Underwater Communication in Acoustic Particle Velocity Channels via Vector Transducers

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***Abstract*—Acoustic vector transducers can measure or generate signals in underwater acoustic particle velocity channels. They utilize the vector components of the acoustic field, such as the three components of particle velocity. In this paper the principles of data transmission and reception via underwater particle velocity communication channels are developed. Proper modulation and equalization methods are devised to transmit and receive data using vector projectors and vector sensors, respectively. Performance gain of the vector communication system compared to a multiple-input multiple-output (MIMO) system with scalar transducers demonstrates the usefulness of compact vector transducers for underwater communication.**

***Index Terms*—Acoustic particle velocity, MIMO systems, underwater communication, vector transducers**

I. INTRODUCTION

Acoustic vector transducers are devices that can measure or stimulate the vector components of the acoustic field such as acoustic particle velocity [1]. In the past they have been used for beamforming and SONAR applications. When used as sound sources or receivers, they are called vector projectors or vector sensors, respectively. In the context of underwater communication, acoustic particle velocity channels can be used for data communication, in addition to the scalar acoustic pressure channel [2]. A multi- channel vector sensor equalizer is developed in [2] and [3]. On the other hand, modeling and characterization of particle velocity channels are studied in [4] and [5]. In [6] the capacity and statistics of measured underwater acoustic particle velocity channels are investigated.

In this paper data transmission using a vector transducer is investigated. It is well known that particle velocity is the spatial gradient of the acoustic pressure field [1]. On the other hand, a dipole composed of two closely-spaced scalar transducers is a vector transducer that can measure particle velocity [1]. By modulating the signal on a dipole in a certain way proposed in the paper, the received signal becomes the convolution of the transmit signal with particle velocity channels. This can be considered as modulation in particle velocity channels.

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To explain the basic concepts, in Section II first a system with a dipole transmitter and a vector transducer receiver is developed. Then it is extended to a system with a vector transducer at the transmit side. Section III includes analytical and simulation-based system performance results. Concluding remarks are given in Section IV, and proofs and detailed derivations are provided in appendices.

Notation: we use j for $\sqrt{-1}$, $E[\cdot]$ for mathematical expectation, ^{T} for transpose, * for complex conjugate, † for complex conjugate transpose, $\oplus$ for convolution, $\otimes$ for kronecker product and $\delta(\cdot)$ for the Dirac delta function.

II. SIGNALS AND CHANNELS IN THE PROPOSED SYSTEM

In the two-dimensional *y*-*z* (range-depth) plane, a vector transducer measures or stimulates three components of the acoustic field: pressure as well as y and z components of the particle velocity. A scalar transducer deals with the acoustic pressure. The y and z components of acoustic particle velocity, on the other hand, can be measured or stimulated by two dipoles in y and z directions, respectively. This is because particle velocity in a certain direction is the spatial derivative of the acoustic field in that direction. In Fig. 1 there is one vector transducer at the transmit side and one vector transducer at the receive side, where each consists of three scalar transducers. The three scalar transducers at the *Tx* are labeled as 1, 2 and 3, with the scalar pairs {2, 3} and {2, 1} acting as two dipoles in y and z directions, respectively. At the *Rx*, there are two dipoles in y and z directions as well. They measure the y and z components of the acoustic particle velocity at the point (y_0, z_0) , whereas scalar transducer #2 measures the acoustic pressure at (y_0, z_0) . Overall there are nine pressure channels in Fig.1, where p_{iq} stands for the channel from the scalar transducer i at *Tx* to the scalar transducer q at *Rx*, $i = 1,2,3$ and $q = 1,2,3$. A list of all the channels defined and used in the paper is provided in Table I.

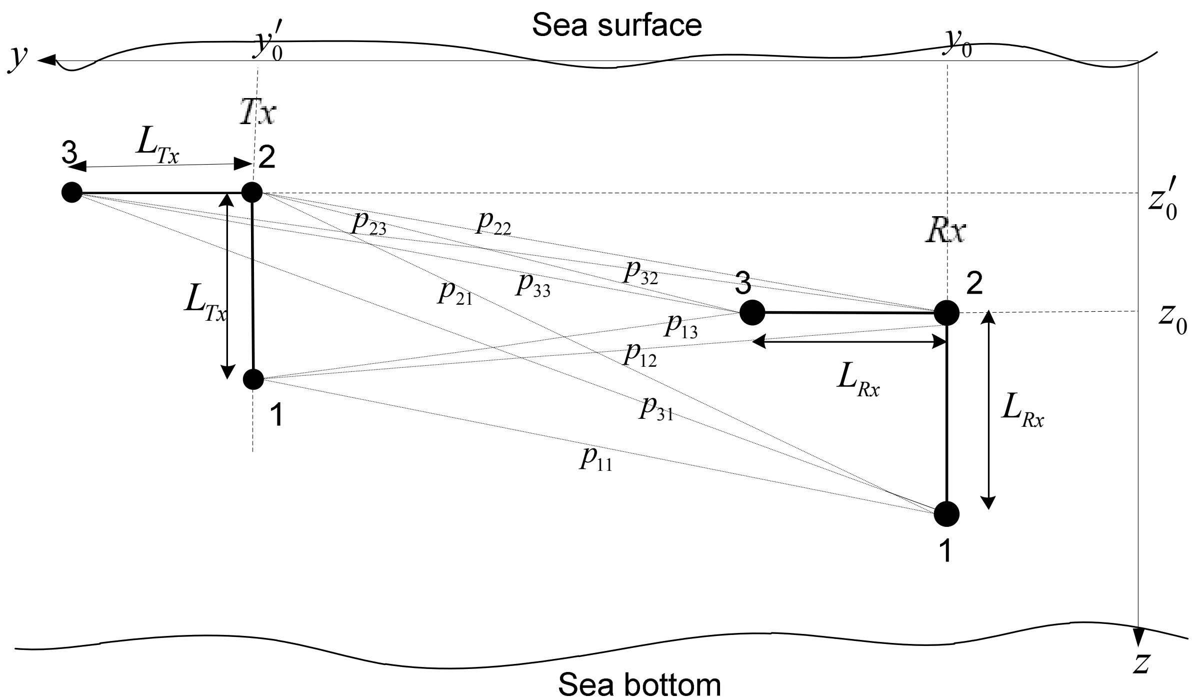

Fig. 1. The proposed vector transducer underwater acoustic communication system. There is one vector transmitter and one vector receiver.

