Lecture 5

Instability

- Why can some solar features stay long time like prominences and coronal loops, while some produce quick events such as flares? This is the problem of instability.

- Basic Methods
  - Linearization method

    Consider one dimension potential energy wave

    force : $F = -\frac{dW}{dx}$ $\quad m\ddot{x} = F = -\frac{dW}{dx}$

    for small displacement

$$m\ddot{x} = -x\left(\frac{d^2W}{dx^2}\right)_{x=0} = F_1(x)$$

$F_1(x)$ is first order approximation of F.

Assume $x = x_0 e^{i\omega t}$ (normal mode solution)

$$\omega^2 = \frac{1}{m}\left(\frac{d^2W}{dx^2}\right)_0$$

if $\left(\frac{d^2W}{dx^2}\right)_0 > 0$ $\quad \omega^2 > 0$ $\quad$ Oscillation, Stable

< 0 $\quad \omega^2 < 0$ $\quad$ Unstable

$= 0$ $\quad$ Neutrally Stable

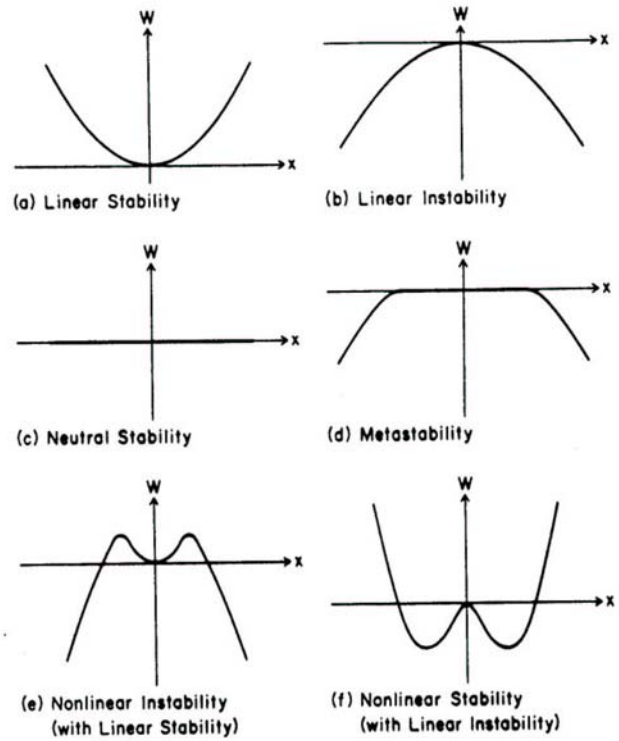


Fig. 7.1. Potential energy curves for a one-dimensional system that is in equilibrium at $x = 0$.

An alternative approach for tackling particle stability is to consider the change (δW) in *potential energy* due to a displacement (x) from equilibrium. To first order in x, $\delta W = x(\mathrm{d}W/\mathrm{d}x)_0$, which vanishes by assumption. To second order,

$$\delta W \equiv W(x) - W(0) = \frac{x^2}{2}\left(\frac{\mathrm{d}^2 W}{\mathrm{d}x^2}\right)_0,$$

which may be derived alternatively by noting that the change in potential energy is just minus the work done by the linear force ($F_1(x)$), namely,

$$\delta W = -\int_0^x F_1(x)\,\mathrm{d}x = -\tfrac{1}{2}xF_1(x). \tag{7.1}$$

The particle is in *stable* equilibrium if $\delta W > 0$ for *all* small displacements from $x = 0$, both with $x > 0$ and $x < 0$. It is *unstable* if $\delta W < 0$ for *at least one* small displacement, either with $x > 0$ or $x < 0$. The frequency (ω) can be written in terms of δW by

- Energy Method

$$\delta W = W(x) - W(0) = \frac{x^2}{2}\left(\frac{d^2W}{dx^2}\right)_0 = -\frac{1}{2}xF_1(x)$$

$\delta W > 0$ for all small displacement - stable

$\delta W < 0$ for one point - unstable

- Linearized Equations
  - This is very similar to the study of waves

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B}\right)$$

$$\rho \frac{Dv}{dt} = -\nabla P + \vec{j} \times \vec{B} + \rho \vec{g}$$

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \vec{v}\right) = 0$$

$$\vec{j} = \frac{\nabla \times \vec{B}}{\mu}, \qquad \nabla \cdot \vec{B} = 0, \qquad \frac{D}{Dt}\left(\frac{P}{\rho r}\right) = 0$$

