Week 13: Chapter 13

Universal Gravitation

Newton's Law of Universal Gravitation

- Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the distance between them

$$F_g = G\frac{m_1 m_2}{r^2}$$

- G is the ***universal gravitational constant*** and equals 6.673 x 10^{-11} N·m^2 / kg^2

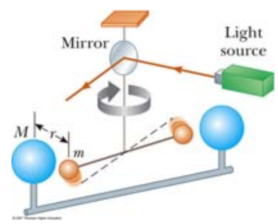
Clicker Question

A planet has two moons of equal mass. Moon 1 is in a circular orbit of radius r. Moon 2 is in a circular orbit of radius 2r. What is the magnitude of gravitational force exerted by the planet on Moon 2 comparing that on Moon 1?

A. 4 time as large as that on Moon 1
B. Twice as large
C. Equal
D. ½
E. 1/4

Finding the Value of G

- In 1789 Henry Cavendish measured G
- The two masses are fixed at the ends of a light horizontal rod
- Two large masses were placed near the small ones
- The angle of rotation was measured



Law of Gravitation, cont

- This is an example of an ***inverse square law***
  - The magnitude of the force varies as the inverse square of the separation of the particles
- The law can also be expressed in vector form

$$\vec{\mathbf{F}}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

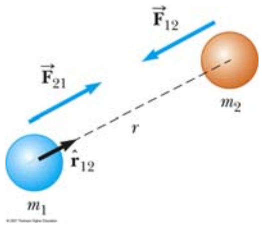
  - The negative sign indicates an attractive force

Notation

- $\vec{\mathbf{F}}_{12}$ is the force exerted by particle 1 on particle 2
- The negative sign in the vector form of the equation indicates that particle 2 is attracted toward particle 1
- $\vec{\mathbf{F}}_{21}$ is the force exerted by particle 2 on particle 1

More About Forces

- $\vec{F}_{12} = -\vec{F}_{21}$
  - The forces form a Newton's Third Law action-reaction pair
- Gravitation is a field force that always exists between two particles, regardless of the medium between them
- The force decreases rapidly as distance increases
  - A consequence of the inverse square law



Gravitational Force Due to a Distribution of Mass

- The gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distribution were concentrated at the center
- The force exerted by the Earth on a particle of mass m near the surface of the Earth is

$$F_g = G\frac{M_E m}{R_E^2}$$

G vs. g

- Always distinguish between *G* and *g*
- *G* is the universal gravitational constant
  - It is the same everywhere
- *g* is the acceleration due to gravity
  - $g = 9.80$ m/s² at the surface of the Earth
  - *g* will vary by location

Finding *g* from *G*

- The magnitude of the force acting on an object of mass *m* in freefall near the Earth's surface is *mg*
- This can be set equal to the force of universal gravitation acting on the object

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$

g Above the Earth's Surface

- If an object is some distance *h* above the Earth's surface, *r* becomes $R_E + h$

$$g = \frac{GM_E}{(R_E + h)^2}$$

- This shows that *g* decreases with increasing altitude
- As $r \to \infty$, the weight of the object approaches zero

Variation of *g* with Height

TABLE 13.1
Free-Fall Acceleration *g* at Various Altitudes Above the Earth's Surface

Altitude *h* (km)	*g* (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

Johannes Kepler

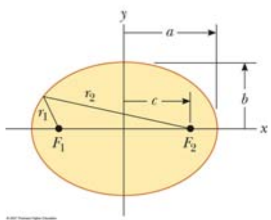
- 1571 – 1630
- German astronomer
- Best known for developing laws of planetary motion
  - Based on the observations of Tycho Brahe

Kepler's Laws

- Kepler's First Law
  - All planets move in elliptical orbits with the Sun at one focus
- Kepler's Second Law
  - The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals
- Kepler's Third Law
  - The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit

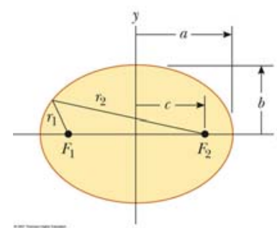
Notes About Ellipses

- F_1 and F_2 are each a **focus** of the ellipse
  - They are located a distance *c* from the center
  - The sum of r_1 and r_2 remains constant
    - Use the active figure to vary the values defining the ellipse
- The longest distance through the center is the **major axis**
  - *a* is the *semimajor axis*



