Week 10
Chapter 10 section 6-9

Rotation Part II

Review: Comparison Between Rotational and Linear Equations

TABLE 10.1

Kinematic Equations for Rotational and Translational Motion Under Constant Acceleration

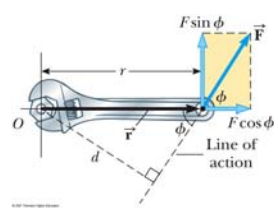
Rotational Motion About a Fixed Axis	Translational Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Torque

- Torque, τ, is the tendency of a force to rotate an object about some axis
  - Torque is a vector, but we will deal with its magnitude here
  - $\tau = r F \sin \phi = F d$
    - F is the force
    - ϕ is the angle the force makes with the horizontal
    - d is the *moment arm* (or lever arm) of the force

Torque, cont

- The moment arm, d, is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force
  - $d = r \sin \Phi$

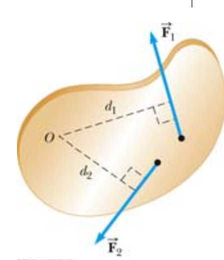


Torque, final

- The horizontal component of the force ($F \cos \phi$) has no tendency to produce a rotation
- Torque will have direction
  - If the turning tendency of the force is counterclockwise, the torque will be positive
  - If the turning tendency is clockwise, the torque will be negative

Net Torque

- The force $\vec{F}_1$ will tend to cause a counterclockwise rotation about O
- The force $\vec{F}_2$ will tend to cause a clockwise rotation about O
- $\Sigma\tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$



Torque vs. Force

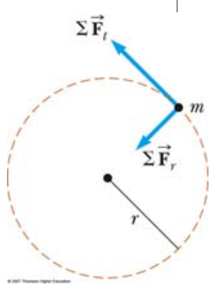
- Forces can cause a change in translational motion
  - Described by Newton's Second Law
- Forces can cause a change in rotational motion
  - The effectiveness of this change depends on the force and the moment arm
  - The change in rotational motion depends on the torque

Torque Units

- The SI units of torque are N·m
  - Although torque is a force multiplied by a distance, it is very different from work and energy
  - The units for torque are reported in N·m and not changed to Joules

Torque and Angular Acceleration

- Consider a particle of mass *m* rotating in a circle of radius *r* under the influence of tangential force $\vec{F}_t$
- The tangential force provides a tangential acceleration:
  - $F_t = ma_t$
- The radial force, $\vec{F}_r$ causes the particle to move in a circular path

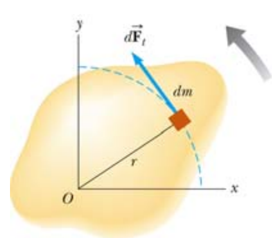


Torque and Angular Acceleration, Particle cont.

- The magnitude of the torque produced by $\sum\vec{F}_t$ around the center of the circle is
  - $\Sigma\tau = \Sigma F_t\, r = (ma_t)\, r$
- The tangential acceleration is related to the angular acceleration
  - $\Sigma\tau = (ma_t)\, r = (mr\alpha)\, r = (mr^2)\, \alpha$
- Since mr^2 is the moment of inertia of the particle,
  - $\Sigma\tau = I\alpha$
  - The torque is directly proportional to the angular acceleration and the constant of proportionality is the moment of inertia

Torque and Angular Acceleration, Extended

- Consider the object consists of an infinite number of mass elements *dm* of infinitesimal size
- Each mass element rotates in a circle about the origin, *O*
- Each mass element has a tangential acceleration



Torque and Angular Acceleration, Extended cont.

- From Newton's Second Law
  - $dF_t = (dm)\, a_t$
- The torque associated with the force and using the angular acceleration gives
  - $d\tau = r\, dF_t = a_t r\, dm = \alpha r^2\, dm$
- Finding the net torque
  - $\sum\tau = \int \alpha r^2 dm = \alpha \int r^2 dm$
  - This becomes $\Sigma\tau = I\alpha$

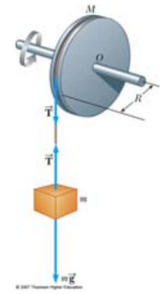
Clicker Question

You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is Δt. You replace the bit with a larger one that results in a doubling of the moment of inertia of the drill's entire rotating mechanism. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. What is the time interval for this second bit to come to rest?

A. 4 Δt
B. 2 Δt
C. Δt
D. 0.5 Δt
E. 0.25 Δt

Torque and Angular Acceleration, Wheel Example

- Analyze:
- The wheel is rotating and so we apply $\Sigma\tau = I\alpha$
  - The tension supplies the tangential force
- The mass is moving in a straight line, so apply Newton's Second Law
  - $\Sigma F_y = ma_y = mg - T$

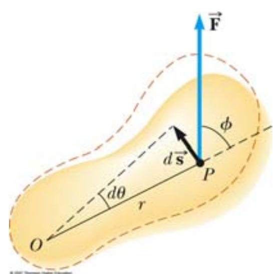


Work in Rotational Motion

- Find the work done by $\vec{\mathbf{F}}$ on the object as it rotates through an infinitesimal distance $ds = r\,d\theta$

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$$
$$= (F \sin\phi)\, r\, d\theta$$