According to the linearized equation for time-harmonic waves [1] at frequency f_0, for each transducer at Tx we define two particle velocity channels at Rx at (y_0, z_0). More specifically, for Tx_1 there are $v_z^{Tx_1}$ and $v_y^{Tx_1}$, for Tx_2 we have $v_z^{Tx_2}$ and $v_y^{Tx_2}$ and finally for Tx_3 we have $v_z^{Tx_3}$ and $v_y^{Tx_3}$. These six channels are defined by the following equations in (1), with $L_{Rx} \to 0$, based on the fact that velocity is the spatial derivative of the pressure.

$$\begin{aligned} v_z^{Tx_1} &= -(j\rho_0\omega_0)^{-1}(p_{11} - p_{12}) / L_{Rx}, \\ v_y^{Tx_1} &= -(j\rho_0\omega_0)^{-1}(p_{13} - p_{12}) / L_{Rx}, \\ v_z^{Tx_2} &= -(j\rho_0\omega_0)^{-1}(p_{21} - p_{22}) / L_{Rx}, \\ v_y^{Tx_2} &= -(j\rho_0\omega_0)^{-1}(p_{23} - p_{22}) / L_{Rx}, \\ v_z^{Tx_3} &= -(j\rho_0\omega_0)^{-1}(p_{31} - p_{32}) / L_{Rx}, \\ v_y^{Tx_3} &= -(j\rho_0\omega_0)^{-1}(p_{33} - p_{32}) / L_{Rx}. \end{aligned} \tag{1}$$

Here ρ_0 is the fluid density and $\omega_0 = 2\pi f_0$ and L_{Rx} is the small spacing between the transducers at the Rx side.

A. Single Dipole Transmitter

To modulate a signal on the z velocity channel, we use the vertical dipole at the Tx side in Fig. 1. More specifically, to transmit the symbol s, we modulate $s / (jkL_{Tx})$ and $-s / (jkL_{Tx})$ on Tx_1 and Tx_2, respectively, where L_{Tx} denotes the spacing between the two transmit scalar transducers, $k = 2\pi / \lambda$ is the wavenumber and λ is the wavelength. Since z velocity is the spatial gradient of the pressure field in the z direction, this transmission scheme means that the symbol s is modulated on the z velocity channel. This can be better understood by writing the equation for the received signal at each scalar transducer at the Rx side

$$\begin{aligned} r_1 &= ((p_{11} - p_{21}) / (jkL_{Tx})) \oplus s, \\ r_2 &= ((p_{12} - p_{22}) / (jkL_{Tx})) \oplus s, \\ r_3 &= ((p_{13} - p_{23}) / (jkL_{Tx})) \oplus s. \end{aligned} \tag{2}$$

As proved in [7] [8], each channel $(p_{1q} - p_{2q}) / (jkL_{Tx})$ in (2), $q = 1, 2, 3$, is a z velocity channel with which the symbol s is convolved and measured at the point q. The role of the scaling factor jk is explained in the next few lines.

The z and y particle velocity signals at Rx are spatial gradients of the received signals r_1, r_2, and r_3, measured by the two receive dipoles as $L_{Rx} \to 0$

$$\begin{aligned} \eta_z &= -(j\rho_0\omega_0)^{-1}(r_1 - r_2) / L_{Rx}, \\ \eta_y &= -(j\rho_0\omega_0)^{-1}(r_3 - r_2) / L_{Rx}. \end{aligned} \tag{3}$$

By combining (1), (2) and (3), the z and y velocity components at Rx can be written as

$$\eta_z = ((v_z^{Tx_1} - v_z^{Tx_2}) / (jkL_{Tx})) \oplus s,$$
$$\eta_y = ((v_y^{Tx_1} - v_y^{Tx_2}) / (jkL_{Tx})) \oplus s. \quad (4)$$

Now we convert velocity channels and signals to pressure-equivalent quantities. Corresponding to the six particle velocity channels in (1), there are six pressure-equivalent particle velocity channels $p_z^{Tx_1}$, $p_z^{Tx_2}$, $p_z^{Tx_3}$, $p_y^{Tx_1}$, $p_y^{Tx_2}$ and $p_y^{Tx_3}$, obtained by multiplying (1) by the negative of the acoustic impedance $-\rho_0 c$ [3] and using the fact that $\omega_0 = ck$, with c as the sound speed

$$\begin{aligned}
p_z^{Tx_1} &= -\rho_0 c v_z^{Tx_1} = (jk)^{-1}(p_{11} - p_{12}) / L_{Rx},\\
p_y^{Tx_1} &= -\rho_0 c v_y^{Tx_1} = (jk)^{-1}(p_{13} - p_{12}) / L_{Rx},\\
p_z^{Tx_2} &= -\rho_0 c v_z^{Tx_2} = (jk)^{-1}(p_{21} - p_{22}) / L_{Rx},\\
p_y^{Tx_2} &= -\rho_0 c v_y^{Tx_2} = (jk)^{-1}(p_{23} - p_{22}) / L_{Rx},\\
p_z^{Tx_3} &= -\rho_0 c v_z^{Tx_3} = (jk)^{-1}(p_{31} - p_{32}) / L_{Rx},\\
p_y^{Tx_3} &= -\rho_0 c v_y^{Tx_3} = (jk)^{-1}(p_{33} - p_{32}) / L_{Rx}.
\end{aligned} \quad (5)$$

Similarly, the pressure-equivalent z and y velocity signals at Rx are η_z and η_y multiplied by $-\rho_0 c$

$$r_z = -\rho_0 c \eta_z, \quad r_y = -\rho_0 c \eta_y. \quad (6)$$

By substituting (4) into (6) and upon using the channel definitions in (5) we obtain

$$r_z = ((p_z^{Tx_1} - p_z^{Tx_2}) / (jkL_{Tx})) \oplus s,$$
$$r_y = ((p_y^{Tx_1} - p_y^{Tx_2}) / (jkL_{Tx})) \oplus s. \quad (7)$$

To include the effect of noise, pressure-equivalent ambient velocity noise terms ξ_z and ξ_y should be added to (7), whereas the ambient pressure noise ξ_2 needs to be added to the middle equation in (2). The final set of noisy signals received at Rx is given by

$$r_2 = p^z \oplus s + \xi_2, \quad r_y = p_y^z \oplus s + \xi_y, \quad r_z = p_z^z \oplus s + \xi_z, \quad (8)$$

where $p^z = (jk)^{-1}(p_{12} - p_{22}) / L_{Tx}$, $p_y^z = (jk)^{-1}(p_y^{Tx_1} - p_y^{Tx_2}) / L_{Tx}$ and $p_z^z = (jk)^{-1}(p_z^{Tx_1} - p_z^{Tx_2}) / L_{Tx}$ are pressure-equivalent impulse responses of the three channels in the system. Note that the superscript z indicates that the transmitter is a dipole in the z direction, whereas the subscripts y and z mean that a horizontal and a vertical dipole are used at the Rx side, respectively.

B. Vector Transmitter

Now we extend the vertical dipole transmitter to a vector transmitter which includes two dipoles in z and y directions using the following method, three data streams can be modulated on the pressure, z-velocity and y-velocity channels. Let the symbols modulated on the three scalar transducers 1, 2 and 3 at Tx be $s_1 + s_2 / (jkL_{Tx})$, $s_3 / (kL_{Tx}) - s_2 / (jkL_{Tx})$ and $-s_3 / (kL_{Tx})$, respectively. Since velocity is the spatial gradient of the pressure, one can intuitively see that symbol s_1 is sent via the pressure channel, whereas symbols s_2 and s_3 are transmitted via the z and y acoustic velocity channels, respectively.

