Week 7, Chapter 7, section 6-8 Chapter 8, section 1-5

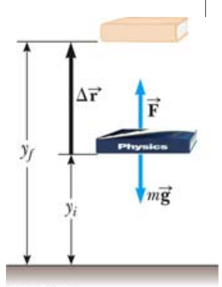
Potential Energy
and
Energy Conservation

Potential Energy

- Potential energy is energy related to the configuration of a system in which the components of the system interact by forces
  - The forces are internal to the system
  - Can be associated with only specific types of forces acting between members of a system

Gravitational Potential Energy

- The system is the Earth and the book
- Do work on the book by lifting it slowly through a vertical displacement
  $\Delta\vec{\mathbf{r}} = \Delta y\hat{\mathbf{j}}$
- The work done on the system must appear as an increase in the energy of the system



Gravitational Potential Energy, cont

- There is no change in kinetic energy since the book starts and ends at rest
- Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth

$$W = \left(\vec{\mathbf{F}}_{\text{app}}\right)\cdot\Delta\vec{\mathbf{r}}$$

$$W = (mg\hat{\mathbf{j}})\cdot\left[\left(y_f - y_i\right)\hat{\mathbf{j}}\right]$$

$$W = mgy_f - mgy_i$$

Gravitational Potential Energy, final

- The quantity *mgy* is identified as the gravitational potential energy, U_g
  - $U_g = mgy$
- Units are joules (J)
- Is a scalar
- Work may change the gravitational potential energy of the system
  - $W_{\text{net}} = \Delta U_g$

Click Question

- John and Tom ski down from the same point of a mountain peak. John chose a slope of 10 degrees while Tom chose a slope of 5 degrees. They both reach the same ground level. What one has a higher potential energy in the end?

A. John
B. Tom
C. Same
D. Can not be determined

Gravitational Potential Energy, Problem Solving

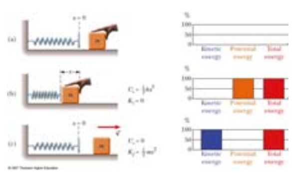
- The gravitational potential energy depends only on the vertical height of the object above Earth's surface
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero
  - The choice is arbitrary because you normally need the difference in potential energy, which is independent of the choice of reference configuration

Elastic Potential Energy

- ***Elastic Potential Energy*** is associated with a spring
- The force the spring exerts (on a block, for example) is $F_s = - kx$
- The work done by an external applied force on a spring-block system is
  - $W = ½ kx_f^2 – ½ kx_i^2$
  - The work is equal to the difference between the initial and final values of an expression related to the configuration of the system

Elastic Potential Energy, cont

- This expression is the elastic potential energy: $U_s = ½ kx^2$
- The elastic potential energy can be thought of as the energy stored in the deformed spring
- The stored potential energy can be converted into kinetic energy
- Observe the effects of different amounts of compression of the spring

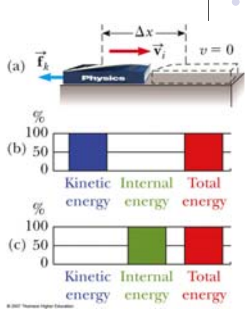


Elastic Potential Energy, final

- The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$)
  - The energy is stored in the spring only when the spring is stretched or compressed
- The elastic potential energy is a maximum when the spring has reached its maximum extension or compression
- The elastic potential energy is always positive
  - x^2 will always be positive

Internal Energy

- The energy associated with an object's temperature is called its *internal energy*, E_{int}
- In this example, the surface is the system
- The friction does work and increases the internal energy of the surface



Conservative Forces

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle
- The work done by a conservative force on a particle moving through any closed path is zero
  - A closed path is one in which the beginning and ending points are the same

Conservative Forces, cont

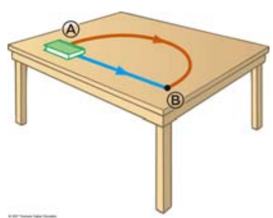
- Examples of conservative forces:
  - Gravity
  - Spring force
- We can associate a potential energy for a system with any conservative force acting between members of the system
  - This can be done only for conservative forces
  - In general: $W_C = -\Delta U$

Nonconservative Forces

- A nonconservative force does not satisfy the conditions of conservative forces
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system

Nonconservative Forces, cont

- The work done against friction is greater along the brown path than along the blue path
- Because the work done depends on the path, friction is a nonconservative force



