Week2: Chapter 2

Motion in One Dimension

Clicker Testing

- What is your SAT Scores (all 3 parts Math+Reading+Writing)
- A. Above 2200
- B. 1900 to 2200
- C. 1600 to 1900
- D. Below 1600
- E. Unknown or do not want to tell

Lecture Quiz

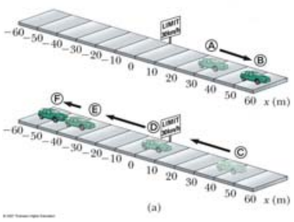
- Starting from one oasis, a camel walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a second oasis. What distance separates the two oases?
- **A.** 15 km
- **B.** 48 km
- **C.** 28 km
- **D.** 53 km
- **E.** 55 km

Kinematics

- Describes motion while ignoring the agents that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

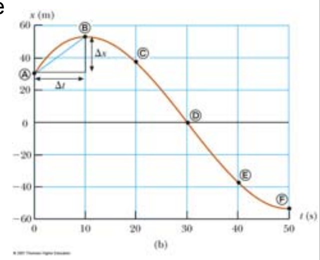
Position

- The object's position is its location with respect to a chosen reference point
  - Consider the point to be the origin of a coordinate system
- In the diagram, allow the road sign to be the reference point



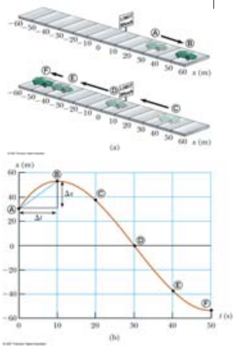
Position-Time Graph

- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



Motion of Car

- Note the relationship between the position of the car and the points on the graph
- Compare the different representations of the motion



Data Table

- The table gives the actual data collected during the motion of the object (car)
- Positive is defined as being to the right

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Displacement

- Defined as the change in position during some time interval
  - Represented as Δx
    $\Delta x \equiv x_f - x_i$
  - SI units are meters (m)
  - Δx can be positive or negative
- Different than distance – the length of a path followed by a particle

Distance vs. Displacement – An Example

- Assume a player moves from one end of the court to the other and back
- Distance is twice the length of the court
  - Distance is always positive
- Displacement is zero
  - $\Delta x = x_f - x_i = 0$ since $x_f = x_i$

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

Vectors and Scalars

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions
- Scalar quantities are completely described by magnitude only

Average Velocity

- The **average velocity** is rate at which the displacement occurs

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

  - The x indicates motion along the x-axis
- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position – time graph

Average Speed

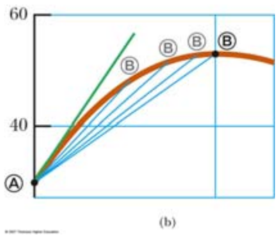
- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time: $v_{avg} \equiv \frac{d}{t}$
- The speed has no direction and is always expressed as a positive number
- Neither average velocity nor average speed gives details about the trip described

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero
- The instantaneous velocity indicates what is happening at every point of time

Instantaneous Velocity, graph

- The instantaneous velocity is the slope of the line tangent to the *x* vs. *t* curve
- This would be the green line
- The light blue lines show that as Δt gets smaller, they approach the green line



Instantaneous Velocity, equations

- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero

Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- The instantaneous speed has no direction associated with it

Particle Under Constant Velocity

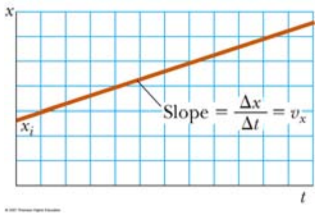
- Constant velocity indicates the instantaneous velocity at any instant during a time interval is the same as the average velocity during that time interval
  - $v_x = v_{x,\,avg}$
  - The mathematical representation of this situation is the equation

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \quad or \quad x_f = x_i + v_x \Delta t$$

  - Common practice is to let $t_i = 0$ and the equation becomes: $x_f = x_i + v_x t$ (for constant v_x)

Particle Under Constant Velocity, Graph

- The graph represents the motion of a particle under constant velocity
- The slope of the graph is the value of the constant velocity
- The y-intercept is x_i



Average Acceleration

- Acceleration is the rate of change of the velocity

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

- Dimensions are L/T^2
- SI units are m/s^2
- In one dimension, positive and negative can be used to indicate direction