TABLE I

A LIST OF CHANNELS AND THEIR PARAMETERS

p_{iq}	Pressure channel response from the transducer i at Tx to the transducer q at Rx, $i = 1,2,3$ and $q = 1,2,3$
$v_z^{Tx_i}$ and $v_y^{Tx_i}$	Vertical and horizontal components of particle velocity channels from Tx_i to Rx, $i = 1,2,3$
$p_z^{Tx_i}$ and $p_y^{Tx_i}$	Vertical and horizontal components of pressure-equivalent particle velocity channels from Tx_i to Rx, $i = 1,2,3$
p^d, p_z^d, p_y^d	Pressure, as well as vertical and horizontal components of pressure-equivalent particle velocity channels from a transmit dipole to Rx_2, d=z indicates a vertical transmit dipole and d=y means a horizontal transmit dipole
$p_{Rx_q}^d$	Pressure channel response from a dipole transmitter to the receive scalar transducer q, $q = 1,2,3$ and d=z,y
Ω^d, Ω_y^d, Ω_z^d	Powers of the channels p^d, p_y^d and p_z^d, d=z,y
Ω^p, Ω_y^p, Ω_z^p	Powers of the channels p_{12}, $p_y^{Tx_1}$ and $p_z^{Tx_1}$
ρ_y^d, ρ_z^d, ρ_{zy}^d	Correlations among the channel pairs $\{p^d, p_y^d\}$, $\{p^d, p_z^d\}$ and $\{p_y^d, p_z^d\}$, d=z,y
ρ^y, ρ^z, ρ^{zy}	Correlations among the channel pairs $\{p^y, p_{12}\}$, $\{p^z, p_{12}\}$ and $\{p^z, p^y\}$
ρ_{12}, ρ_{13}, ρ_{23}	Correlations among the channel pairs $\{p_{12}, p_{22}\}$, $\{p_{12}, p_{32}\}$ and $\{p_{22}, p_{32}\}$

To show this analytically, similarly to (2) we write the received signal of each scalar transducer at Rx

$$
\begin{aligned}
r_1 &= p_{11} \oplus s_1 + ((p_{11} - p_{21}) / (jkL_{Tx})) \oplus s_2 \\
&\quad - ((p_{31} - p_{21}) / (kL_{Tx})) \oplus s_3, \\
r_2 &= p_{12} \oplus s_1 + ((p_{12} - p_{22}) / (jkL_{Tx})) \oplus s_2 \\
&\quad - ((p_{32} - p_{22}) / (kL_{Tx})) \oplus s_3, \\
r_3 &= p_{13} \oplus s_1 + ((p_{13} - p_{23}) / (jkL_{Tx})) \oplus s_2 \\
&\quad - ((p_{33} - p_{23}) / (kL_{Tx})) \oplus s_3.
\end{aligned} \tag{9}
$$

By combining the channels $p_z^{Tx_i}$ and $p_y^{Tx_i}$ from (5) with (9), i=1,2,3, the received z-velocity and y-velocity signals at Rx can be written as

$$
\begin{aligned}
r_z &= (r_1 - r_2) / (jkL_{Rx}) = p_z^{Tx_1} \oplus s_1 + \\
&\quad ((p_z^{Tx_1} - p_z^{Tx_2}) / (jkL_{Tx})) \oplus s_2 - ((p_z^{Tx_3} - p_z^{Tx_2}) / (kL_{Tx})) \oplus s_3, \\
r_y &= (r_3 - r_2) / (jkL_{Rx}) = p_y^{Tx_1} \oplus s_1 + \\
&\quad ((p_y^{Tx_1} - p_y^{Tx_2}) / (jkL_{Tx})) \oplus s_2 - ((p_y^{Tx_3} - p_y^{Tx_2}) / (kL_{Tx})) \oplus s_3.
\end{aligned} \tag{10}
$$

Upon including the noise terms, the received pressure, y-velocity and z-velocity signals, respectively, can be written as

$$
\begin{aligned}
r_2 &= p_{12} \oplus s_1 + p^z \oplus s_2 - jp^y \oplus s_3 + \xi_2, \\
r_y &= p_y^{Tx_1} \oplus s_1 + p_y^z \oplus s_2 - jp_y^y \oplus s_3 + \xi_y, \\
r_z &= p_z^{Tx_1} \oplus s_1 + p_z^z \oplus s_2 - jp_z^y \oplus s_3 + \xi_z.
\end{aligned} \tag{11}
$$

The three new channels in (11) are defined as $p^y = (jk)^{-1}(p_{32} - p_{22}) / L_{Tx}$, $p_y^y = (jk)^{-1}(p_y^{Tx_3} - p_y^{Tx_2}) / L_{Tx}$ and $p_z^y = (jk)^{-1}(p_z^{Tx_3} - p_z^{Tx_2}) / L_{Tx}$. They represent the three pressure-equivalent impulse responses from the y-oriented dipole at Tx to the Rx in the system.

III. System Performance Analysis

Consider the proposed system with a vector transmitter and a vector receiver, whose input-output equations are derived in (11). Suppose the maximum number of taps in each channel is M and there are K symbols to transmit from each transmitter. Then $\mathbf{p}^z = [p^z(0) \ldots p^z(M-1)\ 0 \cdots 0]^T$, $\mathbf{p}_y^z = [p_y^z(0) \ldots p_y^z(M-1)\ 0 \cdots 0]^T$ and $\mathbf{p}_z^z = [p_z^z(0) \ldots p_z^z(M-1)\ 0 \cdots 0]^T$ are the taps of the three channel impulse responses from the vertical transmit dipole to Rx_2 in (11), appended by $(K-1)$ zeros. Similarly, $\mathbf{p}^y = [p^y(0) \ldots p^y(M-1)\ 0 \cdots 0]^T$, $\mathbf{p}_y^y = [p_y^y(0) \ldots p_y^y(M-1)\ 0 \cdots 0]^T$, $\mathbf{p}_z^y = [p_z^y(0) \ldots p_z^y(M-1)\ 0 \cdots 0]^T$ are the taps of the three channel impulse responses in (11) from the horizontal transmit dipole to Rx_2 appended by $(K-1)$ zeros. Also $\mathbf{p}_{12} = [p_{12}(0) \ldots p_{12}(M-1)\ 0 \cdots 0]^T$, $\mathbf{p}_y^{Tx_1} = [p_y^{Tx_1}(0) \ldots p_y^{Tx_1}(M-1)\ 0 \cdots 0]^T$ and $\mathbf{p}_z^{Tx_1} = [p_z^{Tx_1}(0) \ldots p_z^{Tx_1}(M-1)\ 0 \cdots 0]^T$ are the taps of the three scalar channel impulse responses in (11) from Tx_1 to Rx_2. Then the channel matrices from a z-dipole, a y-dipole and a scalar pressure transmitter can be written as $\mathbf{H}^z = [\mathbf{H}_1{}^T\ \mathbf{H}_2{}^T\ \mathbf{H}_3{}^T]^\dagger$, $\mathbf{H}^y = j[\mathbf{H}_4{}^T\ \mathbf{H}_5{}^T\ \mathbf{H}_6{}^T]^\dagger$ and $\mathbf{H}^p = [\mathbf{H}_7{}^T\ \mathbf{H}_8{}^T\ \mathbf{H}_9{}^T]^\dagger$, respectively. Each matrix contains three submatrices because the receiver has a scalar pressure transducer, as well as a y-dipole and z-dipole, as shown in Fig.1. Also each $\mathbf{H}_l$, $l = 1, 2, \cdots, 9$, is a circulant matrix of size $(K+M-1) \times (K+M-1)$ which has following structure

