ECE 744: Homework

1. (This is a simulation assignment. You can use Matlab or any other language.) Write the code for the projection gradient descent algorithm that solves the following problem:

   min $\qquad x_1^2 + 9x_2^2$

   subject to

$$
\begin{aligned}
&2x_1 + x_2 \geq 1 \\
&2x_1 + 3x_2 \geq 1 \\
&x_1 \geq 0 \\
&x_2 \geq 0
\end{aligned}
$$

   apply the projection gradient method to both the primal and the dual problems. Choose different stepsizes to show how the algorithm converges and/or diverges. What is the optimal stepsize in your opinion?

2. (Optimal power and bandwidth allocation in a Gaussian broadcast channel). We consider a communication system in which a central node transmits messages to n receivers. Each receiver channel is characterized by its (transmit) power level $P_i \geq 0$ and its bandwidth $W_i \geq 0$. The power and bandwidth of a receiver channel determine its bit rate R_i via

$$R_i = \alpha_i W_i \log(1 + \beta_i P_i P_i / W_i)$$

   where α_i and β_i are known positive constants. For $W_i = 0$, we take $R_i = 0$ (which is what you get if you take the limit as $W_i \to 0$).

   The powers must satisfy a total power constraint, which has the form

$$\sum_{i=1}^{n} P_i \leq P_0,$$

   where $P_0 \geq 0$ is a given total power available to allocate among the channels. Similarly, the bandwidths must satisfy

$$\sum_{i=1}^{n} W_i \leq W_0,$$

   where $W_0 \geq 0$ is the given total available bandwidth. The optimization variables in this problem are the powers P_i and the bandwidths W_i. The objective is to maximize the total rate-sum,

$$\sum_{i=1}^{n} R_i$$

(a) Pose this problem as a convex optimization problem. You should justify why it is a convex problem.

(b) Associate a Lagrange multiplier λ for the constraint $\sum P_i \leq P_0$. (Do NOT associate a Lagrange multiplier for the constraint $\sum Wi = W_0$.) Write down the conditions to optimize the Lagrangian for a given λ.

(c) Using the condition in Part (b), show that the optimal power and bandwidth allocation must satisfy the following property: there must exist a positive number λ such that

$$P_i = \begin{cases} \frac{W_i}{\beta_i}(\frac{\alpha_i\beta_i}{\lambda} - 1) & \frac{\alpha_i\beta_i}{\lambda} \geq 1 \\ 0 & \text{Else} \end{cases}$$

Further, substitute this value of P_i into the condition in Part (b). Show that among the subset J of receivers with $\frac{\alpha_i\beta_i}{\lambda} > 1$, only those receivers i with

$$\alpha_i \log(\frac{\alpha_i\beta_i}{\lambda}) - \frac{\lambda}{\beta_i}(\frac{\alpha_i\beta_i}{\lambda} - 1) = \max_{j \in J}\{\alpha_j \log(\frac{\alpha_j\beta_j}{\lambda}) - \frac{\lambda}{\beta_j}(\frac{\alpha_j\beta_j}{\lambda} - 1)\}$$

will be allocated non-zero bandwidth Wi.

(d) Develop an algorithm to iteratively compute the optimal value of λ. Describe how to adaptively control P_i and W_i based on this iterative algorithm.