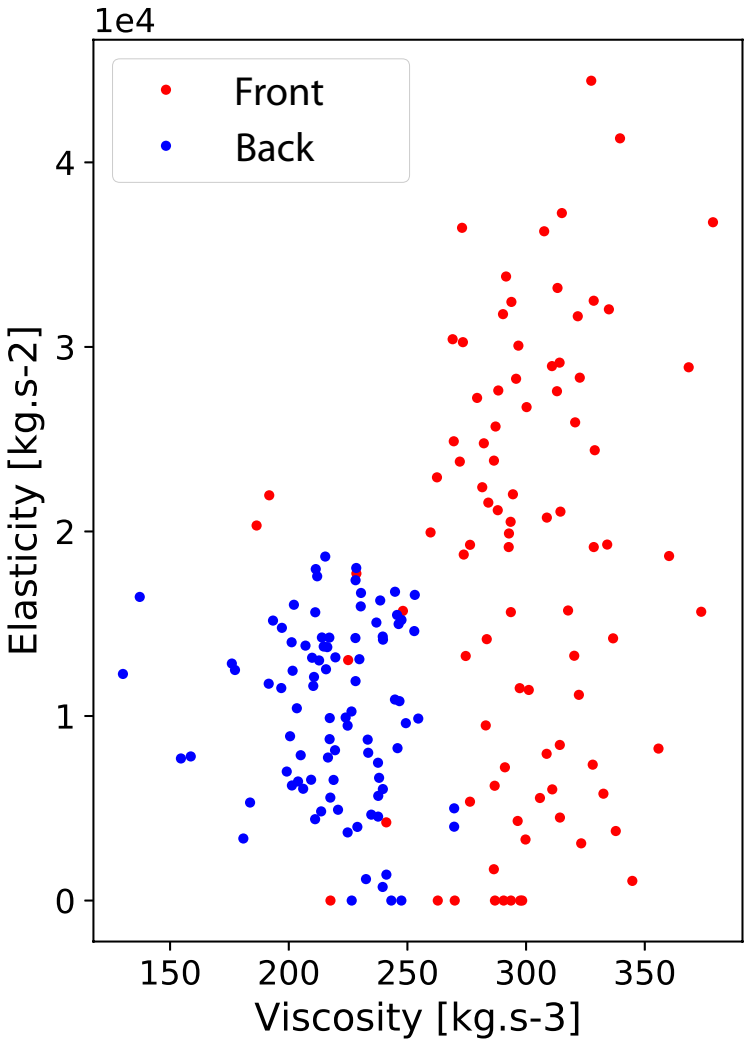
~~$F = ku + \mu\dot{u}$~~

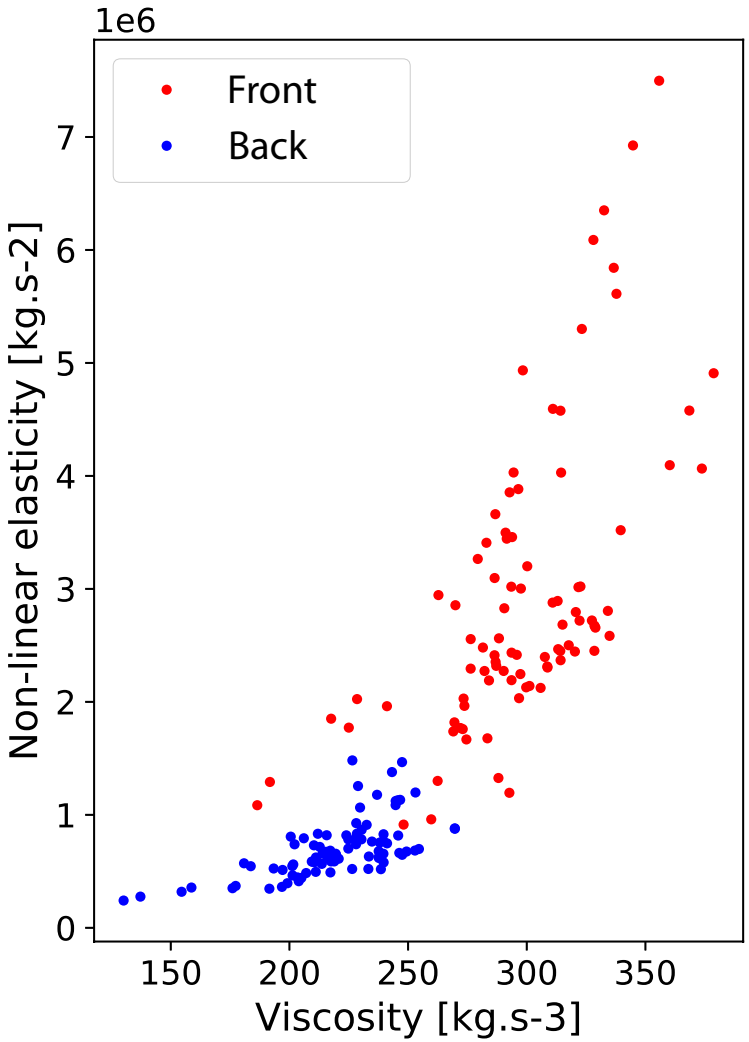
$$F = k(u) + \mu\dot{u}$$

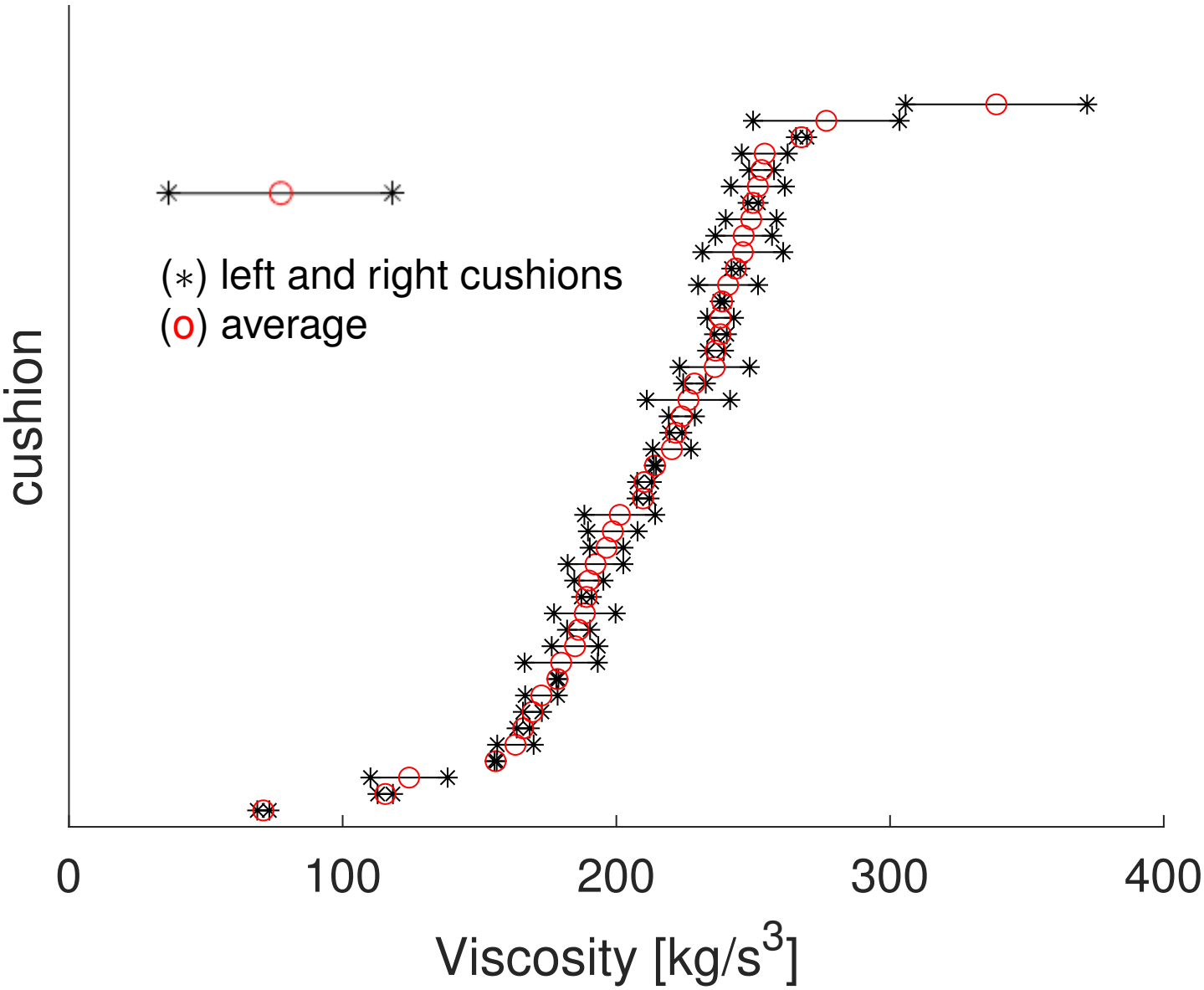