$$
\mathbf{H}_l = \begin{pmatrix}
h_l(0) & 0 & \cdots & 0 & h_l(M-1) & \cdots & h_l(1) \\
h_l(1) & h_l(0) & \cdots & 0 & 0 & \cdots & h_l(2) \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & h_l(M-1) & \vdots & \ddots & h_l(0)
\end{pmatrix}, \quad l = 1, 2, \cdots, 9, \tag{12}
$$

where $h_1 = p^z$, $h_2 = p_y^z$ and $h_3 = p_z^z$, $h_4 = p^y$, $h_5 = p_y^y$, $h_6 = p_z^y$, $h_7 = p_{12}$, $h_8 = p_y^{Tx_1}$ and $h_9 = p_z^{Tx_1}$. Let the noise vector associated with Rx_2 and y and z receive dipole signal in (11) be $\mathbf{N} = [\mathbf{N}_1{}^T\ \mathbf{N}_2{}^T\ \mathbf{N}_3{}^T]^T$, where

$$
\mathbf{N}_\ell = [\xi_\ell(0)\ \ \xi_\ell(1)\ \ \cdots\ \ \xi_\ell(K+M-2)]^T,\ \ell = 2,\ z,\ y. \tag{13}
$$

Since the vector transmitter has a scalar transducer and two dipoles, we use a space-time block code (STBC) for three transmitters [9]. There are three K-symbol data streams, defined by $\mathbf{S}_i = [s_i(0)\ s_i(1)\ s_i(2) \cdots s_i(K-1)\ 0 \cdots 0]^T$, i = 1, 2, 3, each composed of K independent and identically distributed symbols and $(M-1)$ zeros. Upon applying a rate ¾ STBC [9] [10] to our vector transmitter, the received data over four time intervals can be written as

$$
\begin{bmatrix} \overline{\overline{\mathbf{R}}}^{(1)} \\ \{\overline{\overline{\mathbf{R}}}^{(2)}\}^* \\ \{\overline{\overline{\mathbf{R}}}^{(3)}\}^* \\ \{\overline{\overline{\mathbf{R}}}^{(4)}\}^* \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{H}}^p & \dot{\mathbf{H}}^z & \dot{\mathbf{H}}^y \\ \{\dot{\mathbf{H}}^z\}^* & -\{\dot{\mathbf{H}}^p\}^* & \mathbf{0} \\ \{\dot{\mathbf{H}}^y\}^* & \mathbf{0} & -\{\dot{\mathbf{H}}^p\}^* \\ \mathbf{0} & \{\dot{\mathbf{H}}^y\}^* & -\{\dot{\mathbf{H}}^z\}^* \end{bmatrix} \begin{bmatrix} \overline{\overline{\mathbf{S}}}_1 \\ \overline{\overline{\mathbf{S}}}_2 \\ \overline{\overline{\mathbf{S}}}_3 \end{bmatrix} + \begin{bmatrix} \overline{\overline{\mathbf{N}}}^{(1)} \\ \{\overline{\overline{\mathbf{N}}}^{(2)}\}^* \\ \{\overline{\overline{\mathbf{N}}}^{(3)}\}^* \\ \{\overline{\overline{\mathbf{N}}}^{(4)}\}^* \end{bmatrix}. \tag{14}
$$

Here $\overline{\overline{\mathbf{S}}}_i = \mathbf{Q}\mathbf{S}_i$, i = 1, 2, 3, is the orthogonal discrete Fourier transform (DFT) of $\mathbf{S}_i$, with $\mathbf{Q}$ as the DFT matrix [10]. Additionally we have $\overline{\overline{\mathbf{R}}}^{(m)} = (\mathbf{I}_3 \otimes \mathbf{Q})\mathbf{R}^{(m)}$ and $\overline{\overline{\mathbf{N}}}^{(m)} = (\mathbf{I}_3 \otimes \mathbf{Q})\mathbf{N}^{(m)}$, m = 1, 2, 3, 4, where $\mathbf{I}_3$ is the 3×3 identity matrix. Note that $\mathbf{R}^{(m)}$ is the received signal vector during the m-th time interval, m=1,2,3,4, defined by $\mathbf{R}^{(m)} = [r_2^{(m)}(0) \cdots r_2^{(m)}(K+M-2)\ r_y^{(m)}(0) \cdots r_y^{(m)}(K+M-2)$

$r_z^{(m)}(0)\cdots r_z^{(m)}(K+M-2)]^T$, where $r_2^{(m)}$, $r_y^{(m)}$ and $r_z^{(m)}$ are the received pressure, y-velocity and z-velocity signals in (11) during the m-th time interval. Similarly, $\mathbf{N}^{(m)} = [\xi_2^{(m)}(0)\ \cdots\ \xi_2^{(m)}(K+M-2)\ \xi_y^{(m)}(0)\cdots\xi_y^{(m)}(K+M-2)$ $\xi_z^{(m)}(0)\ \cdots\xi_z^{(m)}(K+M-2)]^T$, m = 1, 2, 3, 4, denotes the receiver noise vector over the m-th time interval. Additionally we have $\dot{\mathbf{H}}^z = [\dot{\mathbf{H}}_1{}^T\ \dot{\mathbf{H}}_2{}^T\ \dot{\mathbf{H}}_3{}^T]^\dagger$, $\dot{\mathbf{H}}^y = j[\dot{\mathbf{H}}_4{}^T\ \dot{\mathbf{H}}_5{}^T\ \dot{\mathbf{H}}_6{}^T]^\dagger$ and $\dot{\mathbf{H}}^p = [\dot{\mathbf{H}}_7{}^T\ \dot{\mathbf{H}}_8{}^T\ \dot{\mathbf{H}}_9{}^T]^T$, where $\dot{\mathbf{H}}_l = \mathbf{Q}\mathbf{H}_l\mathbf{Q}^*$, $l = 1, 2, \cdots, 9$. Note that $\mathbf{H}_l$ is circulant and $\dot{\mathbf{H}}_l$ is diagonal. Finally, $\mathbf{0}$ is all-zero matrix.