Notes About Ellipses, cont

- The shortest distance through the center is the **minor axis**
  - *b* is the *semiminor axis*
- The **eccentricity** of the ellipse is defined as $e = c / a$
  - For a circle, $e = 0$
  - The range of values of the eccentricity for ellipses is $0 < e < 1$
  - The higher the value of e, the longer and thinner the ellipse



Notes About Ellipses, Planet Orbits

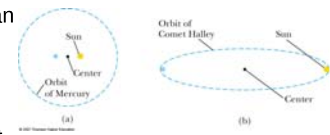
- The Sun is at one focus
  - Nothing is located at the other focus
- **Aphelion** is the point farthest away from the Sun
  - The distance for aphelion is $a + c$
    - For an orbit around the Earth, this point is called the apogee
- **Perihelion** is the point nearest the Sun
  - The distance for perihelion is $a – c$
    - For an orbit around the Earth, this point is called the perigee

Kepler's First Law

- A circular orbit is a special case of the general elliptical orbits
- Is a direct result of the inverse square nature of the gravitational force
- Elliptical (and circular) orbits are allowed for **bound** objects
  - A bound object repeatedly orbits the center
  - An **unbound** object would pass by and not return
    - These objects could have paths that are parabolas ($e = 1$) and hyperbolas ($e > 1$)

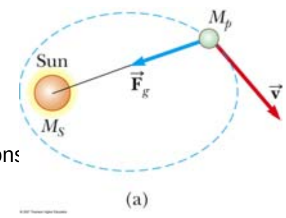
Orbit Examples

- Mercury has the highest eccentricity of any planet (a)
  - $e_{Mercury} = 0.21$
- Halley's comet has an orbit with high eccentricity (b)
  - $e_{Halley's\ comet} = 0.97$
- Remember nothing physical is located at the second focus
  - The small blue dot



Kepler's Second Law

- Is a consequence of conservation of angular momentum
- The force produces no torque, so angular momentum is conserved
- $\vec{L} = \vec{r} \times \vec{p} = M_P \vec{r} \times \vec{v}$ = cons
- Use the active figure to vary the value of e and observe the orbit

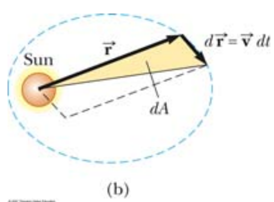


Kepler's Second Law, cont.

- Geometrically, in a time *dt*, the radius vector *r* sweeps out the area *dA*, which is half the area of the parallelogram

  $|\vec{r} \times d\vec{r}|$
- Its displacement is given by

  $d\vec{r} = \vec{v}dt$



Kepler's Second Law, final

- Mathematically, we can say

$$\frac{dA}{dt} = \frac{L}{2M_p} = \text{constant}$$

- The radius vector from the Sun to any planet sweeps out equal areas in equal times
- The law applies to any central force, whether inverse-square or not

Kepler's Third Law

- Can be predicted from the inverse square law
- Start by assuming a circular orbit
- The gravitational force supplies a centripetal force
- K_s is a constant

$$\frac{GM_{Sun}M_{Planet}}{r^2} = \frac{M_{Planet}v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$T^2 = \left(\frac{4\pi^2}{GM_{Sun}}\right) r^3 = K_S r^3$$

Kepler's Third Law, cont

- This can be extended to an elliptical orbit
- Replace *r* with *a*
  - Remember *a* is the semimajor axis

$$T^2 = \left(\frac{4\pi^2}{GM_{Sun}}\right) a^3 = K_S a^3$$

- K_S is independent of the mass of the planet, and so is valid for any planet

Kepler's Third Law, final

- If an object is orbiting another object, the value of *K* will depend on the object being orbited
- For example, for the Moon around the Earth, K_{Sun} is replaced with K_{Earth}

Example, Mass of the Sun

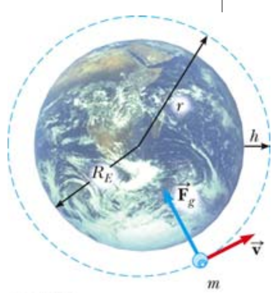
- Using the distance between the Earth and the Sun, and the period of the Earth's orbit, Kepler's Third Law can be used to find the mass of the Sun

$$M_{Sun} = \frac{4\pi^2 r^3}{GT^2}$$

- Similarly, the mass of any object being orbited can be found if you know information about objects orbiting it