Nonlinear Results





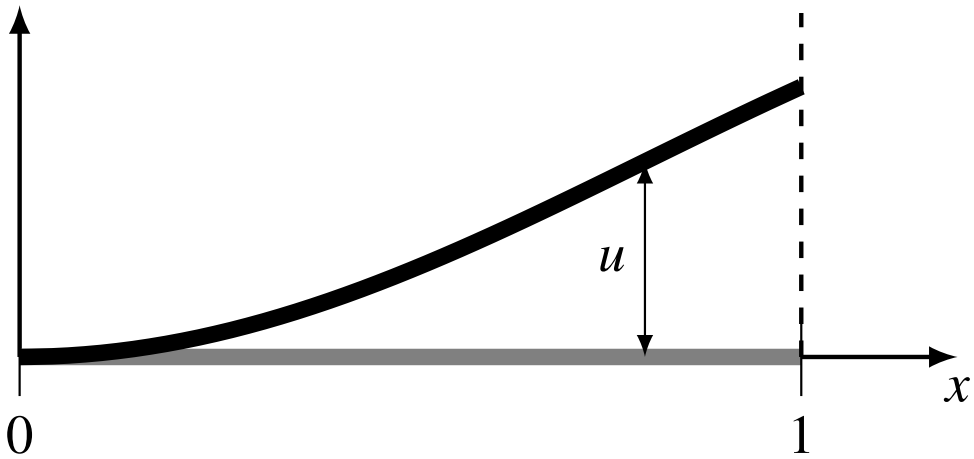

1e4
Front
Back
Elasticity [kg.s-2]
0
1
2
3
4
150
200
250
300
350
Viscosity [kg.s-3]