Assuming perfect channel estimate at the receiver, here we use a zero forcing equalizer to investigate the feasibility of symbol detection in the proposed system in Fig. 1. Let us define

$$\mathcal{R} = \begin{bmatrix} \bar{\bar{\mathbf{R}}}^{(1)} \\ \{\bar{\bar{\mathbf{R}}}^{(2)}\}^* \\ \{\bar{\bar{\mathbf{R}}}^{(3)}\}^* \\ \{\bar{\bar{\mathbf{R}}}^{(4)}\}^* \end{bmatrix}, \quad \mathcal{H} = \begin{bmatrix} \dot{\mathbf{H}}^p & \dot{\mathbf{H}}^z & \dot{\mathbf{H}}^y \\ \{\dot{\mathbf{H}}^z\}^* & -\{\dot{\mathbf{H}}^p\}^* & \mathbf{0} \\ \{\dot{\mathbf{H}}^y\}^* & \mathbf{0} & -\{\dot{\mathbf{H}}^p\}^* \\ \mathbf{0} & \{\dot{\mathbf{H}}^y\}^* & -\{\dot{\mathbf{H}}^z\}^* \end{bmatrix},$$

$$\mathcal{S} = \begin{bmatrix} \bar{\bar{\mathbf{S}}}_1 \\ \bar{\bar{\mathbf{S}}}_2 \\ \bar{\bar{\mathbf{S}}}_3 \end{bmatrix}, \quad \mathcal{N} = \begin{bmatrix} \bar{\bar{\mathbf{N}}}^{(1)} \\ \{\bar{\bar{\mathbf{N}}}^{(2)}\}^* \\ \{\bar{\bar{\mathbf{N}}}^{(3)}\}^* \\ \{\bar{\bar{\mathbf{N}}}^{(4)}\}^* \end{bmatrix}. \quad (15)$$

Then the received data vector in (14) can be written as $\mathcal{R} = \mathcal{H}\mathcal{S} + \mathcal{N}$. The minimum variance unbiased estimate of the transmitted symbol vector is [11]

$$\hat{\mathcal{S}} = \left(\mathcal{H}^\dagger \mathcal{D}^{-1} \mathcal{H}\right)^{-1} \mathcal{H}^\dagger \mathcal{D}^{-1} \mathcal{R}, \quad (16)$$

where $\mathcal{D} = E[\mathcal{N}\mathcal{N}^\dagger]$ is the covariance matrix of the noise vector $\mathcal{N}$. The estimation error covariance matrix [11] for the vector $\mathcal{S}$ can be written as

$$\mathcal{W} = E[(\hat{\mathcal{S}} - \mathcal{S})(\hat{\mathcal{S}} - \mathcal{S})^\dagger] = \left(\mathcal{H}^\dagger \mathcal{D}^{-1} \mathcal{H}\right)^{-1}. \quad (17)$$

Let $\mathbf{\Sigma} = E[\mathbf{N}^{(m)}\{\mathbf{N}^{(m)}\}^\dagger]$ be the covariance matrix of the $3(K+M-1)\times 1$ noise vector $\mathbf{N}^{(m)}$, $m = 1, 2, 3, 4$. Then we have $\dot{\mathbf{\Sigma}} = E[\bar{\bar{\mathbf{N}}}^{(m)}\{\bar{\bar{\mathbf{N}}}^{(m)}\}^\dagger] = (\mathbf{I}_3 \otimes \mathbf{Q})\mathbf{\Sigma}(\mathbf{I}_3 \otimes \mathbf{Q}^\dagger)$. Additionally, $\mathcal{D}$ can be written as

$$\mathcal{D} = \begin{bmatrix} \dot{\mathbf{\Sigma}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dot{\mathbf{\Sigma}}^* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dot{\mathbf{\Sigma}}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dot{\mathbf{\Sigma}}^* \end{bmatrix}, \quad (18)$$

which results in

$$\mathcal{W} = \begin{bmatrix} \begin{matrix}\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p \\ +(\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y)^* \\ +(\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z)^*\end{matrix} & \begin{matrix}\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z \\ -(\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p)^*\end{matrix} & \begin{matrix}\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y \\ -(\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p)^*\end{matrix} \\ \begin{matrix}\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p \\ -(\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z)^*\end{matrix} & \begin{matrix}(\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p)^* \\ +(\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y)^* \\ +\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z\end{matrix} & \begin{matrix}\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y \\ -(\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z)^*\end{matrix} \\ \begin{matrix}\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p \\ -(\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y)^*\end{matrix} & \begin{matrix}\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z \\ -(\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y)^*\end{matrix} & \begin{matrix}(\{\dot{\mathbf{H}}^p\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^p)^* \\ +\{\dot{\mathbf{H}}^y\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^y \\ +(\{\dot{\mathbf{H}}^z\}^\dagger \dot{\mathbf{\Sigma}}^{-1} \dot{\mathbf{H}}^z)^*\end{matrix} \end{bmatrix}^{-1}. \quad (19)$$

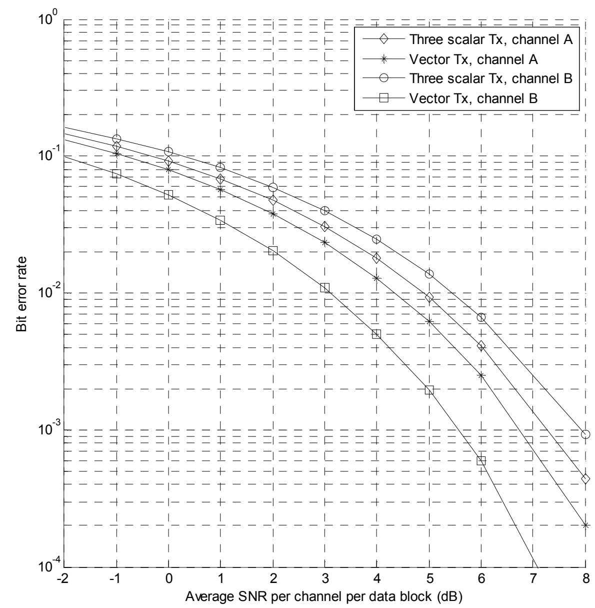

Fig. 2. Performance of the proposed system with a vector transmitter and a vector receiver, compared to a system with three scalar transmitters and a vector receiver, with $L_{Tx} = L_{Rx} = 0.225\lambda$. Parameters of channel A are $z_0' = 25$ m, $z_0 = 63$ m, $D_0 = 81.158$ m, $D_y = 1000$ m, $|\beta| = 0.7$ and $\alpha = 0.08$. Channel B parameters are $\sigma_B = \sigma_S = \sigma_b = \sigma_s = \pi/120\ (1.5^\circ)$, $\Lambda_B = \Lambda_b = 0.1$, $\mu_B = 11\pi/12\ (165^\circ)$, $\mu_S = 10\pi/9\ (200^\circ)$, $\mu_b = \pi/18\ (10^\circ)$ and $\mu_s = 348\pi/180\ (348^\circ)$.

It is shown in Appendix that $\mathcal{W}$ is a block diagonal matrix. Furthermore, the following closed-form expression is derived in Appendix for the estimation error covariance matrix of the data vector $\mathbf{S}_i$, $i = 1, 2, 3$

$$\mathbf{W} = E[(\hat{\mathbf{S}}_i - \mathbf{S}_i)(\hat{\mathbf{S}}_i - \mathbf{S}_i)^\dagger] = (\{\mathbf{H}^p\}^\dagger \mathbf{\Sigma}^{-1}\mathbf{H}^p + \{\mathbf{H}^y\}^T \mathbf{\Sigma}^{-1}\{\mathbf{H}^y\}^* + \{\mathbf{H}^z\}^T \mathbf{\Sigma}^{-1}\{\mathbf{H}^z\}^*)^{-1} = \Delta(\mathbf{H}_1^\dagger \mathbf{H}_1 + 2\mathbf{H}_2^\dagger \mathbf{H}_2 + 2\mathbf{H}_3^\dagger \mathbf{H}_3 + \mathbf{H}_4^\dagger \mathbf{H}_4 + 2\mathbf{H}_5^\dagger \mathbf{H}_5 + 2\mathbf{H}_6^\dagger \mathbf{H}_6 + \mathbf{H}_7^\dagger \mathbf{H}_7 + 2\mathbf{H}_8^\dagger \mathbf{H}_8 + 2\mathbf{H}_9^\dagger \mathbf{H}_9)^{-1}. \quad (20)$$

For binary phase shift keying (BPSK) modulation and similarly to [12] and [13], the average bit error rate (BER) for a block of size K, after the zero forcing equalization in (19) can be written as

$$\overline{P}_e = \frac{1}{K}\sum_{\kappa=1}^{K} Q(\sqrt{2\Omega_s / w_{\kappa\kappa}}). \quad (21)$$

Here $\Omega_s = E[|s_i(k)|^2] = 1$ is the average symbol power, i = 1, 2, 3, $w_{\kappa\kappa}$ is the κ-th diagonal element of $\mathbf{W}$ in (20) and $Q(x) = (2\pi)^{-1/2}\int_x^{+\infty} \exp(-x^2/2)dx$.

