35^{th} Annual Workshop on Mathematical Problems in Industry: Analyzing Viscoelastic Materials

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Executive Summary

Viscoelastic materials can exhibit interesting phenomena and describe mechanical properties such as “fading memory”, stress relaxation, and hysteresis during loading and unloading. Force-displacement curves are often generated during mechanical testing to characterize the properties of a viscoelastic system (e.g., a memory foam mattress) and are used to visualize the hysteresis (given by the area contained within the force-displacment curve). Viscoelastic systems are often used as components in a larger system, and the performance of the larger system is often highly dependent on the properties of its viscoelastic components (e.g., a vehicle’s performance is dependent on the properties of its tires). In the case of vehicle performance, the hysteresis should be reduced so that most of the energy can be returned to the system.

During the course of the 35^{th} Workshop on Mathematical Problems in Industry, the group divided into two large teams. The first team was concerned with analyzing a set of data files containing force and displacement data, or torque and angular displacement data from a variety of different viscoelastic systems. For this team, the objective was to calculate features from the data sets that effectively describe the variability between the different viscoelastic systems. Part of the data analysis was to categorize the data by clustering techniques and extracting some features that were easy to identify as the ones that mostly affect the variance of the data. Examples of this include but are not limited to maximum force, maximum displacement, and the time to peak force. Moreover, dimensionality reduction was performed

to reduce the number of features to work with, according to a Principal Component Analysis (PCA). Most of the variability in the data was explained by only two principal components. Based on the factor loading of each dominant component, an interpretation of the relevancy of the measured quantities was then able to be completed.

The second group was concerned with constructing and analyzing mathematical models that are capable to describe the constitutive law of the viscoelastic material involved, to best represent the shape of the hysteresis curve of the system. Within the mathematical modelling group, two subgroups formed: one that analyzed how the material can support an impact-test, and one that studied the response of the material to a flex-test. For the impact group, the first modelling attempts used a combination of traditional linear springs and dashpots, but these models always yield an underlying linear dependence in the force-displacement loading curves. Different approaches have been considered to describe the non-linearity in the spring components, the limiting cases of which are the quasi-linear viscoelastic (QLV) Maxwell model and the QLV Kelvin-Voigt model.

To capture the experimentally observed nonlinear behavior it was decided to treat the spring as a nonlinear element but to leave the dashpot as a linear element. Two approaches were taken at this point. The first assumed a parametric model that treats the spring elements as linear for small displacements, but increasingly nonlinear as the displacement increases. The second approach allowed for a spring response curve that was defined by a spline, matching the experimentally observed force for a few key displacements. In both cases the parameters were chosen to optimize the fit to the resulting time series of both the predicted force curve and the displacement curve to their respective experimentally observed values. This latter model was capable of classifying the cushions in terms of dissipation and non-linearity. Moreover, the dashpot coefficient is larger the front portion of each cushion as compared to the back. Further experiments that increased the resolution of the spline not surprisingly resulted in increasingly faithful force-displacement curves. However only the most simple case a spline with only one interior knot was required to sufficiently classify the cushions and identify outliers.

The team analyzing the material response to a flex-test modeled the system as a cantilevered beam, and studied the torque dependence on angular momentum. The torque as a function of time was fit as a sum of sines from the given data, and was implemented as a boundary condition at the free end of the beam. The team wrote a finite difference scheme to model the angular displacement of the beam, plotted it against the curve fit of torque, and compared it to the plot of the observed experimental data. The model prediction differed from the data in two key ways: (i) it has an opposite parameterization in time compared to the data and (ii) and it shows a non-smooth tip in the torque-angular displacement curve. Despite the limitations of the linear model chosen, suitable for small displacements only, the team considered that it would have served as a benchmark for a possible future model that allows for large deformations.