Example, Geosynchronous Satellite

- A geosynchronous satellite appears to remain over the same point on the Earth
- The gravitational force supplies a centripetal force
- You can find *h* or *v*



The Gravitational Field

- A **gravitational field** exists at every point in space
- When a particle of mass *m* is placed at a point where the gravitational field is $\vec{\mathbf{g}}$, the particle experiences a force $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$
- The field exerts a force on the particle

The Gravitational Field, 2

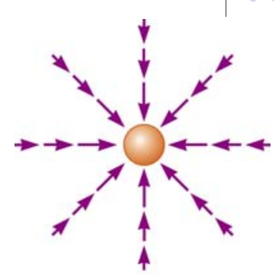
- The gravitational field $\vec{\mathbf{g}}$ is defined as

$$\vec{\mathbf{g}} \equiv \frac{\vec{\mathbf{F}}_g}{m}$$

- The gravitational field is the gravitational force experienced by a test particle placed at that point divided by the mass of the test particle
- The presence of the test particle is not necessary for the field to exist
- The *source particle* creates the field

The Gravitational Field, 3

- The gravitational field vectors point in the direction of the acceleration a particle would experience if placed in that field
- The magnitude is that of the freefall acceleration at that location



(a)

The Gravitational Field, final

- The gravitational field describes the "effect" that any object has on the empty space around itself in terms of the force that *would* be present *if* a second object were somewhere in that space

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$

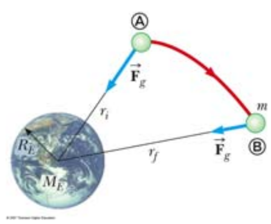
Gravitational Potential Energy

- The gravitational force is conservative
- The change in gravitational potential energy of a system associated with a given displacement of a member of the system is defined as the negative of the work done by the gravitational force on that member during the displacement

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\, dr$$

Gravitational Potential Energy, cont

- As a particle moves from A to B, its gravitational potential energy changes by ΔU



Gravitational Potential Energy for the Earth

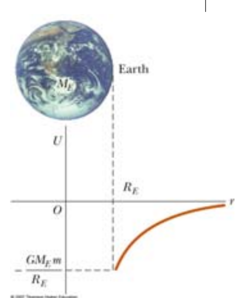
- Choose the zero for the gravitational potential energy where the force is zero
  - This means $U_i = 0$ where $r_i = \infty$

$$U(r) = -\frac{GM_E m}{r}$$

  - This is valid only for $r \geq R_E$ and not valid for $r < R_E$
  - U is negative because of the choice of U_i

Gravitational Potential Energy for the Earth, cont

- Graph of the gravitational potential energy U versus r for an object above the Earth's surface
- The potential energy goes to zero as r approaches infinity



Gravitational Potential Energy, General

- For any two particles, the gravitational potential energy function becomes

$$U = -\frac{Gm_1m_2}{r}$$

- The gravitational potential energy between any two particles varies as $1/r$
  - Remember the force varies as $1/r^2$
- The potential energy is negative because the force is attractive and we chose the potential energy to be zero at infinite separation

Gravitational Potential Energy, General cont

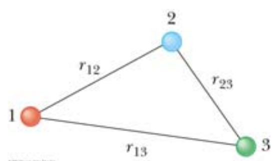
- An external agent must do positive work to increase the separation between two objects
  - The work done by the external agent produces an increase in the gravitational potential energy as the particles are separated
    - *U* becomes less negative

Binding Energy

- The absolute value of the potential energy can be thought of as the ***binding energy***
- If an external agent applies a force larger than the binding energy, the excess energy will be in the form of kinetic energy of the particles when they are at infinite separation

Systems with Three or More Particles

- The total gravitational potential energy of the system is the sum over all pairs of particles
- Gravitational potential energy obeys the *superposition principle*



Systems with Three or More Particles, cont

- Each pair of particles contributes a term of *U*
- Assuming three particles:

$$U_{total} = U_{12} + U_{13} + U_{23}$$
$$= -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

- The absolute value of U_{total} represents the work needed to separate the particles by an infinite distance

Energy and Satellite Motion

- Assume an object of mass *m* moving with a speed *v* in the vicinity of a massive object of mass *M*
  - *M* >> *m*
- Also assume *M* is at rest in an inertial reference frame
- The total energy is the sum of the system's kinetic and potential energies