Conservative Forces and Potential Energy

- Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system
- The work done by such a force, *F*, is

$$W_C = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

  - ΔU is negative when *F* and *x* are in the same direction

Conservative Forces and Potential Energy

- The conservative force is related to the potential energy function through

$$F_x = -\frac{dU}{dx}$$

- The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*
  - Can be extended to three dimensions

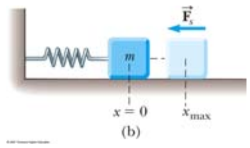
Conservative Forces and Potential Energy – Check

- Look at the case of a deformed spring

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

  - This is Hooke's Law and confirms the equation for U
- U is an important function because a conservative force can be derived from it

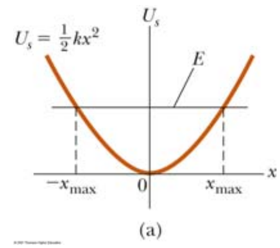
Energy Diagrams and Equilibrium



- Motion in a system can be observed in terms of a graph of its position and energy
- In a spring-mass system example, the block oscillates between the turning points, $x = \pm x_{max}$
- The block will always accelerate back toward $x = 0$

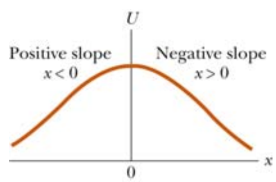
Energy Diagrams and Stable Equilibrium

- The $x = 0$ position is one of ***stable equilibrium***
- Configurations of stable equilibrium correspond to those for which $U(x)$ is a minimum
- $x = x_{max}$ and $x = -x_{max}$ are called the turning points



Energy Diagrams and Unstable Equilibrium

- $F_x = 0$ at $x = 0$, so the particle is in equilibrium
- For any other value of x, the particle moves away from the equilibrium position
- This is an example of ***unstable equilibrium***
- Configurations of unstable equilibrium correspond to those for which $U(x)$ is a maximum



Neutral Equilibrium

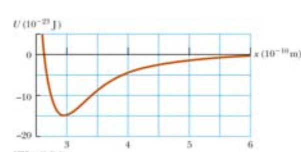
- ***Neutral equilibrium*** occurs in a configuration when U is constant over some region
- A small displacement from a position in this region will produce neither restoring nor disrupting forces

Potential Energy in Molecules

- There is potential energy associated with the force between two neutral atoms in a molecule which can be modeled by the *Lennard-Jones* function

$$U(x) = 4\in\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

Potential Energy Curve of a Molecule



- Find the minimum of the function (take the derivative and set it equal to 0) to find the separation for stable equilibrium
- The graph of the Lennard-Jones function shows the most likely separation between the atoms in the molecule (at minimum energy)

Types of Systems

- Nonisolated systems
  - Energy can cross the system boundary in a variety of ways
  - Total energy of the system changes
- Isolated systems
  - Energy does not cross the boundary of the system
  - Total energy of the system is constant

Ways to Transfer Energy Into or Out of A System

- ***Work*** – transfers by applying a force and causing a displacement of the point of application of the force
- ***Mechanical Waves*** – allow a disturbance to propagate through a medium
- ***Heat*** – is driven by a temperature difference between two regions in space

More Ways to Transfer Energy Into or Out of A System

- ***Matter Transfer*** – matter physically crosses the boundary of the system, carrying energy with it
- ***Electrical Transmission*** – transfer is by electric current
- ***Electromagnetic Radiation*** – energy is transferred by electromagnetic waves

Examples of Ways to Transfer Energy

- a) Work

- b) Mechanical Waves

- c) Heat

Examples of Ways to Transfer Energy, cont.

- d) Matter transfer

- e) Electrical Transmission

- f) Electromagnetic radiation

Conservation of Energy

- ***Energy is conserved***
  - This means that energy cannot be created nor destroyed
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer

Conservation of Energy, cont.