At equilibrium

$$0 = \nabla P_0 + \overrightarrow{j_0} \times \overrightarrow{B} + \rho_0 \overrightarrow{g}$$

$$\overrightarrow{j_0} = \frac{\nabla \times \overrightarrow{B}}{\mu} \qquad \nabla \cdot B_0 = 0$$

Perturb the equilibrium by

$$\rho = \rho_0 + \rho_1, \quad \vec{v} = \overrightarrow{v_1}, \quad P = P_0 + P_1, \quad \vec{B} = \overrightarrow{B_0} + \overrightarrow{B_1}, \quad \vec{j} = \vec{j}_0 + \overrightarrow{j_1}$$

$$Then: \quad \frac{\partial \overrightarrow{B_1}}{\partial t} = \nabla \times \left(\overrightarrow{v_1} \times \overrightarrow{B_0} \right)$$

$$\rho_0 \frac{\partial \overrightarrow{v_1}}{\partial t} = -\nabla P_1 + \overrightarrow{j_1} \times \overrightarrow{B_0} + \overrightarrow{j_0} \times \overrightarrow{B_1} + \rho_1 \overrightarrow{g}$$

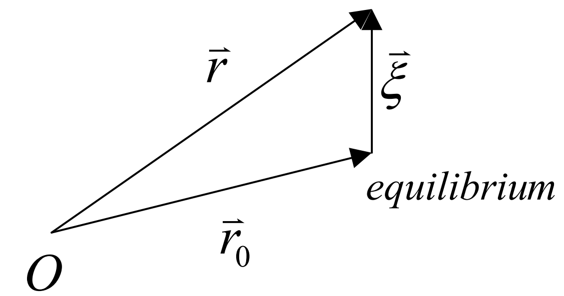
$$\frac{\partial \rho_1}{\partial t} = \nabla \cdot \left(\rho_0 \overrightarrow{v_1} \right) = 0$$

$$\overrightarrow{j_1} = \frac{\nabla \times \overrightarrow{B_1}}{\mu}, \quad \nabla \overrightarrow{B_1} = 0$$

$$\frac{\partial P_1}{\partial t} = \frac{\partial P_0}{\rho_0} \frac{\partial \rho_1}{\partial t} - P0 \left(v1 \cdot \nabla \right) \log_e \left(\frac{P_0}{\rho_0^{\gamma}} \right)$$

To lowest order $\nabla_0 = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

Define $\vec{v}_1 = \frac{\partial \vec{\xi}}{\partial t}$



$\vec{B}_1 = \nabla_0 \times \left(\vec{\xi} \times \vec{B}_0\right) \qquad \rho_1 = -\nabla \cdot \left(\rho_0 \vec{\xi}\right)$

Equation of motion becomes

$$\rho_0 \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}\left(\xi\left(r_0, t\right)\right)$$

$$\vec{F}\left(\xi\left(r_0, t\right)\right) = -\nabla P_1 + \rho_1 \vec{g} + \vec{j}_1 \times \vec{B} + \vec{j}_0 \times \vec{B}_1$$

$$= -\nabla P_1 + \nabla\left(\rho_0 \vec{\xi}\right)\vec{g} + \left(\nabla \times \left(\nabla \times \left(\vec{\xi} \times \vec{B}_0\right)\right)\right) \times \frac{\vec{B}_0}{\mu}$$

$$+ \left(\nabla \times \vec{B}_0\right) \times \left(\nabla \times \left(\vec{\xi} \times \vec{B}_0\right)\right) / \mu$$

solution $\vec{\xi}\left(r_0, t\right) = \vec{\xi}\left(r_0\right) e^{i\omega t}$

$$-\omega^2 \rho_0 \xi\left(r_0\right) = \vec{F}\left(\xi\left(r_0\right)\right)$$

As a special cose $\rho_0 \quad P_0 \quad B_0$ are constants.

$$\vec{F} = \rho_0 c_s^2 \nabla\left(\nabla \cdot \xi\right) - \nabla\left(\rho_0 \vec{\xi}\right)\vec{g} + \rho_0 v_A^2 \left(\nabla \times \left(\nabla \times \left(\vec{\xi} \times \vec{B}_0\right)\right)\right) \times \vec{B}_0$$

$\omega^2 > 0$ system oscillates - stable

$\omega^2 < 0$ unstable

- Rayleigh-Taylor Instability
  - See Fig 7.3 interface between two medium

$$\text{if} \quad B \equiv 0 \qquad \rho_0^{(+)} < \rho_0^{(-)} \quad \text{stable}$$

$$\rho_0^{(+)} > \rho_0^{(-)} \quad \textit{un}\text{stable}$$

  - With B field, magnetic tension provides stabilizing force.