1e6
Front
Back
Non-linear elasticity [kg.s-2]
0
1
2
3
4
5
6
7
150
200
250
300
350
Viscosity [kg.s-3]


(∗) left and right cushions
(o) average
cushion
0
100
200
300
400
Viscosity [kg/s^3]

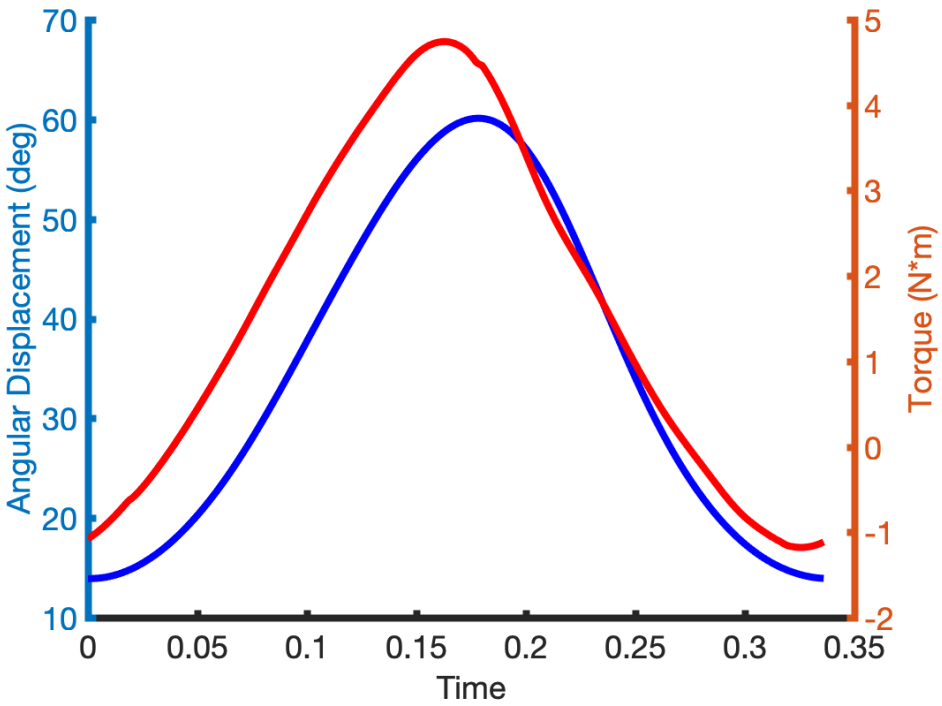
Flex Model

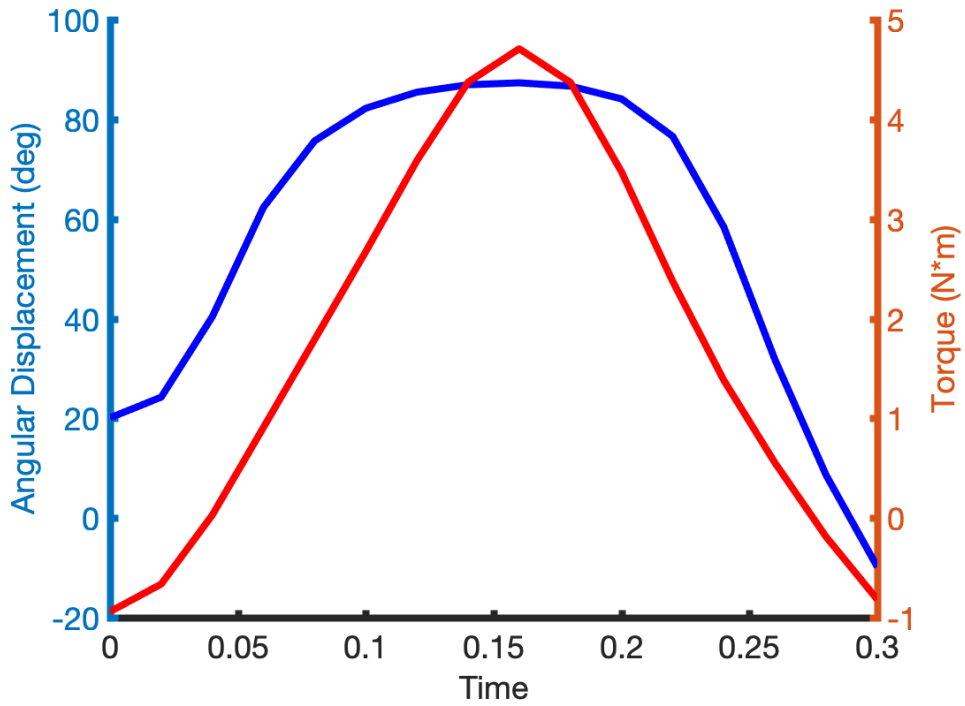
Cantilevered Beam Model



$$\begin{cases} u_{tt} = -u_{xxxx} - \alpha u_t \\ u(0,t) = u_x(0,t) = 0 \\ u_{xxx}(1,t) = 0, u_{xx}(1,t) = m(t) \end{cases}$$