Channel Powers: Let $h(\tau)$ be a channel impulse response, with $H(f)$ as its Fourier transform. According to the properties of Bello functions [14] [15], the power of this channel is $\Omega = \int_{-\infty}^{\infty} E[|h(\tau)|^2]d\tau = E[|H(f)|^2]$. The powers of the channels in (11) are then given by $\Omega^d = E[|P^d(f)|^2]$, $\Omega_y^d = E[|P_y^d(f)|^2]$, $\Omega_z^d = E[|P_z^d(f)|^2]$, $d = y, z$, as well as $\Omega^p = E[|P_{12}(f)|^2]$, $\Omega_y^p = E[|P_y^{Tx_1}(f)|^2]$, and $\Omega_z^p = E[|P_z^{Tx_1}(f)|^2]$. As shown in [7] [8], we have $\Omega^d = \Omega_y^d + \Omega_z^d$, $d = y, z$. Also according to [3] and [4], $\Omega^p = \Omega_y^p + \Omega_z^p$.

Noise Powers: As shown in [3], for the isotropic noise model, the noise terms in (11) are uncorrelated, that is, $E[\xi_2\xi_y^*] = E[\xi_2\xi_z^*] = E[\xi_z\xi_y^*] = 0$. Moreover, for the powers of the noise terms in (11) we have $\Delta = E[|\xi_2|^2]$ and $\Delta_y = E[|\xi_y|^2] = \Delta_z = E[|\xi_z|^2] = \Delta/2$ [3].

Signal-to-Noise Ratios (SNRs): We notice that in the first time slot, $t = 1$, data is modulated on p, y and z channels by the transmit vector transducer and received by the p sensor of the receive vector transducer. So, the average received SNR for the p-receiver and the first time slot is $\zeta(1) = (\Omega^p + \Omega^y + \Omega^z)/(3\Delta)$. For $t = 1$, the average received SNRs for the y and z sensors of the vector receiver are, respectively, $\zeta_y(1) = (\Omega_y^p + \Omega_y^y + \Omega_y^z)/(3\Delta_y)$ and $\zeta_z(1) = (\Omega_z^p + \Omega_z^y + \Omega_z^z)/(3\Delta_z)$. In the second time slot and for the STBC used, data is modulated on p and z channels and received by p, y and z sensors. So, for $t = 2$, average received SNRs by p, y and z sensors are $\zeta(2) = (\Omega^p + \Omega^z)/(2\Delta)$, $\zeta_y(2) = (\Omega_y^p + \Omega_y^z)/(2\Delta_y)$ and $\zeta_z(2) = (\Omega_z^p + \Omega_z^z)/(2\Delta_z)$, respectively. For $t = 3$ and $t = 4$ we similarly have $\zeta(3) = (\Omega^p + \Omega^y)/(2\Delta)$, $\zeta_y(3) = (\Omega_y^p + \Omega_y^y)/(2\Delta_y)$, $\zeta_z(3) = (\Omega_z^p + \Omega_z^y)/(2\Delta_z)$, and $\zeta(4) = (\Omega^y + \Omega^z)/(2\Delta)$, $\zeta_y(4) = (\Omega_y^y + \Omega_y^z)/(2\Delta_y)$, $\zeta_z(4) = (\Omega_z^y + \Omega_z^z)/(2\Delta_z)$. Since the vector receiver has three p, y and z sensors, the average SNR per receive channel over the t-th data block is $\overline{\zeta}(t) = (\zeta(t) + \zeta_y(t) + \zeta_z(t))/3$, $t = 1,2,3,4$. Using the derived channel power relations $\Omega^p = \Omega_y^p + \Omega_z^p$, $\Omega^y = \Omega_y^y + \Omega_z^y$, $\Omega^z = \Omega_y^z + \Omega_z^z$, and the noise power relation $\Delta_y = \Delta_z = \Delta/2$, it can be shown that $\overline{\zeta}(1) = (\Omega^p + \Omega^y + \Omega^z)/(3\Delta)$, $\overline{\zeta}(2) = (\Omega^p + \Omega^z)/(2\Delta)$, $\overline{\zeta}(3) = (\Omega^p + \Omega^y)/(2\Delta)$ and $\overline{\zeta}(4) = (\Omega^y + \Omega^z)/(2\Delta)$. Let us define the average SNR per channel per data block by $\overline{\zeta} = (\overline{\zeta}(1) + \overline{\zeta}(2) + \overline{\zeta}(3) + \overline{\zeta}(4))/4$, which can be shown to be $\overline{\zeta} = (\Omega^p + \Omega^y + \Omega^z)/(3\Delta)$. With $\Omega^y + \Omega^z = \Omega^p$, $\overline{\zeta}$ for the proposed system with one vector transmitter and one vector receiver becomes $\overline{\zeta} = 2\Omega^p/(3\Delta)$.