Case 1: Plasma supported by magnetic fields

$$\text{assume} \qquad P_0^{(-)} << \left(B_0^{(-)}\right)^2 \big/ 2\mu$$

$$P_0^{(+)} = \left(B_0^{(-)}\right)^2 \big/ 2\mu$$

$$v_1 = e^{i\omega t} v_1(z) e^{iky} \qquad \text{similar for all other variables}$$

Motion equation in z direction

$$\rho_0 \frac{\partial^2 v_{1z}}{\partial t^2} = -\frac{\partial}{\partial t}\left(\frac{\partial P_1}{\partial z}\right) - \frac{\partial j_{1y}}{\partial t} B_{0x} - j_{0y} \frac{\partial B_{1x}}{\partial t} - \frac{\partial \rho_1}{\partial t} g$$

$$\frac{\partial \rho_1}{\partial t} = -\frac{d}{dz}\left(\rho_0 v_{1z}\right), \qquad j_{1y} = \frac{1}{\mu}\frac{\partial B_{1x}}{\partial z}, \quad \frac{\partial B_{1x}}{\partial t} = 0$$

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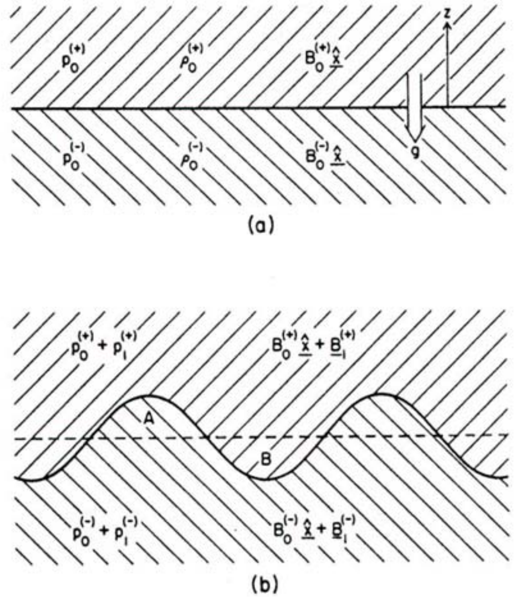

Fig. 7.3. (a) The interface between two plasmas in equilibrium. (b) The perturbed interface.

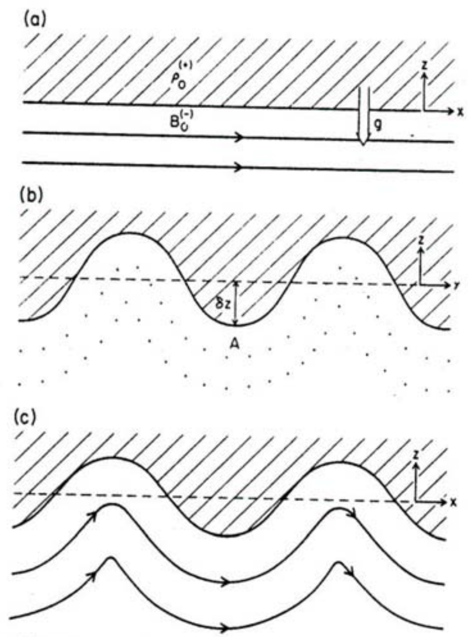

Fig. 7.4. Plasma (shaded) supported by a magnetic field ($B_0^{(-)}\hat{x}$). (a) Equilibrium configuration. (b) Perturbations rippled in the y-direction. (c) Perturbations rippled in the x-direction.

$$-\rho_0\omega^2 v_{1z} = -\omega\frac{dP_1}{dz} + g\frac{d}{dz}(\rho_0 v_{1z})$$

integrate over interface

$$0 = -i\omega[P_1] + g[\rho_0 v_{1z}]$$

$$[P_1] = P_1^{(+)} - P_1^{(-)} \qquad \longleftarrow [\ ]\ jump$$

from 7.10 $\dfrac{dv_{1z}}{dz} + ikv_{1y} = 0, \quad i\rho\omega v_{1y} = -ikP_1$

So $P_1 = -\dfrac{i\omega\rho_0}{k^2}\dfrac{dv_{1z}}{dz}$

$$0 = -\frac{\omega^2}{k^2}\left[\rho_0\frac{dv_{1z}}{dz}\right] + gv_{1z}[\rho_0]$$

$$0 = -\frac{\omega^2}{k^2}\rho_0^{(+)}\left(\frac{dv_{1z}}{dz}\right)^{(+)} + gv_{1z}^{(+)}\rho_0^{(+)}$$

if initial equilibrium value $B_0,\ P_0,\ \rho_0$ are uniform

$$\frac{\partial B_1}{\partial t} = 0 \qquad \frac{\partial \rho_1}{\partial t} = 0, \quad \nabla\times\vec{v}_1 = 0, \ \nabla^2\vec{v} = 0$$

$$\frac{d^2 v_{1z}}{dz^2} = +k^2 v_{1z} \qquad v_{1z} = \begin{cases} e^{-kz} & z > 0 \\ e^{kz} & z < 0 \end{cases}$$