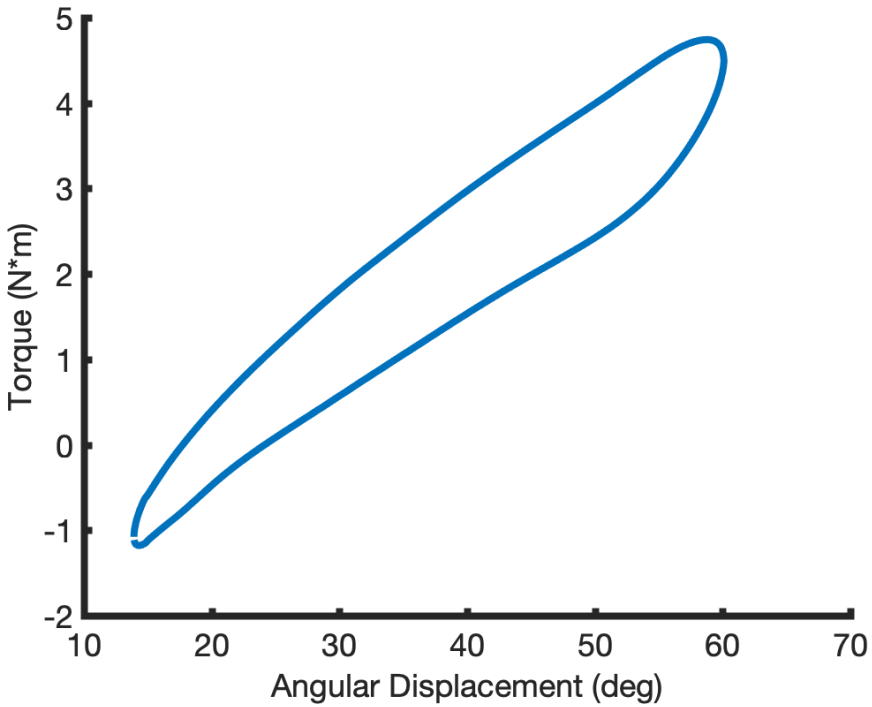
Flex Model

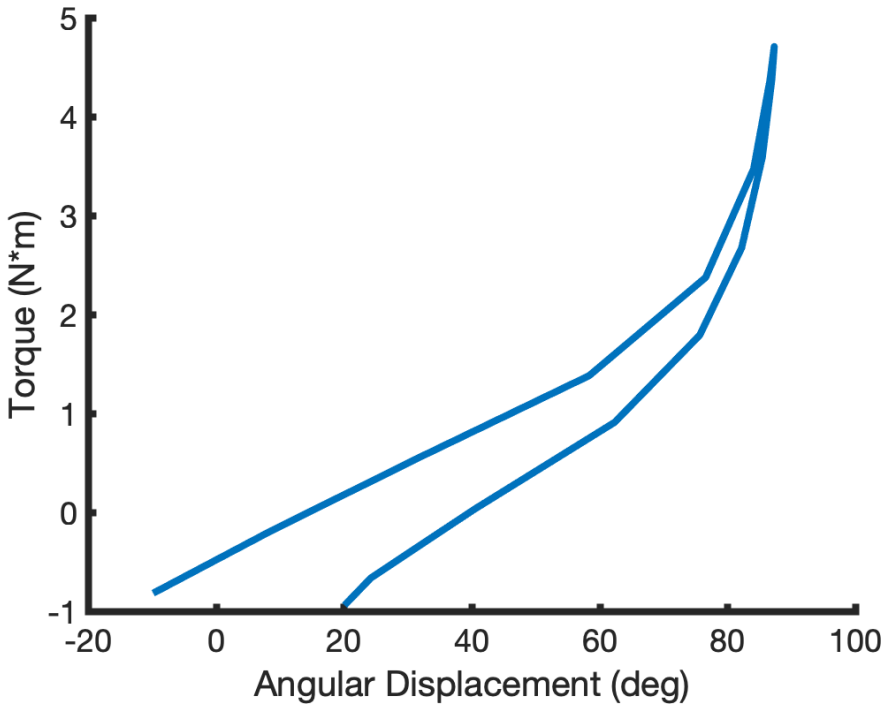




Left: Data, Right: Model

Flex Model



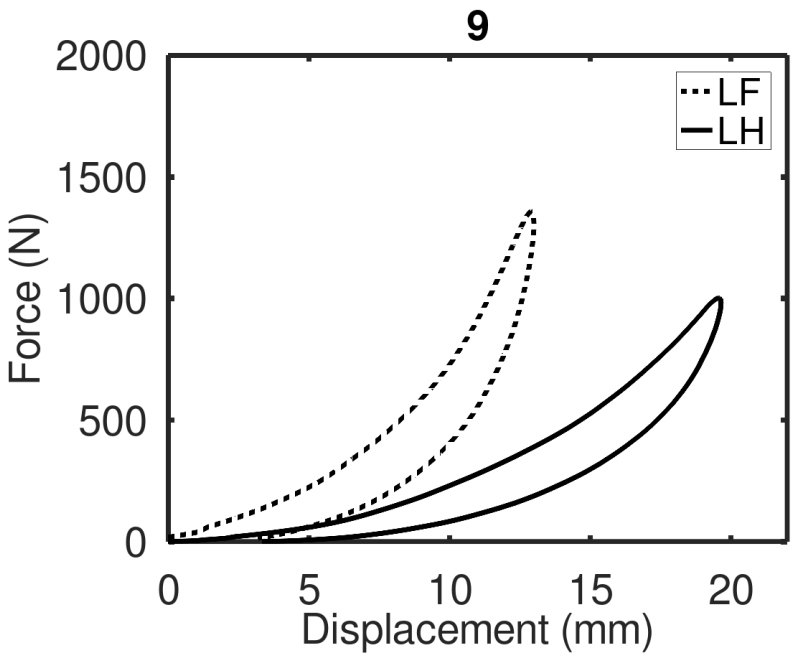