In simulation we choose the two channel models described in [7] and [8], where channel model A is suitable for shallow waters with smooth boundaries, where reflections from the bottom and surface are the dominant propagation mechanism. On the other hand, channel model B is appropriate for shallow water channels with random rough boundaries, where scattering is the dominant mechanism and precise deterministic modeling is not feasible. Using (21) and with $\Omega^p = 3/2$, average BER $\overline{P}_e$ is plotted in Fig. 2 versus $\overline{\zeta} = 1/\Delta$ for both channel types, with $K = 200$ symbols, at a rate of 2400 bits/sec. As a reference, average BER of a system with three scalar transmitters and vector receiver is shown in Fig. 2 for both channels as well. The parameters for channel A are $z_0' = 25$ m, $z_0 = 63$ m, channel depth $D_0 = 81.158$ m, transmission range $D_v = 1000$ m, and the bottom reflection coefficient magnitude $|\beta| = 0.7$. For channel B the bottom ray contribution ratios at the transmitter and receiver sides are $\Lambda_B = \Lambda_b = 0.1$, respectively, the mean of angle departure (AOD) of rays hitting the sea bottom at the transmitter is $\mu_B = 11\pi/12$ (165°), the mean AOD of rays hitting the sea surface at the transmitter is $\mu_S = 10\pi/9$ (200°), the mean angle of arrival (AOA) of rays from the sea bottom at the receiver is $\mu_b = \pi/18$ (10°), the mean AOA of rays from the sea surface at the receiver is $\mu_s = 348\pi/180$ (348°) and angle spreads are $\sigma_B = \sigma_S = \sigma_b = \sigma_s = \pi/120$ (1.5°) for the two AODs and AOAs. The average SNR in system is defined by first representing the powers of the channels stimulated by the three scalar transmitters Tx_1, Tx_2, Tx_3 in Fig. 1 as $\Omega^p = E[|P_{12}(f)|^2]$, $\Omega^{Tx_2} = E[|P_{22}(f)|^2]$ and $\Omega^{Tx_3} = E[|P_{32}(f)|^2]$, respectively. Then upon replacing Ω^y and Ω^z in the above relations by Ω^{Tx_2} and Ω^{Tx_3}, respectively, and using these channel power relations for a scalar transmitter and a vector receiver $\Omega^p = \Omega_v^p + \Omega_z^p$, $\Omega^{Tx_2} = \Omega_y^{Tx_2} + \Omega_z^{Tx_2}$ and $\Omega^{Tx_3} = \Omega_y^{Tx_3} + \Omega_z^{Tx_3}$ [3], it can be similarly shown that $\bar{\zeta} = (\Omega^p + \Omega^{Tx_2} + \Omega^{Tx_3})/(3\Delta)$. With $\Omega^{Tx_2} = \Omega^{Tx_3} = \Omega^p/2$ and $\Omega^p = 3/2$, the average SNR per channel per data block is $\bar{\zeta} = 1/\Delta$, used to plot the average BER of the reference system in Fig. 2.

According to Fig. 2, BER of the proposed vector transmitter system is lower than the BER of the system with three scalar transmitters. When translated into SNR, the proposed system offers 2.5 dB and 0.5 dB gains in channel B and channel A, respectively.

IV. CONCLUSION

In this paper, data transmission in particle velocity channels via acoustic vector transducers is proposed and investigated in detail. Based on the derived system and channel equations, as well as extensive analytical studies and simulation, feasibility and utility of signal modulation in particle velocity channels is demonstrated. The results provide new opportunities in communication via acoustic vector transducers.

APPENDIX
CLOSED-FORM EXPRESSION FOR SYMBOL ESTIMATION ERROR

For the isotropic noise model it can be shown that the noise terms in (11) are uncorrelated [3], i.e., $E[\xi_2^{(m)}(n)\{\xi_y^{(\tilde{m})}(\tilde{n})\}^*] = E[\xi_2^{(m)}(n)\{\xi_z^{(\tilde{m})}(\tilde{n})\}^*] = E[\xi_z^{(m)}(n)\{\xi_y^{(\tilde{m})}(\tilde{n})\}^*] = 0$, $\forall m, \tilde{m} = 1,2,3,4$ and $\forall n, \tilde{n} = 0,1,...,K+M-2$. Additionally, for the noise powers we have [3] $\Delta = E[|\xi_2^{(m)}(n)|^2]$, $\Delta_y = E[|\xi_y^{(m)}(n)|^2] = \Delta/2$ and $\Delta_z = E[|\xi_z^{(m)}(n)|^2] = \Delta/2$, $\forall m,n$. The noise covariance matrices can therefore be reduced to

$$\mathbf{\Sigma} = \dot{\mathbf{\Sigma}} = \begin{bmatrix} \Delta & 0 & 0 \\ 0 & \Delta/2 & 0 \\ 0 & 0 & \Delta/2 \end{bmatrix} \otimes \mathbf{I}_{K+M-1}, \quad (22)$$

where the property $\mathbf{Q}\mathbf{Q}^\dagger = \mathbf{I}_{K+M-1}$ is used and $\mathbf{I}_{K+M-1}$ is the $(K+M-1)\times(K+M-1)$ identity matrix. Now we compute the elements of the large matrix in (19). We start by the {1,1} sub-matrix, for which we have (23) - (25). The last expressions in (23) - (25) hold true because using $\dot{\mathbf{H}}_l = \mathbf{Q}\mathbf{H}_l\mathbf{Q}^*$, it is straightforward to verify that $\dot{\mathbf{H}}_l^\dagger\dot{\mathbf{H}}_l = \mathbf{Q}^T\mathbf{H}_l^\dagger\mathbf{H}_l\mathbf{Q}^*$, $l = 1,2,...,9$. Similar expressions can be obtained for {2,2} and {3,3} sub-matrices in (19). The off-diagonal sub-matrices in (19) can be shown to be $\mathbf{0}$. As an example, for the {1,2} sub-matrix we have (26).

$$\begin{aligned}\{\dot{\mathbf{H}}^p\}^\dagger\dot{\mathbf{\Sigma}}^{-1}\dot{\mathbf{H}}^p &= \Delta^{-1}[\dot{\mathbf{H}}_7^\dagger\ \dot{\mathbf{H}}_8^\dagger\ \dot{\mathbf{H}}_9^\dagger]\left(\begin{bmatrix}1&0&0\\0&2&0\\0&0&2\end{bmatrix}\otimes\mathbf{I}_{K+M-1}\right)\begin{bmatrix}\dot{\mathbf{H}}_7\\\dot{\mathbf{H}}_8\\\dot{\mathbf{H}}_9\end{bmatrix}\\ &= \Delta^{-1}(\dot{\mathbf{H}}_7^\dagger\dot{\mathbf{H}}_7 + 2\dot{\mathbf{H}}_8^\dagger\dot{\mathbf{H}}_8 + 2\dot{\mathbf{H}}_9^\dagger\dot{\mathbf{H}}_9)\\ &= \Delta^{-1}\mathbf{Q}^T(\mathbf{H}_7^\dagger\mathbf{H}_7 + 2\mathbf{H}_8^\dagger\mathbf{H}_8 + 2\mathbf{H}_9^\dagger\mathbf{H}_9)\mathbf{Q}^*,\end{aligned} \quad (23)$$

$$\begin{aligned}(\{\dot{\mathbf{H}}^y\}^\dagger\dot{\mathbf{\Sigma}}^{-1}\dot{\mathbf{H}}^y)^* &= \left(-j[\dot{\mathbf{H}}_4^T\ \dot{\mathbf{H}}_5^T\ \dot{\mathbf{H}}_6^T]\dot{\mathbf{\Sigma}}^{-1}j\begin{bmatrix}\dot{\mathbf{H}}_4^*\\\dot{\mathbf{H}}_5^*\\\dot{\mathbf{H}}_6^*\end{bmatrix}\right)^*\\ &= \Delta^{-1}(\dot{\mathbf{H}}_4^\dagger\dot{\mathbf{H}}_4 + 2\dot{\mathbf{H}}_5^\dagger\dot{\mathbf{H}}_5 + 2\dot{\mathbf{H}}_6^\dagger\dot{\mathbf{H}}_6)\\ &= \Delta^{-1}\mathbf{Q}^T(\mathbf{H}_4^\dagger\mathbf{H}_4 + 2\mathbf{H}_5^\dagger\mathbf{H}_5 + 2\mathbf{H}_6^\dagger\mathbf{H}_6)\mathbf{Q}^*,\end{aligned} \quad (24)$$