- Mathematically, $\Delta E_{system} = \Sigma T$
  - E_{system} is the total energy of the system
  - T is the energy transferred across the system boundary
    - Established symbols: $T_{work} = W$ and $T_{heat} = Q$
    - Others just use subscripts
- The Work-Kinetic Energy theorem is a special case of Conservation of Energy
  - The full expansion of the above equation gives $\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$

Isolated System

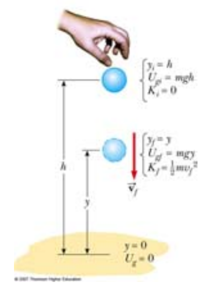
- For an isolated system, $\Delta E_{mech} = 0$
  - Remember $E_{mech} = K + U$
  - This is ***conservation of energy*** for an isolated system with no nonconservative forces acting
- If nonconservative forces are acting, some energy is transformed into internal energy
- Conservation of Energy becomes $\Delta E_{system} = 0$
  - E_{system} is all kinetic, potential, and internal energies
  - This is the most general statement of the isolated system model

Isolated System, cont

- The changes in energy can be written out and rearranged
- $K_f + U_f = K_i + U_i$
  - Remember, this applies only to a system in which conservative forces act

Example – Free Fall

- Determine the speed of the ball at y above the ground
- Conceptualize
  - Use energy instead of motion
- Categorize
  - System is isolated
  - Only force is gravitational which is conservative



Example – Free Fall, cont

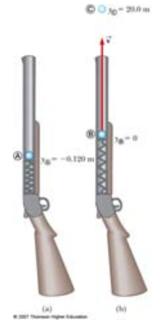
- Analyze
  - Apply Conservation of Energy
  - $K_f + U_{gf} = K_i + U_{gi}$
    - $K_i = 0$, the ball is dropped
    - Solving for v_f

$$v_f = \sqrt{v_i^2 + 2g(h - y)}$$

- Finalize
  - The equation for v_f is consistent with the results obtained from kinematics

Example – Spring Loaded Gun

- Conceptualize
  - The projectile starts from rest
  - Speeds up as the spring pushes against it
  - As it leaves the gun, gravity slows it down
- Categorize
  - System is projectile, gun, and Earth
  - Model as a system with no nonconservative forces acting

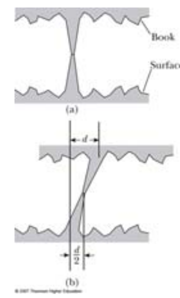


Example – Spring Gun, cont

- Analyze
  - Projectile starts from rest, so $K_i = 0$
  - Choose zero for gravitational potential energy where projectile leaves the gun
  - Elastic potential energy will also be 0 here
  - After the gun is fired, the projectile rises to a maximum height, where its kinetic energy is 0
- Finalize
  - Did the answer make sense
  - Note the inclusion of two types of potential energy

Kinetic Friction

- Kinetic friction can be modeled as the interaction between identical teeth
- The frictional force is spread out over the entire contact surface
- The displacement of the point of application of the frictional force is not calculable



Work – Kinetic Energy Theorem

- Is valid for a particle or an object that can be modeled as an object
- When a friction force acts, you cannot calculate the work done by friction
- However, Newton's Second Law is still valid even though the work-kinetic energy theorem is not

Work – Kinetic Energy With Friction

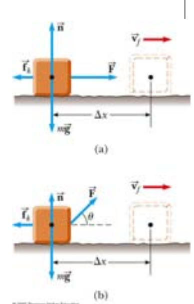
- In general, if friction is acting in a system:
  - $\Delta K = \Sigma W_{\text{other forces}} - f_k d$
  - This is a modified form of the work – kinetic energy theorem
    - Use this form when friction acts on an object
  - If friction is zero, this equation becomes the same as Conservation of Mechanical Energy

Including Friction, final

- A friction force transforms kinetic energy in a system to internal energy
- The increase in internal energy of the system is equal to its decrease in kinetic energy
- $\Delta E_{int} = f_k d$

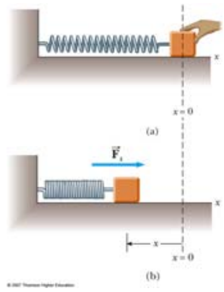
Example – Block on Rough Surface

- The block is pulled by a constant force over a rough horizontal surface
- Conceptualize
  - The rough surface applies a friction force on the block
  - The friction force is in the direction opposite to the applied force



Example – Block-spring System

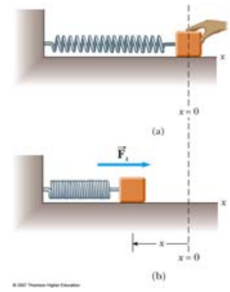
- The problem
  - The mass is attached to a spring, the spring is compressed and then the mass is released
  - A constant friction force acts
- Conceptualize
  - The block will be pushed by the spring and move off with some speed
- Categorize
  - Block and surface is the system
  - System is nonisolated



Clicker Problem

- For the system shown on right, after the block is released from x=-a, the maximum kinetic energy occurs at