Energy and Satellite Motion, 2

- Total energy *E* = *K* + *U*

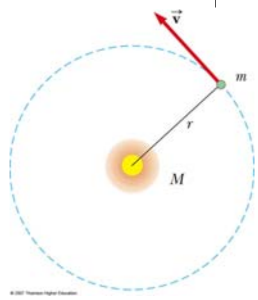
$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

- In a bound system, *E* is necessarily less than 0

Energy in a Circular Orbit

- An object of mass *m* is moving in a circular orbit about *M*
- The gravitational force supplies a centripetal force

$$E = -\frac{GMm}{2r}$$



Energy in a Circular Orbit, cont

- The total mechanical energy is negative in the case of a circular orbit
- The kinetic energy is positive and is equal to half the absolute value of the potential energy
- The absolute value of *E* is equal to the binding energy of the system

Energy in an Elliptical Orbit

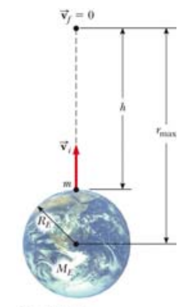
- For an elliptical orbit, the radius is replaced by the semimajor axis

$$E = -\frac{GMm}{2a}$$

- The total mechanical energy is negative
- The total energy is constant if the system is isolated

Escape Speed from Earth

- An object of mass *m* is projected upward from the Earth's surface with an initial speed, v_i
- Use energy considerations to find the minimum value of the initial speed needed to allow the object to move infinitely far away from the Earth



Escape Speed From Earth, cont

- This minimum speed is called the ***escape speed***

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

- Note, v_{esc} is independent of the mass of the object
- The result is independent of the direction of the velocity and ignores air resistance

Escape Speed, General

- The Earth's result can be extended to any planet

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- The table at right gives some escape speeds from various objects

TABLE 13.3
Escape Speeds from the Surfaces of the Planets, Moon, and Sun

Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Moon	2.3
Sun	618

Escape Speed, Implications

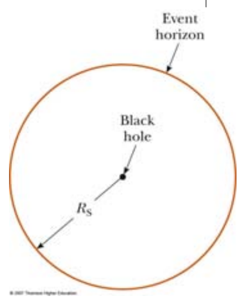
- Complete escape from an object is not really possible
  - The gravitational field is infinite and so some gravitational force will always be felt no matter how far away you can get
- This explains why some planets have atmospheres and others do not
  - Lighter molecules have higher average speeds and are more likely to reach escape speeds

Black Holes

- A black hole is the remains of a star that has collapsed under its own gravitational force
- The escape speed for a black hole is very large due to the concentration of a large mass into a sphere of very small radius
  - If the escape speed exceeds the speed of light, radiation cannot escape and it appears black

Black Holes, cont

- The critical radius at which the escape speed equals *c* is called the **Schwarzschild radius**, R_S
- The imaginary surface of a sphere with this radius is called the **event horizon**
  - This is the limit of how close you can approach the black hole and still escape

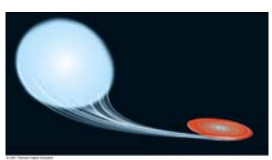


Black Holes and Accretion Disks

- Although light from a black hole cannot escape, light from events taking place near the black hole should be visible
- If a binary star system has a black hole and a normal star, the material from the normal star can be pulled into the black hole

Black Holes and Accretion Disks, cont

- This material forms an ***accretion disk*** around the black hole
- Friction among the particles in the disk transforms mechanical energy into internal energy

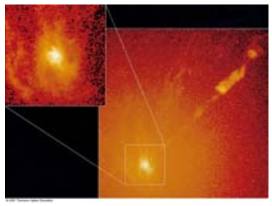


Black Holes and Accretion Disks, final

- The orbital height of the material above the event horizon decreases and the temperature rises
- The high-temperature material emits radiation, extending well into the x-ray region
- These x-rays are characteristics of black holes

Black Holes at Centers of Galaxies

- There is evidence that supermassive black holes exist at the centers of galaxies
- Theory predicts jets of materials should be evident along the rotational axis of the black hole



An HST image of the galaxy M87. The jet of material in the right frame is thought to be evidence of a supermassive black hole at the galaxy's center.