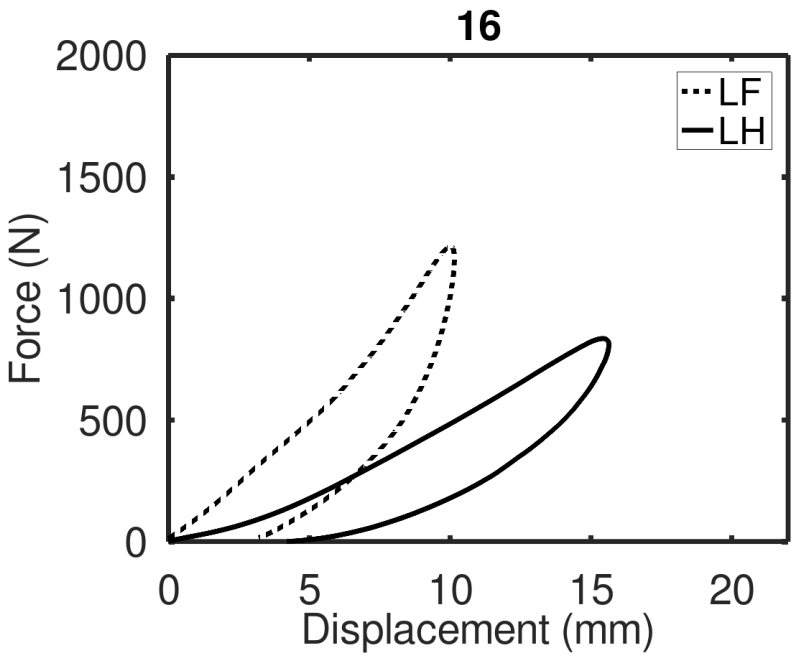
Left: Data, Right: Model

Data Analysis Overview

- Identify important features.
- Run dimensional reduction analysis.
- Interpret results.