$\omega^2 = -gk$ grow like $e^{\sqrt{gk}\ t}$

more general form $e^{i\omega t}v_1(z)e^{-(k_x x + k_y y)}$

$$\omega^2 = -gk + \frac{k_x^2\left(B_0^{(-)}\right)^2}{\mu\rho_0^{(+)}} \qquad k_{critical} = \frac{g\mu\rho_0^{(+)}}{\left(B_0^{(-)}\right)^2}$$

$k > k_{critical}$ stable --- small wavelength

$k < k_{critical}$ unstable

Case 2: Uniform field (Fig. 7.3)

It can be shown that

$$\omega^2 = -gk\frac{\rho_0^{(+)} - \rho_0^{(-)}}{\rho_0^{(+)} + \rho_0^{(-)}} + \frac{2B_0^2 k_x^2}{\mu\left(\rho_0^{(+)} + \rho_0^{(-)}\right)}$$

$$k_c = \frac{\left(\rho_0^{(+)} - \rho_0^{(-)}\right)g\mu}{2B_0^2}$$

interface is unstable $\quad 0 < k < k_c$

fastest growing mode $\quad k = \frac{1}{2}k_c$

- Energy Method Applications

$$\delta W > 0 \qquad \textit{stable}$$

$$\delta W < 0 \qquad \text{in at least one point, unstable}$$

perturbation as above

$$\rho_0 + \rho_1, \quad P_0 + P_1, \quad B_0 + B_1, \quad \xi(r_0, t) = \xi(r_0) e^{+i\omega t}$$

$$\delta W = \int \left(\frac{B_0^2}{2\mu} + P_0 U_0 + \rho_0 g z \right) dV$$

$$= -\frac{1}{2} \int (\xi \cdot F) dV$$

$$= \frac{1}{2} \int \left[\xi \cdot \nabla P - \xi \cdot \rho_1 g - \xi \cdot \left(\vec{j}_1 \times \vec{B}_0 + \vec{j}_0 \times \vec{B}_1 \right) \right] dV$$

$$= \frac{1}{2} \left[\frac{B_1^2}{\mu} - \vec{j}_0 \left(\vec{B}_1 \times \xi \right) + \gamma \frac{P_0}{\rho_0} \nabla \cdot \left(\rho_0 \vec{\xi} \right) \left(\nabla \vec{\xi} \right) + \left(\vec{\xi} \cdot \vec{g} \right) \nabla \cdot \left(\rho_0 \vec{\xi} \right) \right] dV$$

- Kink instability
  - Consider a cylindrically symmetric magnetic flux tube. If it is kinked, point A has stronger field than point B. ---- perturbation can go further.
    In polar coordinate system

$$\vec{B}_0 = B_{0\phi}(R)\hat{\phi} + B_{0z}(R)\hat{z}$$

$$\vec{j}_0 = -\frac{dB_{0z}}{dR}\hat{\phi} + \frac{1}{R}\frac{d}{dR}(RB_{0z})\hat{z}$$

force free condition $\vec{j}_0 \times \vec{B}_0 = 0$

$$\frac{d}{dR}\left(B_{0\phi}^2 + B_{0z}^2\right) = -\frac{2B_{0\phi}^2}{R}$$

$$\delta W = (2\mu)^{-1}\int B_1^2 - \vec{B}\cdot\vec{\xi}\times\left(\nabla\times\vec{B}_0\right)dV \quad \text{ignore } g \,\&\, P$$

$$B_1 = \nabla\times(\xi\times B_0)$$

several complicated steps

$$\delta \mathrm{W} = \int_0^\infty F\left(\frac{d\xi^R}{dR}\right)^2 - G\xi R^2 + \frac{\left(k^2+h^2\right)R^2+1}{R}\left(B_0\xi - H\right)^2 dR$$

$$F(R) = \frac{R\left(B_{0\phi} + kRB_{0z}\right)^2 + h^2R^3B_0^2}{1+\left(k^2+h^2\right)R^2}$$

$$G(R) = -\frac{\left(B_{0\phi} + RkB_{0z}\right)^2}{R} - \frac{\left(B_{0\phi} - kRB_{0z}\right)^2}{R\left[1+\left(k^2+h^2\right)R^2\right]} - \frac{h^2RB_0^2}{1+\left(k^2+h^2\right)R^2} + h^2RB_0^2 - \frac{d}{dR}\left(\frac{B_0^2 + h^2R^2B_{0\phi}^2}{1+\left(k^2+h^2\right)R^2}\right)$$

