Digital Transmission via Underwater Acoustic Particle Velocity Channels

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Abstract—Recent studies have shown that a compact underwater vector sensor receiver can perform as an array of scalar sensors. Here we propose a scheme to transmit data via underwater particle velocity channels using a dipole vector sensor. System equations are derived and simulations are presented. This demonstrates the possibility of digital transmission using a vector sensor.

I. INTRODUCTION
Vector sensors measure the signal not only in the pressure channel, but also in particle velocity channels. In the past few decades, a large volume of research has been conducted on the theory and design of vector sensors, mainly used in SONAR systems and underwater target localization [1] [2] [3] [4], due to the highly directional information that vector sensors provide. Recent achievements in material science and manufacturing technologies provide low noise and small vector sensors with wide dynamic ranges [4] [5]. Recently vector sensors are proposed to be used at the receive side of underwater acoustic communication systems, for multichannel equalization [6] [7]. More specifically, a multichannel receiver is proposed [6] [7] with one pressure sensor as the transmitter and a vector sensor as the receiver. Statistical characteristics of acoustic particle velocity channels are expressed in [8].

Since a vector sensor has a compact size, small platforms such as unmanned underwater vehicles will significantly benefit if a compact vector sensor can be used for signal transmission as well. The key to this problem is to find out how signal can be transmitted via particle velocity channels. A dipole vector sensor is composed of two closely-placed pressure sensors that measure the spatial gradient of the field [5]. So, the spatial gradient between the data on these two pressure sensors can be viewed as the signal which is modulated on the dipole, in the direction of the axis of the dipole. One interpretation could be that the signal is transmitted via a particle velocity channel which is aligned with the direction of the dipole. The key advantage of the proposed dipole transmitter is that in the reverse (reception) mode, it provides diversity.

In this article, a communication system with one dipole at the transmitter and a vector sensor at the receiver is proposed. In Section II, the schematic of this system is presented and system equations are derived. System performance analysis and simulation results are provided in Section III, and Section IV concludes the paper.

II. SIGNALS AND CHANNELS IN THE PROPOSED SYSTEM
In the two-dimensional $y$-$z$ (range-depth) plane, a vector sensor device measures three components of the field: acoustic pressure, and $y$ and $z$ components of the acoustic particle velocity. A pressure sensor in the device measures the acoustic pressure. The $y$ and $z$ components of particle velocity can be measured by two dipoles in $y$ and $z$ directions, respectively, by spatial differentiation of the acoustic field in each direction. In Fig. 1 there is one dipole at the transmitter and two dipoles at the receiver. The dipole at the transmitter $Tx$ is parallel to the $z$ axis and the two pressure sensors in $Tx$ are labeled as 1 and 2. At the receiver $Rx$, the two dipoles are parallel to $y$ and $z$ axes with their centers at the point $(y_0, z_0)$. They measure the $y$ and $z$ components of the acoustic particle velocity at $(y_0, z_0)$. The four pressure sensors at $Rx$ are labeled as 1, 2, 3 and 4. Another pressure sensor labeled as 5 is located at $(y_0, z_0)$, to measure the acoustic pressure component in the vector sensor device. So, there are ten different pressure channels in Fig. 1, where $p_{iq}$ stands for the channel from sensor $i$ at $Tx$ to sensor $q$ at $Rx$, $i = 1,2$ and $q = 1,2,3,4,5$.

According to the linearized equation for time-harmonic waves [5], at frequency $f_0$, for each sensor in $Tx$, there are two particle velocity channels at $Rx$, the vertical and horizontal components of particle velocity at $(y_0, z_0)$. More specifically, for $Tx_1$ there are $v_{y1}^{Tx_1}$ and $v_{z1}^{Tx_1}$, whereas for $Tx_2$ we have $v_{y2}^{Tx_2}$ and $v_{z2}^{Tx_2}$. These four channels are given by the following equations, where $L_{R_i} \rightarrow 0$

\begin{equation}
\begin{aligned}
v_{y1}^{Tx_1} &= -(j\rho_0 \omega_0)^{-1}(p_{11} - p_{12})/L_{R_2}, \\
v_{z1}^{Tx_1} &= -(j\rho_0 \omega_0)^{-1}(p_{21} - p_{22})/L_{R_2}, \\
v_{y2}^{Tx_2} &= -(j\rho_0 \omega_0)^{-1}(p_{13} - p_{14})/L_{R_4}, \\
v_{z2}^{Tx_2} &= -(j\rho_0 \omega_0)^{-1}(p_{23} - p_{24})/L_{R_4}.
\end{aligned}
\end{equation}

