Comments on "Statistics of the Scattering Cross-Section of a Small Number of Random Scatterers"

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In [1], Daba and Bell have used a random vectors model to investigate the statistical behavior of the scattering cross-section, when the number of scatterers is small. Specifically, they have considered the sum of n random vectors with lengths A_i's and angles Φ_i's. Using a phasor notation, the ith vector can be represented by $A_i \exp(j\Phi_i)$, where $j = \sqrt{-1}$. Thus, the above-mentioned vector sum can be written as $\sum_{i=1}^{n} A_i \exp(j\Phi_i)$. In [1], n is a deterministic variable, A_i's are positive deterministic variables, and Φ_i's are independent random variables, distributed uniformly on $[0,2\pi)$. Based on the above assumptions, Daba and Bell have derived closed-form expressions for the probability density function (PDF) of $S_n = \left|\sum_{i=1}^{n} A_i \exp(j\Phi_i)\right|^2$, defined in [1, (2)], using two different approaches. In Section III.A of [1] and for different A_i's, a recursive method is developed; while in Section III.B of [1] and for equal A_i's, an orthonormal Laguerre polynomial representation is used. However, the following issues concerning these two methods should be mentioned:

1) It seems that the recursive method described in [1] has been used previously in [2], for solving exactly the same mathematical problem, i.e. the PDF of S_n.

2) Laguerre series expansion for the PDF of S_n, presented in [1], holds not only for positive identical constants A_i's, but also is applicable to the general case in which A_i's are dependent positive random variables [3] [4].

3) In [1], the coefficients of the Laguerre series expansion for the PDF of S_n are determined via

Monte-Carlo simulations, assuming positive identical constants A_i's. However, it can be shown that under the more general condition of dependent positive random A_i's, these coefficients can be computed exactly, via closed-form formulas [3] [4].

4) In [1], by the Kolmogorov-Smirnov nonparametric test, it is only confirmed that a Laguerre series expansion, with a finite number of terms, can closely approximate the PDF of S_n. However, to manipulate a series with infinite number of terms efficiently, it is important to have a quantitative relationship between the truncation error and the number of terms used. This important issue in not addressed in [1]. For a similar random vector problem, this subject is discussed completely in [3], and necessary relationships are derived between the truncation error and the number of terms used. These results are reported partly in [5] and [6].

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