Force-Displacement Diagrams

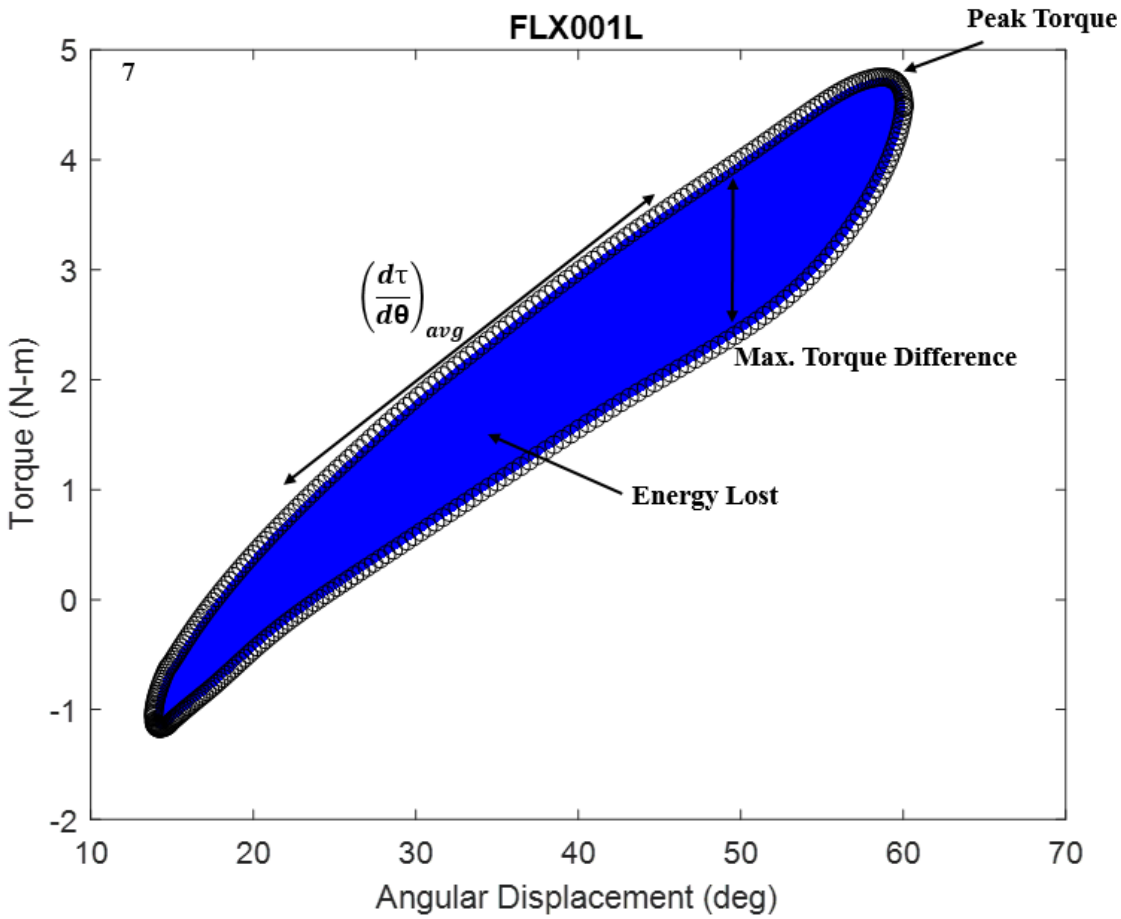




Examples of Features

1. Peak Force/Torque
2. Time to Peak Force/Torque
3. Loading/Unloading Average Stiffness
4. Instantaneous Stiffness at Different Points
5. Variance of Stiffness
6. Energy Efficiency
7. Maximum Difference of Force/Torque at Given Displacement
8. Curvature at Peak Force/Torque
9. Angular Displacement Ratio at Max Torque Difference

Features


FLX001L
7
Peak Torque
$\left(\frac{d\tau}{d\theta}\right)_{avg}$
Max. Torque Difference
Energy Lost
Torque (N-m)
Angular Displacement (deg)
5
4
3
2
1
0
-1
-2
10
20
30
40
50
60
70

Correlation Heatmap - Flex

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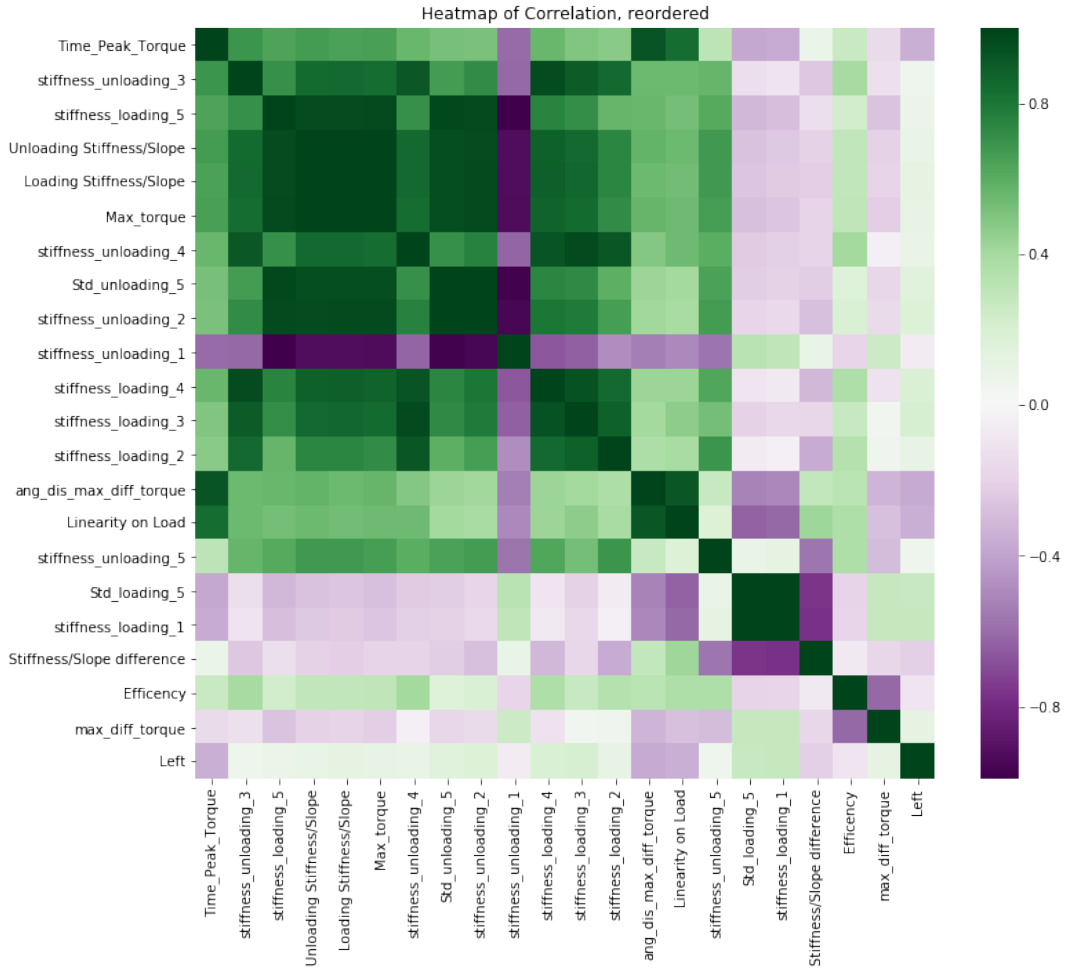
Different measures of stiffness are highly correlated.