$$\begin{aligned}(\{\dot{\mathbf{H}}^z\}^\dagger\dot{\mathbf{\Sigma}}^{-1}\dot{\mathbf{H}}^z)^* &= \Delta^{-1}(\dot{\mathbf{H}}_1^\dagger\dot{\mathbf{H}}_1 + 2\dot{\mathbf{H}}_2^\dagger\dot{\mathbf{H}}_2 + 2\dot{\mathbf{H}}_3^\dagger\dot{\mathbf{H}}_3)\\ &= \Delta^{-1}\mathbf{Q}^T(\mathbf{H}_1^\dagger\mathbf{H}_1 + 2\mathbf{H}_2^\dagger\mathbf{H}_2 + 2\mathbf{H}_3^\dagger\mathbf{H}_3)\mathbf{Q}^*.\end{aligned} \quad (25)$$

$$\begin{aligned}&\{\dot{\mathbf{H}}^p\}^\dagger\dot{\mathbf{\Sigma}}^{-1}\dot{\mathbf{H}}^z - (\{\dot{\mathbf{H}}^z\}^\dagger\dot{\mathbf{\Sigma}}^{-1}\dot{\mathbf{H}}^p)^*\\ &= [\dot{\mathbf{H}}_7^\dagger\ \dot{\mathbf{H}}_8^\dagger\ \dot{\mathbf{H}}_9^\dagger]\dot{\mathbf{\Sigma}}^{-1}\begin{bmatrix}\dot{\mathbf{H}}_1^*\\\dot{\mathbf{H}}_2^*\\\dot{\mathbf{H}}_3^*\end{bmatrix} - \left([\dot{\mathbf{H}}_1^T\ \dot{\mathbf{H}}_2^T\ \dot{\mathbf{H}}_3^T]\dot{\mathbf{\Sigma}}^{-1}\begin{bmatrix}\dot{\mathbf{H}}_7\\\dot{\mathbf{H}}_8\\\dot{\mathbf{H}}_9\end{bmatrix}\right)^*\\ &= \Delta^{-1}(\dot{\mathbf{H}}_7^\dagger\dot{\mathbf{H}}_1^* + 2\dot{\mathbf{H}}_8^\dagger\dot{\mathbf{H}}_2^* + 2\dot{\mathbf{H}}_9^\dagger\dot{\mathbf{H}}_3^* - \dot{\mathbf{H}}_1^\dagger\dot{\mathbf{H}}_7^* - 2\dot{\mathbf{H}}_2^\dagger\dot{\mathbf{H}}_8^* - 2\dot{\mathbf{H}}_3^\dagger\dot{\mathbf{H}}_9^*)\\ &= \mathbf{0}.\end{aligned} \quad (26)$$

The last identity in (26) holds true because $\dot{\mathbf{H}}_l$ is diagonal, $l = 1,2,\cdots,9$, which results in $\dot{\mathbf{H}}_l^\dagger\dot{\mathbf{H}}_{l'}^* = \dot{\mathbf{H}}_{l'}^*\dot{\mathbf{H}}_l^\dagger = \dot{\mathbf{H}}_{l'}^\dagger\dot{\mathbf{H}}_l^*$. Overall, the large matrix in (19) and therefore $\mathcal{W}$ there are block diagonal matrices. This allows the estimation error covariance matrix of the data vector $\mathbf{S}_1$ to be written as

$$\begin{aligned}\mathbf{W} &= E[(\hat{\mathbf{S}}_1 - \mathbf{S}_1)(\hat{\mathbf{S}}_1 - \mathbf{S}_1)^\dagger]\\ &= E[\mathbf{Q}^{-1}(\hat{\bar{\bar{\mathbf{S}}}}_1 - \bar{\bar{\mathbf{S}}}_1)(\hat{\bar{\bar{\mathbf{S}}}}_1 - \bar{\bar{\mathbf{S}}}_1)^\dagger\{\mathbf{Q}^{-1}\}^\dagger]\\ &= E[\mathbf{Q}^*(\{1,1\}\text{ sub-matrix of }\mathcal{W})\mathbf{Q}]\end{aligned}, \quad (27)$$

where we have used these properties of the $\mathbf{Q}$ matrix: $\mathbf{Q}^{-1} = \mathbf{Q}^*$ and $\mathbf{Q}^T = \mathbf{Q}$ [10]. By inserting the sum of the last equations of (23) - (25) into (27) and using (22), (20) can be obtained for the estimation error covariance matrix of the data vector $\mathbf{S}_1$. The same result holds true for $\mathbf{S}_2$ and $\mathbf{S}_3$.

REFERENCES

[1] C. H. Sherman and J. L. Butler, Transducers and Arrays for Underwater Sound. New York: Springer, 2007.

[2] A. Abdi, H,Guo and P. Sutthiwan, "A new vector sensor receiver for underwater acoustic communication," in Proc. Oceans, Vancouver, BC, Candana, 2007, pp. 1-10.

[3] A. Abdi and H. Guo, "A new compact multichannel receiver for underwater wireless communication networks," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 3326-3329, 2009.

[4] A. Abdi and H. Guo, "Signal correlation modeling in acoustic vector sensor arrays," *IEEE Trans. Signal Processing*, vol. 57, pp. 892-903, 2009.

[5] H. Guo, A. Abdi, A. Song and M. Badiey, "Delay and Doppler spreads in underwater acoustic particle velocity channels," *J. Acoustic. Soc. Am*., vol.129, pp.2012-2025, 2011.

[6] H. Guo, C. Chen, A.Abdi, A. Song, M. Badiey and P. Hurskey, "Capacity and ststistics of measured underwater acoustic particle velocity channels, " in *Proc. Meetings on Acoustics*, vol. 14, pp. 1-17, 2012.

[7] C. Chen and A. Abdi, "Channel models for underwater vector transducer communication systems," in *Proc. Oceans*, Hampton Roads, VA, 2012.

[8] C. Chen and A. Abdi, " Utilization of underwater particle velocity channels for data transmission: Signal, channels and system performance," in Proc. *IEEE Int. Conf. Commun*., Ottawa, ON, Canada, 2012, pp.1-5.

[9] N. Yang and M. Salehi, "A new full rate full diversity orthogonal space-time block code for three transmit antennas," in *Proc. IEEE Conf. on Inform. Science and System*, 2007, pp. 851-856.

[10] N. Al-Dhahir, "Single-carrier frequency-domain equalization for space-time block coded transmissions over frequency-selective fading channels," *IEEE Commun. Lett*., vol.5, pp. 304-306, 2001.

[11] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: PTR Prentice-Hall, 1993.

[12] S. Ohno, **"**Performance of single-carrier block transmissions over multipath fading channels with linear equalization," *IEEE Trans. Signal Processing*, vol. 54, pp. 3678 – 3687, 2006.

[13] C. Tepedelenlioglu and Q. Ma, "On the performance of linear equalizers for block transmission systems," in *Proc. IEEE Globol Telecommun. Conf.,* St. Louis, MO, 2005, pp. 3892-3896.

[14] R. J. Urick, *Principles of Underwater Sound*. New York: McGraw-Hill, 1975.

[15] H. Xu, D. Chizhik, H. Huang and R. Valenzuela, "A generalized space-time multiple-input multiple-output (MIMO) channel model," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 966-975, 2004.