Here $\rho_0$ is the density of the fluid, $j^2 = -1$, $\omega_0 = 2\pi f_0$ and $L_{R_i}$ denotes the spacing between the two pressure sensors in each dipole at $Rx$, as shown in Fig. 1. To transmit the symbol $s$, we modulate $s/\Omega_s$ and $-s/\Omega_s$ on $Tx_1$ and $Tx_2$, respectively, where $\Omega_s$ denotes the distance between the two sensors in the dipole at $Tx$. For the average transmit symbol power we choose $\Omega_s = E[s^2 / L_{R_s}^2] = 1$, where $E$ is the expectation.
operator. This means the average transmit power is 1. Since $z$ velocity is the spatial gradient of the pressure field in the $z$ direction, this transmission scheme means that the symbol $s$ is modulated on the $z$ velocity channel. This can be better understood by writing the equation for the received signal of each pressure sensor at $Rx$

\[
\begin{align*}
    r_1 &= ((p_{11} - p_{21}) / L_{r_1}) \oplus s, \\
    r_2 &= ((p_{12} - p_{22}) / L_{r_1}) \oplus s, \\
    r_3 &= ((p_{13} - p_{23}) / L_{r_3}) \oplus s, \\
    r_4 &= ((p_{14} - p_{24}) / L_{r_3}) \oplus s, \\
    r &= ((p_{15} - p_{25}) / L_{r_3}) \oplus s.
\end{align*}
\]

(2)

Here $\oplus$ is the convolution. As shown in Appendix, each channel $((p_{1q} - p_{2q}) / L_{r_3})$ in (2), $q = 1, 2, 3, 4, 5$, is similar to a velocity channel with which the symbol $s$ is convoluted.

The $z$ and $y$ particle velocity signals at $Rx$ are spatial gradients of the received signals $r_1, r_2, r_3$ and $r_4$, measured by the two dipoles, as $L_{r_3} \to 0$

\[
\begin{align*}
    \eta_z &= -(\rho_0 \omega_0)^{-1}(r_1 - r_2) / L_{r_3}, \\
    \eta_y &= -(\rho_0 \omega_0)^{-1}(r_3 - r_4) / L_{r_3}.
\end{align*}
\]

(3)

By combining (1), (2) and (3), the $z$ and $y$ velocity components at $Rx$ can be written as

\[
\begin{align*}
    \eta_z &= ((v^0_z - v^2_z) / L_{r_3}) \oplus s, \\
    \eta_y &= ((v^0_y - v^2_y) / L_{r_3}) \oplus s.
\end{align*}
\]

Now we convert velocity channels and signals to pressure-equivalent quantities. Corresponding to the four particle velocity channels in (1), there are four pressure-equivalent particle velocity channels $p^v_1, p^v_2, p^v_3, p^v_4$, obtained by multiplying (1) with the negative of the acoustic impedance $-\rho_0 c$ [6]

\[
\begin{align*}
    p^v_1 &= -\rho_0 c v^0_z, \\
    p^v_2 &= -\rho_0 c v^0_z, \\
    p^v_3 &= -\rho_0 c v^2_z, \\
    p^v_4 &= -\rho_0 c v^2_z.
\end{align*}
\]

(5)

In the above equations, $c$ is the sound speed.