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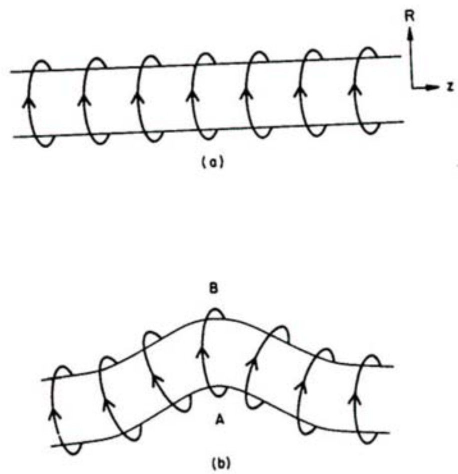

Fig. 7.6. (a) An equilibrium plasma tube surrounded by azimuthal field lines. (b) A lateral kink-like perturbation of the tube.

The effect of line-tying was included by Raadu (1972), that of pressure gradients by Giachetti *et al.* (1977) and that of both together by Hood and Priest (1979b), as outlined in Section 10.2.3. Here the treatment of Raadu is presented, the main object being to illustrate the method rather than obtain particular results.

In the force-free approximation, the gravitational and pressure forces are negligible in the equilibrium. Omitting them also in the perturbed state reduces the general expression (7.39) for the second-order potential energy to

$$\delta W = (2\mu)^{-1} \int B_1^2 - \mathbf{B}_1 \cdot \boldsymbol{\xi} \times (\nabla \times \mathbf{B}_0)\, \mathrm{d}V, \tag{7.42}$$

where

$$\mathbf{B}_1 = \nabla \times (\boldsymbol{\xi} \times \mathbf{B}_0). \tag{7.43}$$

Here, only a certain class of perturbation ($\boldsymbol{\xi}$) will be considered for simplicity. If the resulting smallest value of δW is negative, the system is certainly unstable; but, if it is positive, the system is stable only to that class of perturbation.

The form adopted for the perturbation is

$$\boldsymbol{\xi} = \boldsymbol{\xi}^* f(z), \tag{7.44}$$

where

$$\boldsymbol{\xi}^* = \left[\xi^R(R)\hat{\mathbf{R}} - i\frac{B_{0z}}{B_0}\xi^0(R)\hat{\boldsymbol{\phi}} + i\frac{B_{0\phi}}{B_0}\xi^0(R)\hat{\mathbf{z}} \right] \mathrm{e}^{i(m\phi + kz)}$$

and $f(0) = f(2L) = 0$. This has several important properties. It vanishes at the ends ($z = 0$ and $z = 2L$) of the flux rope and so satisfies rigid boundary conditions there;

$$H\left(R,\xi^R,\frac{d\xi^R}{dR}\right)=\frac{R}{1+\left(k^2+h^2\right)R^2}\left(\frac{d\xi^R}{dR}\left(kR\cdot B_{0\phi}-B_{0z}\right)-\frac{\xi^R}{R}\left(kRB_0+B_{0z}\right)\right)$$

$$h=\frac{\pi}{2L} \qquad \xi\propto e^{kz}$$

$$\delta W=\int_0^\infty\left[F\left(\frac{d\xi^R}{dR}\right)^2-G\left(\xi^R\right)^2\right]dR$$

with boundary conditions, we can solve

$$\frac{d}{dR}\left(F\frac{d\xi^R}{dR}\right)+G\xi^R=0$$

This can be done by getting numerical results such as shown in Figs 7.7 & 7.8

- Summary of Instabilities

    1. Interchange instability

        An interface between two plasma with different P&B. If two neighboring bundle can be interchanged, if the wrinkles have a wavenumber k, radius of curvature Rc interface, then

$$\omega^2 = -\frac{2Pk}{\rho R_c}$$

if magnetic fields are anchored down in two ends with a distance L.

$$\omega^2 = -\frac{2Pk}{\rho R_c} + \frac{v_A^2}{L^2}$$

Long wave: stable

Short wave: unstable

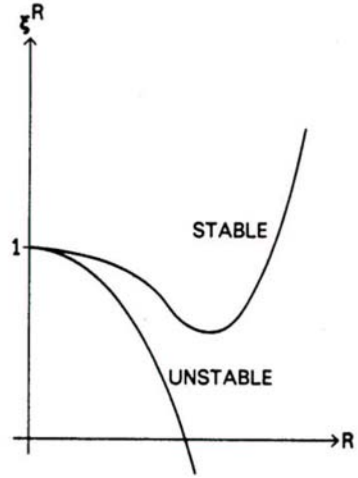

Fig. 7.7. Typical solutions to the Euler–Lagrange equation for the radial component (ζ^R) of the minimising perturbation for a flux tube.