Instantaneous Acceleration

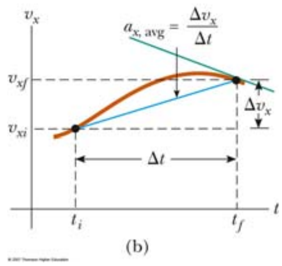
- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

- The term acceleration will mean instantaneous acceleration
  - If average acceleration is wanted, the word average will be included

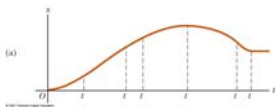
Instantaneous Acceleration -- graph

- The slope of the velocity-time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration



Graphical Comparison

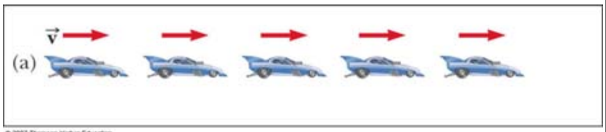
- Given the displacement-time graph (a)
- The velocity-time graph is found by measuring the slope of the position-time graph at every instant
- The acceleration-time graph is found by measuring the slope of the velocity-time graph at every instant



Acceleration and Velocity, 1

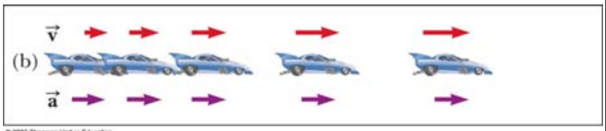
- When an object's velocity and acceleration are in the same direction, the object is speeding up
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down

Acceleration and Velocity, 2



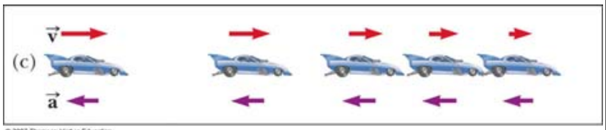
- Images are equally spaced. The car is moving with constant positive velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Acceleration and Velocity, 3



- Images become farther apart as time increases
- Velocity and acceleration are in the same direction
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 4



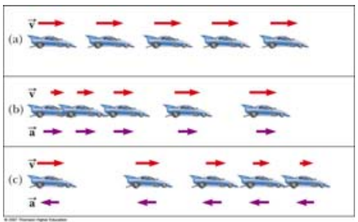
- Images become closer together as time increases
- Acceleration and velocity are in opposite directions
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

Acceleration and Velocity, final

- In all the previous cases, the acceleration was constant
  - Shown by the violet arrows all maintaining the same length
- The diagrams represent motion of a particle under constant acceleration
- A particle under constant acceleration is another useful analysis model

Graphical Representations of Motion

- Observe the graphs of the car under various conditions
- Note the relationships among the graphs
  - Set various initial velocities, positions and accelerations



Kinematic Equations – summary

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
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2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

Kinematic Equations

- The kinematic equations can be used with any particle under uniform acceleration.
- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration
- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

Kinematic Equations, specific

- For constant *a*, $v_{xf} = v_{xi} + a_x t$
- Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration
  - Assumes $t_i = 0$ and $t_f = t$
- Does not give any information about displacement

Kinematic Equations, specific

- For constant acceleration,

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

- The average velocity can be expressed as the arithmetic mean of the initial and final velocities

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + v_{x,avg}\, t = x_i + \frac{1}{2}\left(v_{xi} + v_{fx}\right)t$$

- This gives you the position of the particle in terms of time and velocities
- Doesn't give you the acceleration

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

- Gives final position in terms of velocity and acceleration
- Doesn't tell you about final velocity

Kinematic Equations, specific

- For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

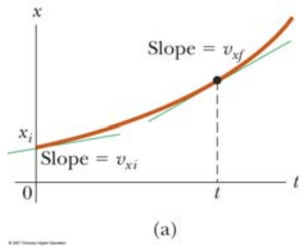
- Gives final velocity in terms of acceleration and displacement
- Does not give any information about the time

When a = 0

- When the acceleration is zero,
  - $v_{xf} = v_{xi} = v_x$
  - $x_f = x_i + v_x t$
- The constant acceleration model reduces to the constant velocity model

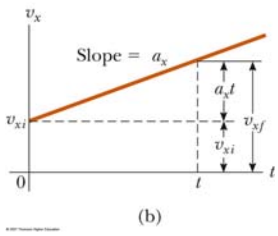
Graphical Look at Motion: displacement – time curve

- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration



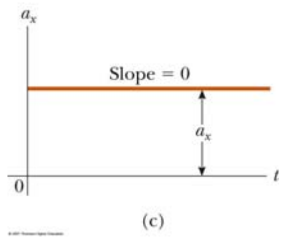
Graphical Look at Motion: velocity – time curve

- The slope gives the acceleration
- The straight line indicates a constant acceleration



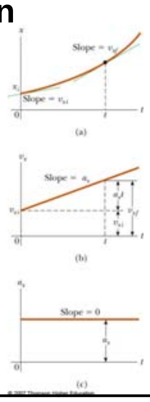
Graphical Look at Motion: acceleration – time curve

- The zero slope indicates a constant acceleration



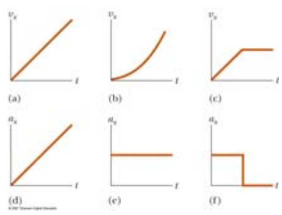
Graphical Motion with Constant Acceleration

- A change in the acceleration affects the velocity and position
- Note especially the graphs when a = 0



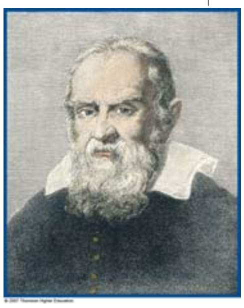
Test Graphical Interpretations

- Match a given velocity graph with the corresponding acceleration graph
- Use Clickers
- A. (a) matches (d)
- B. (a) matches (f)
- C. (b) matches (d)
- D. (b) matches (e)
- E, (c) matches (f)



Galileo Galilei

- 1564 – 1642
- Italian physicist and astronomer
- Formulated laws of motion for objects in free fall
- Supported heliocentric universe



Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object
  - Dropped – released from rest
  - Thrown downward
  - Thrown upward

Acceleration of Freely Falling Object

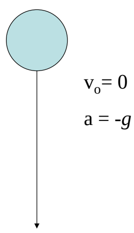
- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s^2
  - g decreases with increasing altitude
  - g varies with latitude
  - 9.80 m/s^2 is the average at the Earth's surface
  - The italicized g will be used for the acceleration due to gravity
    - Not to be confused with g for grams

Acceleration of Free Fall, cont.

- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with $a_y = -g =$ -9.80 m/s^2

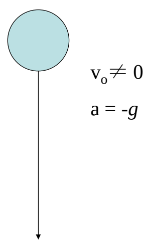
Free Fall – an object dropped

- Initial velocity is zero
- Let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical
- Acceleration is
  - $a_y = -g = -9.80$ m/s^2



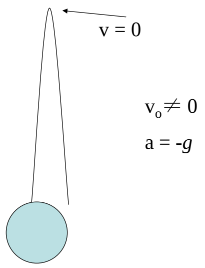
Free Fall – an object thrown downward

- $a_y = -g = -9.80$ m/s^2
- Initial velocity $\neq 0$
  - With upward being positive, initial velocity will be negative



Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- $a_y = -g = -9.80 \text{ m/s}^2$ everywhere in the motion

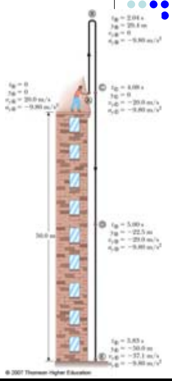


Thrown upward, cont.

- The motion may be symmetrical
  - Then $t_{up} = t_{down}$
  - Then $v = -v_o$
- The motion may not be symmetrical
  - Break the motion into various parts
    - Generally up and down

Free Fall Example

- Initial velocity at A is upward (+) and acceleration is $-g$ (-9.8 m/s^2)
- At B, the velocity is 0 and the acceleration is $-g$ (-9.8 m/s^2)
- At C, the velocity has the same magnitude as at A, but is in the opposite direction
- The displacement is –50.0 m (it ends up 50.0 m below its starting point)

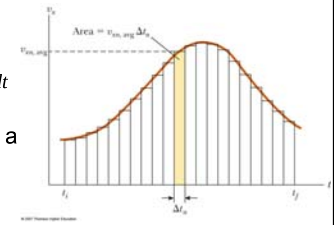


Kinematic Equations from Calculus

- Displacement equals the area under the velocity – time curve

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

- The limit of the sum is a definite integral



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x\,dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x\,dt$$

Kinematic Equations – Calculus Form with Constant Acceleration

- The integration form of $v_f - v_i$ gives

$$v_{xf} - v_{xi} = a_x t$$

- The integration form of $x_f - x_i$ gives

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$