Similarly, the pressure-equivalent $z$ and $y$ velocity components at $Rx$ are $\eta_z$ and $\eta_y$ multiplied by $-\rho_0 c$

\[
\begin{align*}
    r_z &= -\rho_0 c \eta_z, \\
    r_y &= -\rho_0 c \eta_y.
\end{align*}
\]

(6)

By substituting (4) into (6) and upon using the definitions in (5) we obtain

\[
\begin{align*}
    r_z &= ((p^v_z - p^v_2) / L_{r_3}) \oplus s, \\
    r_y &= ((p^v_y - p^v_2) / L_{r_3}) \oplus s.
\end{align*}
\]

(7)

To include the effect of noise, pressure-equivalent ambient velocity noise terms $n_1$ and $n_2$ should be added to (7), whereas the ambient pressure noise $n$ needs to be added to the last equation in (2). The final set of noisy signals received at $Rx$ is given by

\[
\begin{align*}
    r_z &= ((p^v_z - p^v_2) / L_{r_3}) \oplus s + n_z, \\
    r_y &= ((p^v_y - p^v_2) / L_{r_3}) \oplus s + n_y, \\
    r &= ((p_{15} - p_{25}) / L_{r_3}) \oplus s + n.
\end{align*}
\]

(8)

As shown in [6], for the isotropic noise model, the noise terms in (8) are uncorrelated, that is $E[n_1 n_2^*] = E[n_1 n_y^*] = E[n_1^* n_y] = 0$. Moreover, for the powers of the noise terms in (8) we have $\Omega = E[n^2] = \Omega_y^* = E[n_y^2] = \Omega_y / 2$.

III. SYSTEM PERFORMANCE AND SIMULATION RESULTS

Let $\tilde{p}_r = [\tilde{p}_r(0) \ldots \tilde{p}_r(M - 1)]^T$, $\tilde{p}_d = [\tilde{p}_d(0) \ldots \tilde{p}_d(M - 1)]^T$ and $\tilde{p}_d^* = [\tilde{p}_d^*(0) \ldots \tilde{p}_d^*(M - 1)]^T$, with $^T$ as transpose, be the taps of the three channel impulse responses in (8), such that $\tilde{p}_r(\tau) = (p_{13}(\tau) - p_{23}(\tau)) / L_{r_3}$ and $\tilde{p}_d(\tau) = (p^v_{13}(\tau) - p^v_{23}(\tau)) / L_{r_3}$. Here the subscript $d$ indicates that the transmitter is a dipole. The transmitted symbol vector is defined as $S = [s_0, s_1, s_2, \ldots, s_{K-1}]^T$. The channel matrix is $H = [H^r_1, H^r_2, H^r_3]$, where $^*$ denotes the complex conjugate transpose of a matrix and

\[
H = \begin{bmatrix}
    h_1(0) & \cdots & h_1(M-1) \\
    h_2(0) & \cdots & h_2(M-1) \\
    \vdots & \ddots & \vdots \\
    h_3(0) & \cdots & h_3(M-1)
\end{bmatrix}, \quad l = 1, 2, 3,
\]

(9)

where $h_l = \tilde{p}_r$, $h_2 = \tilde{p}_d$, and $h_3 = \tilde{p}_d^*$. Let the noise vector be $N = [N_1^T, N_2^T, N_3^T]^T$, where

\[
N_r = [n_1(0), n_1(1), \ldots, n_1(K + M - 2)]^T, \quad l = 1, 2, 3,
\]

(10)

and $n_1 = n_2 = n_3 = n_4$. These are the noise terms in (8). The vector of received symbols can be written as

\[
R = HS + N.
\]

(11)

Here $R = [r(0) \ldots r(K + M - 2), r_1(0) \ldots r_3(K + M - 2), r_4(0) \ldots r_2(K + M - 2)]^T$ and $r_1, r_2,$ and $r_3$ are the received pressure, $y$-velocity and $z$-velocity signals in (8).