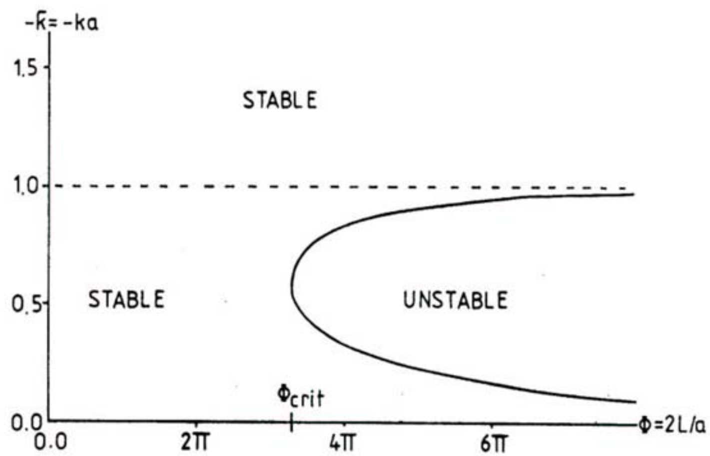

Fig. 7.8. The stability diagram for a uniform-twist force-free flux tube of length $2L$ and effective width $2a$, where k is the wavenumber of the perturbation along the tube and Φ is the twist (from Hood and Priest, 1979b).

2. Reyleigh-Taylor instability (Fig 7.3)

Perturbation grow at rate $e^{i\omega t}$

$$\omega^2 = -gk \frac{\rho_0^{(+)} - \rho_0^{(-)}}{\rho_0^{(+)} + \rho_0^{(-)}}$$

uniform vertical fields, modify growth rate

but do not change instability

$$k = 0 \qquad \omega^2 = -gk \frac{\rho_0^{(+)} - \rho_0^{(-)}}{\rho_0^{(+)} + \rho_0^{(-)}}$$

$$k = \infty \qquad i\omega = \frac{g\sqrt{\mu}}{B_0}\left(\sqrt{\rho_0^+} - \sqrt{\rho_0^-}\right)$$

uniform horizontal field can change instability

$$\omega^2 = -gk \frac{\rho_0^{(+)} - \rho_0^{(-)}}{\rho_0^{(+)} + \rho_0^{(-)}} + \frac{2B_0^2 k_x^2}{\mu\left(\rho_0^{(+)} + \rho_0^{(-)}\right)}$$

critical condition

$$k_c = \frac{\left(\rho_0^{(+)} - \rho_0^{(-)}\right) g\mu}{2B_0^2}$$

if plasma is supported by magnetic fields below, then :

$$\omega^2 = -gk + \frac{k_x^2 \left(B_0^-\right)^2}{\rho\mu P_0^+}$$

most unstable $\qquad k_x = 0 \qquad i\omega = \sqrt{gk}$

$k_x \neq 0$, $\quad$ magnetic tension tries to restore

Long waves $\quad k < \dfrac{v_A^2}{L^2 g} \quad$ are stable

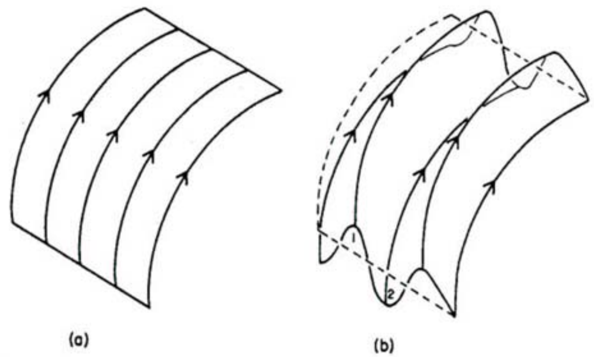

Fig. 7.9. (a) Part of the concave surface of a magnetic field that confines plasma in some region. (b) A flute-like displacement of the interface.