A. x=-a
B. x=0
C. x=a/2
D. kinetic energy is a constant
E. x=a



Example – Spring-block, cont

- Analyze
  - Evaluate f_k d
  - Evaluate $\Sigma W_{\text{other forces}}$
- Finalize
  - Think about the result

Adding Changes in Potential Energy

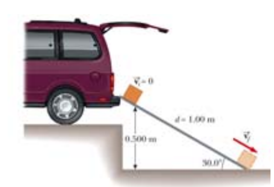
- If friction acts within an isolated system

  $\Delta E_{mech} = \Delta K + \Delta U = -f_k\, d$
  - ΔU is the change in all forms of potential energy
- If friction acts within a nonisolated system

  $\Delta E_{mech} = -f_k\, d + \Sigma W_{\text{other forces}}$

Example – Ramp with Friction

- Problem: the crate slides down the rough ramp
  - Find speed at bottom
- Conceptualize
  - Energy considerations
- Categorize
  - Identify the crate, the surface, and the Earth as the system
  - Isolated system with nonconservative force acting

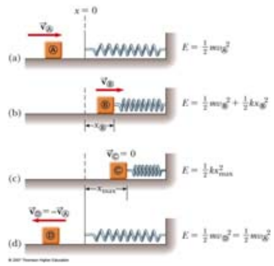


Example – Ramp, cont

- Analyze
  - Let the bottom of the ramp be y = 0
  - At the top: $E_i = K_i + U_{gi} = 0 + mgy_i$
  - At the bottom: $E_f = K_f + U_{gf} = \frac{1}{2} m v_f^2 + mgy_f$
  - Then $\Delta E_{mech} = E_f - E_i = -f_k\, d$
  - Solve for v_f
- Finalize
  - Could compare with result if ramp was frictionless
  - The internal energy of the system increased

Example – Spring Mass Collision

- Without friction, the energy continues to be transformed between kinetic and elastic potential energies and the total energy remains the same
- If friction is present, the energy decreases
  - $\Delta E_{mech} = -f_k d$



Example – Spring Mass, 2

- Conceptualize
  - All motion takes place on a horizontal plane
    - So no changes in gravitational potential energy
- Categorize
  - The system is the block and the system
  - Without friction, it is an isolated system with no nonconservative forces
- Analyze
  - Before the collision, the total energy is kinetic

Problem – Spring Mass 3

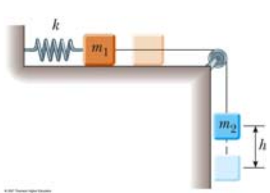
- Analyze
  - Before the collision, the total energy is kinetic
  - When the spring is totally compressed, the kinetic energy is zero and all the energy is elastic potential
  - Total mechanical energy is conserved
- Finalize
  - Decide which root has physical meeting

Problem – Spring Mass 4

- Now add friction
  - Categorize
    - Now is isolated with nonconservative force
  - Analyze
    - Use $\Delta E_{mech} = -f_k d$
  - Finalize
    - The value is less than the case for no friction
      - As expected

Example – Connected Blocks

- Conceptualize
  - Configurations of the system when at rest are good candidates for initial and final points
- Categorize
  - The system consists of the two blocks, the spring, and Earth
  - System is isolated with a nonconservative force acting



Example – Blocks, cont

- Categorize, cont
  - Gravitational and potential energies are involved
  - The kinetic energy is zero if our initial and final configurations are at rest
  - Model the sliding block as a particle in equilibrium in the vertical direction
- Analyze
  - Two forms of potential energy are involved

Connected Blocks, cont

- Analyze, cont
  - Block 2 undergoes a change in gravitational potential energy
  - The spring undergoes a change in elastic potential energy
  - The coefficient of kinetic energy can be measured
- Finalize
  - This allows a method for measuring the coefficient of kinetic energy

Instantaneous Power

- Power is the time rate of energy transfer
- The ***instantaneous power*** is defined as

$$\wp = \frac{dE}{dt}$$

- Using work as the energy transfer method, this can also be written as

$$\wp_{avg} = \frac{W}{\Delta t}$$

Power

- The time rate of energy transfer is called power
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

  - when the method of energy transfer is work

Instantaneous Power and Average Power

- The instantaneous power is the limiting value of the average power as Δt approaches zero

$$\wp = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

- The power is valid for any means of energy transfer

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^2
- A unit of power in the US Customary system is horsepower
  - 1 hp = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J