Assuming perfect channel estimate at $Rx$, here we use a zero forcing equalizer to investigate the feasibility of symbol detection in the proposed system in Fig. 1. The minimum variance unbiased estimate of the transmitted symbol vector $S$ is [9]

\[
\hat{S} = (H^* \Sigma^{-1} H)^{-1} H^* \Sigma^{-1} R.
\]

(12)
where $\Sigma = E[NN']$ is the covariance matrix of the noise in (11). The covariance matrix of the symbol estimation error can be written as [9]

$$W = E[(S - S)(S - S)'] = (H'\Sigma^{-1}H)^{-1}. \quad (13)$$

For BPSK modulation and similarly to [10] and [11], the average bit error rate (BER) for a block of size $K$, after the zero forcing equalization in (12) can be written as

$$\hat{P_e} = \frac{1}{K} \sum_{i=1}^{K} Q(\sqrt{2\Omega_i / w_i}). \quad (14)$$

Here $\Omega_i$ is the average symbol power, $w_i$ is the $i$-th diagonal element of $W$ in (13), and $Q(x) = (2\pi)^{-1/2} \int_{x}^{\infty} \exp(-t^2/2)dt$.

In simulations, channel impulse responses are generated using Bellhop [12] and relevant parameters are given in Table I. The sound speed profile is one obtained during underwater communication experiments conducted in 2002, in waters off San Diego, CA [13] and shown in Fig. 2. $L_{Rs} = L_{Rx} = 0.1\lambda$, $0.2\lambda$ and $K = 200$ BPSK symbols at a bit rate of 2400 bits/sec. are transmitted.

Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth (m)</td>
<td>81.158</td>
</tr>
<tr>
<td>Water density (kg/m$^3$)</td>
<td>1024</td>
</tr>
<tr>
<td>Transmitter depth (m)</td>
<td>$z_1 = 25$</td>
</tr>
<tr>
<td>Transmitter take-off angle (degree)</td>
<td>-30 to 30</td>
</tr>
<tr>
<td>Number of beams</td>
<td>2001</td>
</tr>
<tr>
<td>Bottom type</td>
<td>Coarse silt and very fine sand</td>
</tr>
<tr>
<td>Receiver depth (m)</td>
<td>$z_2 = 63$</td>
</tr>
<tr>
<td>Receiver range (km)</td>
<td>$\sqrt{\tau} - \tau = 1$</td>
</tr>
<tr>
<td>Carrier frequency (Hz)</td>
<td>12000</td>
</tr>
<tr>
<td>Sampling frequency (Hz)</td>
<td>48000</td>
</tr>
<tr>
<td>Data rate (Hz)</td>
<td>2400</td>
</tr>
<tr>
<td>Nominal sound speed (m/s)</td>
<td>1500</td>
</tr>
<tr>
<td>Wavelength (m)</td>
<td>0.125</td>
</tr>
</tbody>
</table>

To define the average signal-to-noise ratio (SNR) per channel in simulations, we first define SNRs in the three channels of the system as $\zeta_d = \Omega_d / \Omega_s$, $\zeta_p = \Omega_p / \Omega_s$, $\zeta_y = \Omega_y / \Omega_s$, where $\Omega_d = (\mathbf{p}_d)'(\mathbf{p}_d)$, $\Omega_p = (\mathbf{p}_p)'(\mathbf{p}_p)$, and $\Omega_y = (\mathbf{p}_y)'(\mathbf{p}_y)$. So, the average SNR per channel is $\zeta = (\zeta_d + \zeta_p + \zeta_y) / 3$.

A. Channel Responses and Parameters

The amplitude of the complex impulse responses of the three channels in (8) are shown in Fig. 3 for two different bottom types with $L_{Rs} = L_{Rx} = 0.2\lambda$. The pressure channel impulse response $p$ from $(y_1', z_1')$ to $(y_2, z_2)$, element no. 5, is shown in Fig. 3 as a reference. Average powers of the four channels in Fig. 3 are normalized to 1. The amplitude of the Fourier transforms of the impulse responses of Fig. 3 are shown in Fig. 4. The amplitude of complex correlations between each two channels of (8) are listed in Table II. Except of the correlation of 0.55, the other channel correlations are very small. This indicates a good level of diversity with compact transmit and receive vector sensors in the proposed system. RMS delay spreads [14] of channels in (8) are also provided in Table II. It appears that for very fine sand, the RMS delay spreads are smaller. Also $\tau_{rms}$ for the channel $(p_{15} - p_{25})/L_{Rs}$ appears to be larger.