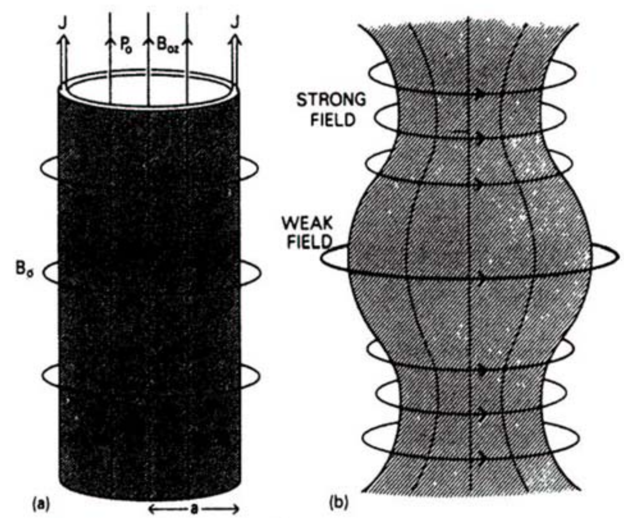

Fig. 7.10. (a) A linear pinch containing plasma at pressure p_0 and magnetic field of strength $B_{0z}\hat{z}$. A current ($J\hat{z}$) flows on the surface and produces a field ($B_\phi\hat{\phi}$) in the surrounding region. (b) A sausage perturbation of the interface.

3. Pinched discharge

vertical current produces azimuthal fields plasma is confined – pinched

$\vec{j} \times \vec{B}$ inwards is balanced by pressure gradient outwards

If there is no magnetic fields inside plasma the growth rate of sausage instability is

$$i\omega = \left(\frac{2P_0 k}{\rho a} \right)^{\frac{1}{2}} \qquad a: \text{radius}$$

for disturbance $k \approx a^{-1}, \quad i\omega = \dfrac{\left(2 P_0 / \rho_0 \right)^{\frac{1}{2}}}{a}$

The pinch can be stabilized by large enough axial field B_z

$$P_0 + \frac{B_{0z}^2}{2\mu} = \frac{B_\phi^2}{2\mu}$$

modified dispersion relation

$$\omega^2 = -\frac{2P_0}{\rho_0 a} + \frac{B_{0z}^2}{\mu \rho_0 a^2}$$

for stability, if requires $B_{0z}^2 > \dfrac{1}{2} B_\phi^2$

4. Flow Instability

Laminar viscous flow between rigid boundaries such as walls of Lab channels becomes unstable and develop into a turbulent flow it occurs when $R_e > R_e^*$ critical Reynolds number

$B \perp$ flow

$$\mathrm{R_e^*} = 50{,}000 \quad \mathrm{Ha}$$

$$\mathrm{Ha} = \frac{\mathrm{B_0 L_0}}{(\sigma/\rho\nu)^{\frac{1}{2}}} : \text{ Hartman number}$$

B// flow

$$\mathrm{R_e^*} = 500 \quad \mathrm{Ha}$$

If we have uniform inviscid flow baring 2 flow speeds

- Kelvin - Helmhotz instability

unstable if $$k > \frac{g\left(\rho^{-^2} - \rho^{+^2}\right)}{\rho^-\rho^+\left(U^{(+)} - U^{(-)}\right)^2}$$

5. Resistive Instability

B field in a current sheet of width l diffused through plasma on a time scale $\tau_{\mathrm{A}} = \dfrac{l^2}{\eta}$

$\eta = (\mu\sigma)^{-1}$ magnetic diffusivity

driving force F_d

restoring force $F_c = \vec{j} \times \vec{B} = -\sigma v_x (\varepsilon B_0)^2 \hat{x}$

if $F_L > F_d$ stable

gravitation mode - gravitation $g\rho\hat{x}$ transverse to current sheet to produce density strafication.

Growth rate $$i\omega = \left(\frac{(kl)^2 \tau_A^2}{\tau_d^4 \tau_G^4} \right)^{\frac{1}{3}}$$

$$\tau_A = \frac{l}{v_A}, \quad \tau_d = \frac{l^2}{\eta}, \quad \eta = (\mu\sigma)^{-1}, \; \tau_G = \left(-\frac{g}{\rho_0} \frac{d\rho_0}{dx} \right)^{-\frac{1}{2}}$$

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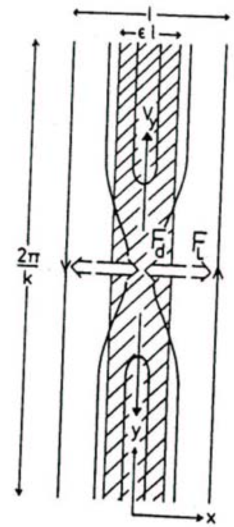

Fig. 7.12. Resistive instability in a current sheet for which the driving force (F_d) exceeds the restoring force (F_L). Significant diffusion takes place over a fraction (ε) of the width (l) of the sheet. Here just one wavelength ($2\pi/k$) is shown, but, in practice, there may be many such features end to end.

