ECE 744: Homework

1. (This is a simulation assignment. You can use Matlab or any other language.) Write the code for the projection gradient descent algorithm that solves the following problem:

$$\min \quad x_1^2 + 9x_2^2$$

subject to

$$2x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 1$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

apply the projection gradient method to both the primal and the dual problems. Choose different stepsizes to show how the algorithm converges and/or diverges. What is the optimal stepsize in your opinion?

2. (Optimal power and bandwidth allocation in a Gaussian broadcast channel). We consider a communication system in which a central node transmits messages to $n$ receivers. Each receiver channel is characterized by its (transmit) power level $P_i \geq 0$ and its bandwidth $W_i \geq 0$. The power and bandwidth of a receiver channel determine its bit rate $R_i$ via

$$R_i = \alpha_i W_i \log(1 + \beta_i P_i P_i / W_i)$$

where $\alpha_i$ and $\beta_i$ are known positive constants. For $W_i = 0$, we take $R_i = 0$ (which is what you get if you take the limit as $W_i \to 0$).

The powers must satisfy a total power constraint, which has the form

$$\sum_{i=1}^{n} P_i \leq P_0,$$

where $P_0 \geq 0$ is a given total power available to allocate among the channels. Similarly, the bandwidths must satisfy

$$\sum_{i=1}^{n} W_i \leq W_0,$$

where $W_0 \geq 0$ is the given total available bandwidth. The optimization variables in this problem are the powers $P_i$ and the bandwidths $W_i$. The objective is to maximize the total rate-sum,

$$\sum_{i=1}^{n} R_i$$
(a) Pose this problem as a convex optimization problem. You should justify why it is a convex problem.

(b) Associate a Lagrange multiplier $\lambda$ for the constraint $\sum P_i \leq P_0$. (Do NOT associate a Lagrange multiplier for the constraint $\sum W_i = W_0$.) Write down the conditions to optimize the Lagrangian for a given $\lambda$.

(c) Using the condition in Part (b), show that the optimal power and bandwidth allocation must satisfy the following property: there must exist a positive number $\lambda$ such that

$$P_i = \begin{cases} \frac{W_i}{\beta_i} \left( \frac{\alpha_i \beta_i}{\lambda} - 1 \right) & \frac{\alpha_i \beta_i}{\lambda} \geq 1 \\ 0 & \text{Else} \end{cases}$$

Further, substitute this value of $P_i$ into the condition in Part (b). Show that among the subset $J$ of receivers with $\frac{\alpha_i \beta_i}{\lambda} > 1$, only those receivers $i$ with

$$\alpha_i \log \left( \frac{\alpha_i \beta_i}{\lambda} \right) - \frac{\lambda}{\beta_i} \left( \frac{\alpha_i \beta_i}{\lambda} - 1 \right) = \max \left\{ \alpha_j \log \left( \frac{\alpha_j \beta_j}{\lambda} \right) - \frac{\lambda}{\beta_j} \left( \frac{\alpha_j \beta_j}{\lambda} - 1 \right) \right\}$$

will be allocated non-zero bandwidth $W_i$.

(d) Develop an algorithm to iteratively compute the optimal value of $\lambda$. Describe how to adaptively control $P_i$ and $W_i$ based on this iterative algorithm.