B. System Performance

The BER of the proposed system is plotted in Fig. 5 using (14) for two different dipole sizes. The dipole size seems to slightly affect the performance. The key advantage of the proposed dipole transmitter is that in the reverse (reception) mode, it provides diversity. This is because it can measure both pressure and particle velocity signals.

IV. Conclusion

In this paper, data transmission in acoustic particle velocity channels for underwater systems is proposed and investigated in detail. Based on the derived ocean acoustical system equations and simulation analytical results, the possibility of signal modulation in particle velocity channels is demonstrated. The results provide new opportunities for underwater communication.

APPENDIX

Characterization of the Acoustic Channels in the Proposed System

Consider Fig. 6, where there are two dipoles, one at the transmitter and the other at the receiver. The size of these two dipoles are $L_{Rs}$ and $L_{Rx}$, respectively. Let the size of each pressure sensor in Fig. 6, the black circles, be much smaller than $\lambda$. Then the baseband acoustic pressure produced by such a
Fig. 3. Channel impulse responses in the proposed system operating in shallow waters with coarse silt or very fine sand sediments

Fig. 4. Channel frequency responses in the proposed system operating in shallow waters with coarse silt or very fine sand sediments
small spherical source of strength $\Gamma$, at the point $T_x$ and measured at the point $R_x$ is given by [15]
\[
p = (j \rho_0 c k \Gamma/((4\pi\ell))) e^{-jk\ell}.
\]  
(15)

Here $\ell$ is the distance between the points $T_x$ and $R_x$ and $k = 2\pi/\lambda$ is the wavenumber, with $\lambda$ is the wavelength. If two small spherical sources of equal strength and a 180 degree phase difference are placed at the points $T_x$ and $T_{x2}$ as shown in Fig. 6, then the acoustic pressure produced by this dipole at the point $R_x$ can be written as [15]
\[
p_d = (jkL_{T_x} + (L_{T_{x2}}/\ell))\cos \theta \times p,
\]  
(16)

where $\theta$ is the departure angle at the transmit side, as shown in Fig. 6. Eq. (16) is derived assuming $L_{T_x} \ll \ell$ and $L_{T_{x2}} \ll \lambda$. Since for cases of interest $\ell$ is very large, say several kilometers, then $L_{T_x}/\ell$ becomes negligible in (16). This results in
\[
p_d / L_{T_x} = jk \cos \theta \times p.
\]  
(17)

Since $L_{T_x} \ll \ell$, one can obtain the following expression for $\ell_2$ in Fig. 6
\[
\ell_2 = \sqrt{(\ell \cos \theta + (L_{T_{x2}}/2))^2 + (\ell \sin \theta)^2} \approx \ell + (L_{T_{x2}} \cos \theta)/2.
\]  
(18)

Using (18), (15) can be re-written as
\[
p = (j \rho_0 c k \Gamma/((4\pi\ell))) e^{-jk(\ell_2 - (L_{T_{x2}} \cos \theta)/2)}.
\]  
(19)

By taking the derivation of $p$ in (19) in the $z$ direction we obtain
\[
\partial p / \partial (L_{T_{x2}}/2) = -(j \rho_0 c k^2 \Gamma/((4\pi\ell))) e^{-jk(\ell_2 - (L_{T_{x2}} \cos \theta)/2)} \cos \theta.
\]  
(20)

Using (15), this can be re-written as
\[
\partial p / \partial (L_{T_{x2}}/2) = jk \cos \theta \times p.
\]  
(21)

Comparison of (17) and (21) shows
\[
p_d / L_{T_x} = \partial p / \partial (L_{T_{x2}}/2).
\]  
(22)

This result shows that the pressure produced by a dipole upon $\Gamma_2/\Gamma$, signaling is the same as the remote vertical spatial gradient of the pressure that would have been produced by a single transmitter of strength $\Gamma$, located at the point $T_x$ in Fig. 6.