- The radial component of the force does no work because it is perpendicular to the displacement



Power in Rotational Motion

- The rate at which work is being done in a time interval dt is

$$\text{Power} = \wp = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$$

- This is analogous to $\wp = Fv$ in a linear system

Work-Kinetic Energy Theorem in Rotational Motion

- The work-kinetic energy theorem for rotational motion states that *the net work done by external forces in rotating a symmetrical rigid object about a fixed axis equals the change in the object's rotational kinetic energy*

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

Work-Kinetic Energy Theorem, General

- The rotational form can be combined with the linear form which indicates *the net work done by external forces on an object is the change in its **total** kinetic energy, which is the sum of the translational and rotational kinetic energies*

Summary of Useful Equations

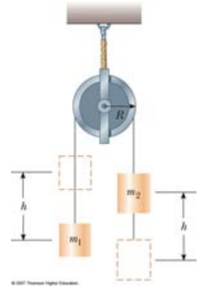
TABLE 10.3

Useful Equations in Rotational and Translational Motion

Rotational Motion About a Fixed Axis	Translational Motion
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If α = constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If a = constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Work $W = \int_{x_i}^{x_f} F_x\, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Energy in an Atwood Machine, Example

- The blocks undergo changes in translational kinetic energy and gravitational potential energy
- The pulley undergoes a change in rotational kinetic energy
- Use the active figure to change the masses and the pulley characteristics



Rolling Object

- The red curve shows the path moved by a point on the rim of the object
  - This path is called a ***cycloid***
- The green line shows the path of the center of mass of the object

Pure Rolling Motion

- In pure rolling motion, an object rolls without slipping
- In such a case, there is a simple relationship between its rotational and translational motions

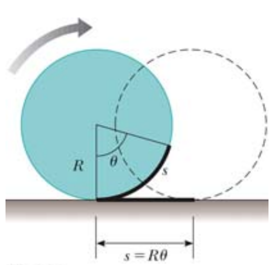
Rolling Object, Center of Mass

- The velocity of the center of mass is

$$v_{CM} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

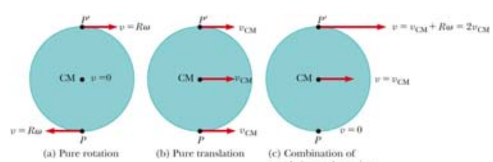
- The acceleration of the center of mass is

$$a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



Rolling Motion Cont.

- Rolling motion can be modeled as a combination of pure translational motion and pure rotational motion
- The contact point between the surface and the cylinder has a translational speed of zero (c)



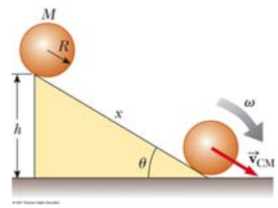
(a) Pure rotation (b) Pure translation (c) Combination of translation and rotation

Total Kinetic Energy of a Rolling Object

- The total kinetic energy of a rolling object is the sum of the translational energy of its center of mass and the rotational kinetic energy about its center of mass
  - $K = \frac{1}{2} I_{CM}\, \omega^2 + \frac{1}{2} M v_{CM}^2$
    - The $\frac{1}{2} I_{CM}\omega^2$ represents the rotational kinetic energy of the cylinder about its center of mass
    - The $\frac{1}{2} Mv^2$ represents the translational kinetic energy of the cylinder about its center of mass

Total Kinetic Energy, Example

- Accelerated rolling motion is possible only if friction is present between the sphere and the incline
  - The friction produces the net torque required for rotation
  - No loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant
  - Use the active figure to vary the objects and compare their speeds at the bottom



Total Kinetic Energy, Example cont

- Apply Conservation of Mechanical Energy
  - Let $U = 0$ at the bottom of the plane
  - $K_f + U_f = K_i + U_i$
  - $K_f = \frac{1}{2}(I_{CM} / R^2)\, v_{CM}^2 + \frac{1}{2} M v_{CM}^2 = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2$
  - $U_i = Mgh$
  - $U_f = K_i = 0$
- Solving for v

$$v = \left[\frac{2gh}{1 + \left(I_{CM} / MR^2\right)}\right]^{1/2}$$

Clicker Question

A basketball rolls across a floor without slipping, with its center of mass moving at a certain velocity. A block of ice of the same mass is set sliding across the floor with the same speed along a parallel line. How do their energies compare?

A. The basketball has more kinetic energy.

B. The ice has more kinetic energy.

C. They have equal kinetic energies.

D. Information is not sufficient to decide.

Sphere Rolling Down an Incline, Example

- **Conceptualize**
  - A sphere is rolling down an incline
- **Categorize**
  - Model the sphere and the Earth as an isolated system
  - No nonconservative forces are acting
- **Analyze**
  - Use Conservation of Mechanical Energy to find v
    - See previous result

Sphere Rolling Down an Incline, Example cont

- **Analyze**, cont
  - Solve for the acceleration of the center of mass
- **Finalize**
  - Both the speed and the acceleration of the center of mass are independent of the mass and the radius of the sphere
- Generalization
  - *All homogeneous solid spheres experience the same speed and acceleration on a given incline*
    - Similar results could be obtained for other shapes