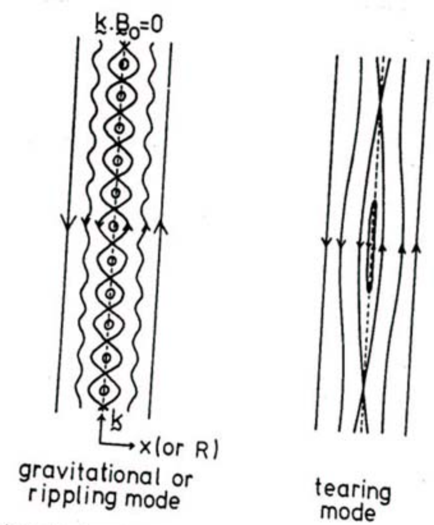

Fig. 7.13. Small- and long- wavelength resistive instabilities in a current sheet or a sheared magnetic field.

rippling mode:

there is a spatial variation across the sheet in magnetic diffusivity $\eta_0(x)$

$$\vec{F}_d = -\frac{v_x}{\omega\eta_0}\frac{d\eta_0}{dx}\frac{\varepsilon B_0^2}{\mu l}\hat{x}$$

$$i\omega = \left[\left(\frac{d\eta_0}{dx}\frac{l}{\eta_0}\right)^4 \frac{(kl)^2}{\tau_A^2\tau_d^3}\right]^{\frac{1}{5}}$$

tearing mode:

only occurs when $kl < 1$

does not require g, or $\frac{d\eta_0}{dx}$

$$i\omega = \left[\tau_d^3\tau_A^2(kl)^2\right]^{-\frac{1}{5}}, \qquad \left(\frac{\tau_A}{\tau_d}\right)^{\frac{1}{4}} < lk < 1$$

longest wavelength has fastest growth rate can lead reconnection, coronal beating.

6. Convective Instability

We have discussed this in chapter 4.

Consider g only

$$N^2 = \frac{g}{T_0}\left(\frac{dT_0}{dz} - \left(\frac{dT_0}{dz}\right)_{ad}\right) \qquad 4.37$$

$\omega = N$ Brunt - Vaisala frequency

if $\frac{dT_0}{dz} > \left(\frac{dT_0}{dz}\right)_{ad}$ $\quad \omega^2 > 0 \quad \rightarrow$ convective instability

7. Radiatively driven thermal instability

If thermal conduction were in effective, thermal instabilities would occur in the corona and upper atmosphere, due to the radiative loss term in energy equation.

Suppose plasma is initially in equilibrium with T_0, ρ_0 balance between mechanical heating $h\rho$(per unit volume) and optically thin radiation $\chi\rho T^\alpha$ leads to $h = \chi\rho_0 T_0^\alpha$

for a perturbation at constant pressure

$$c_p \frac{\partial T}{\partial t} = h - \chi\rho T^\alpha \qquad \rho = \frac{mP_0}{kT}$$

$$= \chi\rho_0 T_0^\alpha \left(1 - \frac{T^{\alpha-1}}{T_0^{\alpha-1}}\right)$$

if $\alpha < 1$, a small decrease in T, $(\mathrm{T} < \mathrm{T}_0)$ makes $\frac{\partial \mathrm{T}}{\partial \mathrm{t}} < 0,$

perturbation continues, - thermal instability

Time scale $\tau_{rad} = \frac{c_P}{\left(\gamma \rho_0 T_0^{\alpha-1}\right)}$

Typically, $\alpha < 1$ for $T > 10^5 K$ see Table 7.1

Thermal instability is prevented by heat conduction along magnetic fields.

$\rho^{-1}\nabla\left(k_{//}\nabla T\right)$ $k_{//} = k_0^{5/2}$

conduction time

$\tau_c = L^2 \rho_0 c_P / k_0 T^{5/2}$

$\tau_c < \tau_{rad}$ stabalized

$$L = \left(\frac{k_0 T_0^{7/2-\alpha}}{\chi \rho_0^2}\right)^{\frac{1}{2}} = L_{\max} \qquad \text{critical length}$$

$L > L_{\max}$ instable

TABLE 7.1

The growth-time in seconds for radiative cooling in a plasma of number density n_0(m^{-3}) and temperature T_0(K).

n_0	T_0				
	10^5	5×10^5	10^6	2×10^6	10^7
10^{14}	440	2200	3.2×10^4	1.3×10^5	3.2×10^6
10^{15}	44	220	3.2×10^3	1.3×10^4	3.2×10^5
10^{16}	4.4	22	320	1.3×10^3	3.2×10^4

Homework

- Try to use normal mode method to analyze the kink instability.