Now we investigate the signals produced by the transmit dipole and measured by the receive dipole. The distance between the points $T_x$ and $R_{x1}$ in Fig. 6 can be written as
\[
\ell_1' = \sqrt{(\ell \cos \theta + (L_{R_x}/2))^2 + (\ell \sin \theta)^2} \approx \ell + (L_{R_x} \cos \theta)/2.
\]  
(23)

Then using (15) and (17), the acoustic pressure produced by the dipole at the point $R_{x1}$ is
\[
p_{d_{R_{x1}}} / L_{T_x} = -(\rho_0 c k^2 \Gamma/((4\pi\ell))) e^{-jk\ell_1'} \cos \theta.
\]  
(24)

For $R_{x2}$ we similarly have
\[
\ell_2' = \sqrt{(\ell \cos \theta + (L_{R_x}/2))^2 + (\ell \sin \theta)^2} \approx \ell + (L_{R_x} \cos \theta)/2,
\]  
(25)

\[
p_{d_{R_{x2}}} / L_{T_x} = -(\rho_0 c k^2 \Gamma/((4\pi\ell))) e^{-jk\ell_2'} \cos \theta.
\]  
(26)

Since $\ell \gg L_{R_x}$ in Fig. 6, for $\ell_1'$ and $\ell_2'$ in the denominators of (24) and (26) we have $\ell_1' \approx \ell_2' \approx \ell$ and also $\ell_1' \approx \ell_2' \approx \theta$. Therefore, the spatial gradient measured by the receive vector sensor can be written as
\[
p_{d_{R_{x1}}} / L_{T_x} - p_{d_{R_{x2}}} / L_{T_x} = -(\rho_0 c k^2 \Gamma/((4\pi\ell))) e^{-jk\ell_1'} \times (e^{-jk\ell_1' \cos \theta/2} - e^{-jk\ell_1' \cos \theta/2}) \cos \theta,
\]  
(27)

where $\ell_1'$ and $\ell_2'$ in the exponents are replaced by their equivalents in (23) and (25). Since $L_{R_x} \ll \ell$, we have $\cos (kL_{R_x} \cos \theta/2) \approx 1$ and $\sin (kL_{R_x} \cos \theta/2) \approx kL_{R_x} \cos \theta/2$. This simplifies (27) to
\[
p_{d_{R_{x1}}} / L_{T_x} - p_{d_{R_{x2}}} / L_{T_x} = -(\rho_0 c k^3 \Gamma_{R_x}/((4\pi\ell))) e^{-jk\ell_1'} \times (e^{-jk\ell_1' \cos \theta/2} - e^{-jk\ell_1' \cos \theta/2}) \cos \theta.
\]  
(28)

Using (15), (28) can be finally written as
\[
(p_{d_{R_{x1}}} / L_{T_x} - p_{d_{R_{x2}}} / L_{T_x}) / L_{R_x} = k^2 \cos \theta \times p.
\]  
(29)

On the other hand, using (25), $p$ in (15) can be written as
\[
p = (j \rho_0 c k \Gamma/((4\pi\ell))) e^{-jk\ell_1} \cos \theta.
\]  
(30)

According to the definition of particle acceleration [16], second spatial derivation of $p$ in (30) gives the particle acceleration at the point $R_x$ produced by the pressure transmitter at the point $T_x$
\[
\partial^2 p / \partial (L_{R_x}/2)^2 = -k^2 \cos \theta \times p.
\]  
(31)

Comparison of (30) and (31) results in
\[
(p_{d_{R_{x1}}} / L_{T_x} - p_{d_{R_{x2}}} / L_{T_x}) / L_{R_x} = -\partial^2 p / \partial (L_{R_x}/2)^2.
\]  
(32)

This means the particle velocity produced by a dipole upon $\Gamma_2/\Gamma$, signaling and measured by a dipole at the receiver is equivalent to the particle acceleration that would have been generated by a single transmitter of strength $\Gamma$, located at the point $T_x$ in Fig. 6.

REFERENCES


Fig. 5. Performance of the proposed system with $L = L_{tx} = L_{rx} = 0.1\lambda, 0.2\lambda$ and two different sediments.

Fig. 6. Geometrical configuration of the system with one transmit dipole and one receive dipole.