Dimensionality Reduction

Principal Component Analysis

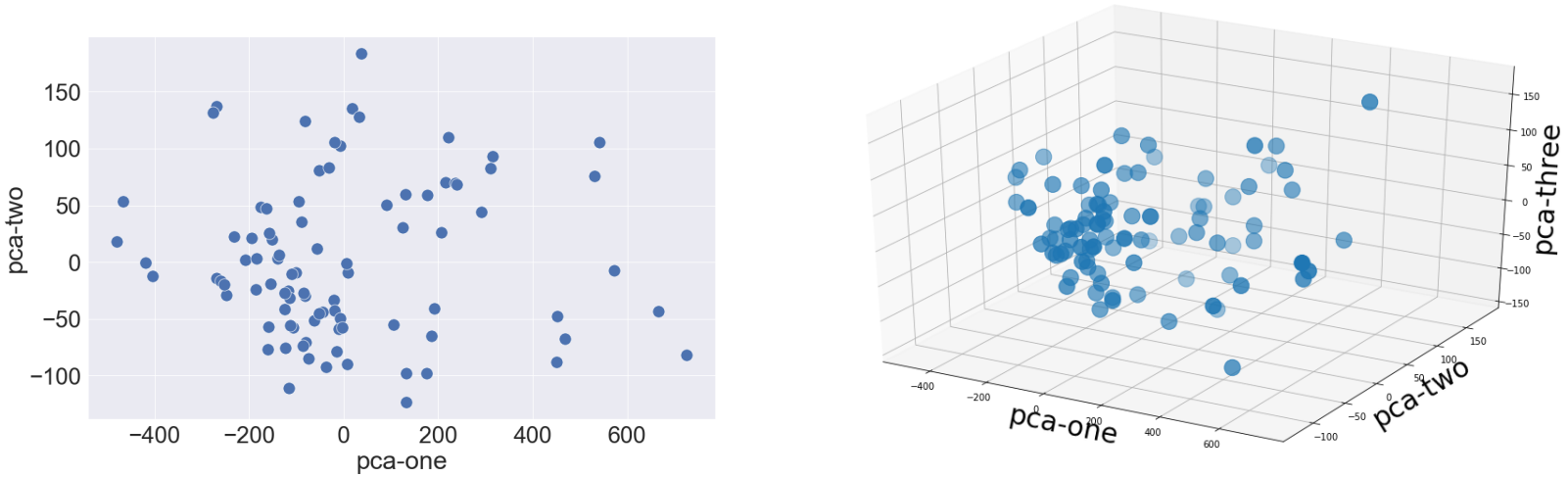
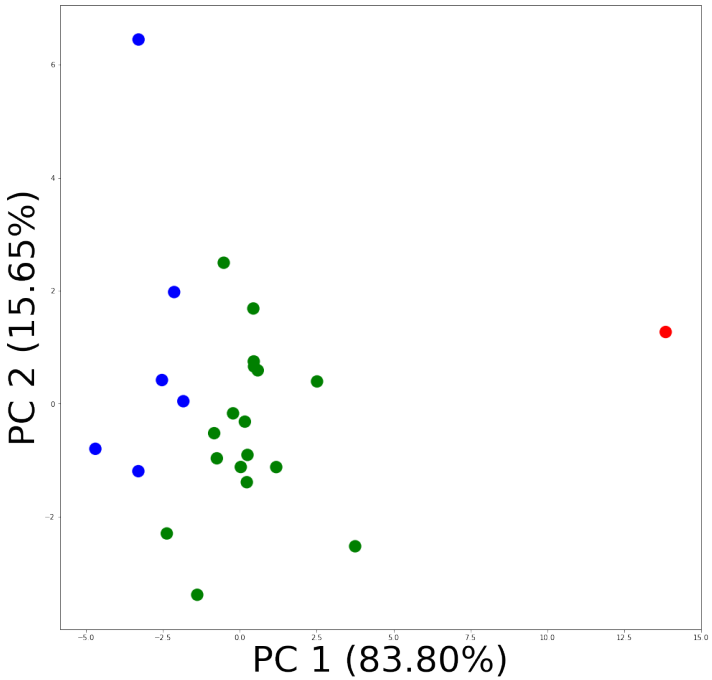
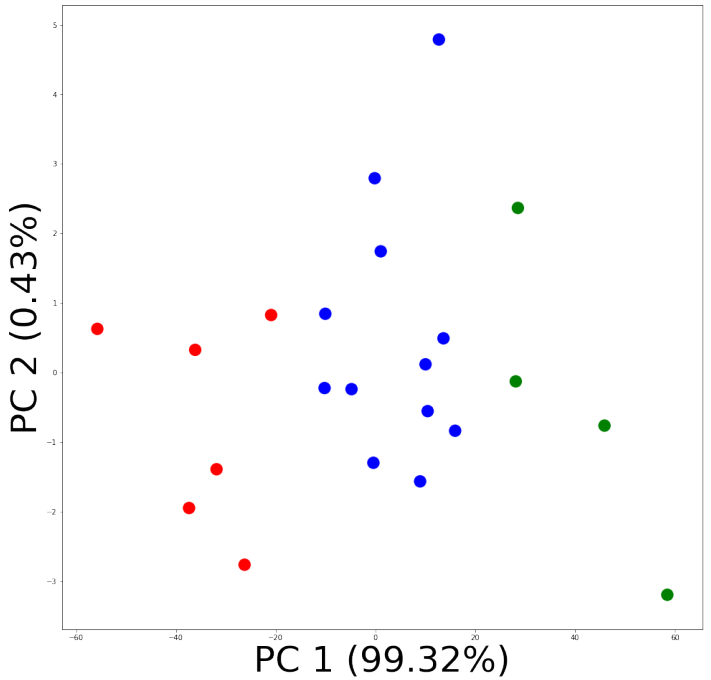


Figure: Data points from front on two principal components and three principal components

K-Means Clustering: Flex

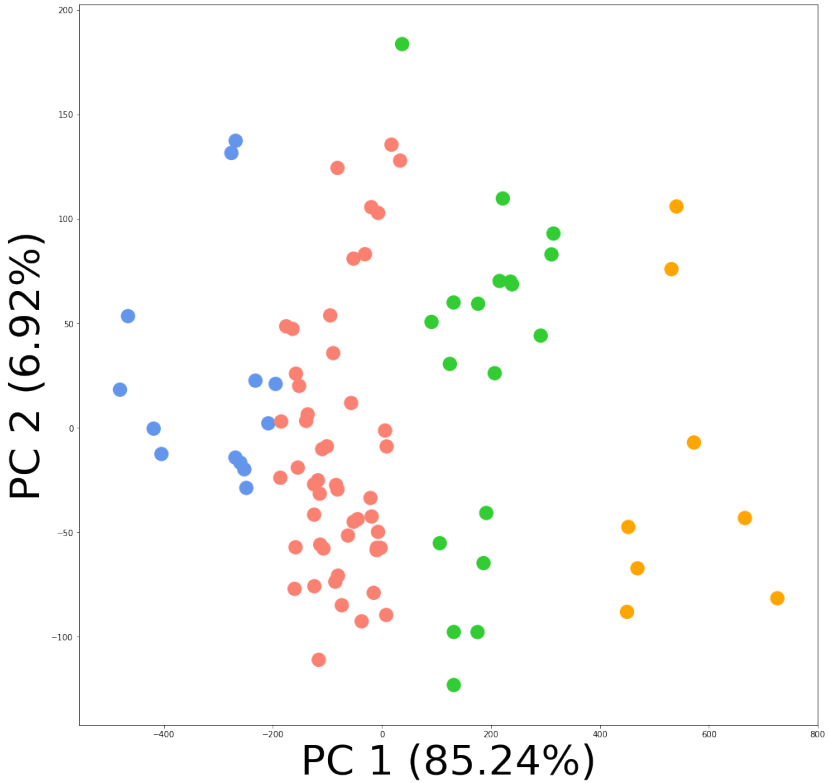
Data points from Flex with coloring from K-Means Clustering.





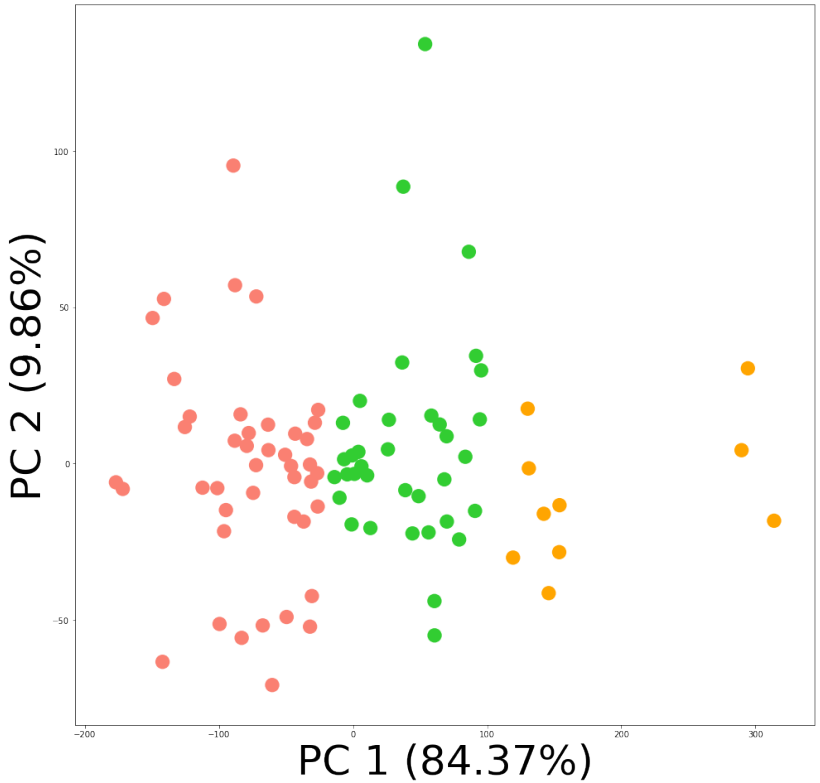
K-Means Clustering: Front

Data points from Front Impact with coloring from K-Means Clustering.



K-Means Clustering: Back

Data points from Back Impact with labels generated from K-Means.



Data Analysis Results

- What's in PC1? Stiffness-related features.
- Analysis **does not** reveal an important role for energy efficiency.
- Analysis **does** reveal a role for where the energy loss is greatest.

Conclusion

- We turned cushions into numbers.
- Both approaches identified similar outliers.
- What can A Large Company use?
  - Many new features to use in PCA.
  - New material models to use in FEA.
- Future work: incorporate customer response data.

Thank you!