Space-Time Doppler Spread Estimation in Mobile Fading Channels

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Abstract*— The Doppler spread, or the mobile speed, reflects how fast in time the fading is experienced by the receiver, and its knowledge is useful for many mobile communication subsystems. In this paper we propose a new speed estimator at the base station, where multiple antennas are available. The estimator is an extension of the single-antenna speed estimation technique proposed in [1]. By taking advantage of the spatial information provided by an antenna array at the base station, the new low-complexity estimator performs very well over a wide range of noise power, nonisotropic scattering, and line-of-sight component, verified by extensive Monte Carlo simulations. The superior performance of the new estimator is further illustrated by comparing with the conventional speed estimation techniques.

I. Introduction

Accurate estimation of the mobile speed, or the maximum Doppler frequency, which indicates the rate of wireless mobile channel variations, is important for many applications such as handoff, adaptive modulation and equalization, power control, etc. (see [1], [2], and references therein). There are three major classes of speed estimation techniques: crossing-based methods [2] [3], covariance-based methods [2] [3], and maximum likelihood (ML) based methods [4] [5]. Crossing-based approaches rely on counting the number of received signal's level crossing which is proportional to the mobile speed, while covariance-based algorithms exploit the maximum Doppler frequency information which exists in the sample autocovariance of the received signal. However, both crossing-based and covariance-based speed estimators are sensitive to noise, especially for small Doppler spreads. Although the ML-based estimators are optimal or near optimal, they are complex to implement, need the knowledge of signal-to-noise-ratio (SNR), and require noise to be Gaussian. Furthermore, the effects of nonisotropic scattering and line-of-sight (LOS) on ML-based estimators are not investigated in [4] and [5].

Recently a new approach has been proposed in [1] , which exploits the unique feature of the Doppler spectrum of mobile fading channels. Compared with conventional crossing-based and covariance-based speed estimators, this technique is not only robust against the noise, including both the Gaussian and impulsive non-Gaussian, but also insensitive to nonisotropic scattering observed at the mobile station (MS). In this paper, we extend the temporal-only speed estimator of [1] to a spatio-temporal estimator, using multiple antennas at the base station (BS) of macrocell-type environments. The new estimator is robust against noise, nonisotropic scattering, and the variations of line of sight (LOS) component, illustrated by Monte Carlo simulation.

The organization of this paper is as follows. The channel model is discussed in Section II, whereas the new speed estimator is presented in Section III. Section IV provides the numerical results and Section V concludes the paper.

II. The Channel Model

Consider a uniform linear antenna array at the elevated BS of a marcocell, composed of L omnidirectional unit-gain elements, with element spacing d. The BS experiences no local scattering, whereas the single antenna MS is surrounded by local scatters. The received lowpass complex envelope at the l-th element of the BS in a noisy Ricean frequency-flat fading channel, in response to an unmodulated carrier transmitted from the MS, is given by

$$z_l(t) = s_l(t) + n_l(t),\ l = 1, 2, ..., L, \tag{1}$$

where $n_l(t)$ represents the additive noise and the complex process $s_l(t)$ includes the random diffuse component $h_l(t)$ and the deterministic LOS component

$$\begin{aligned} s_l(t) =& \sqrt{\frac{P_s}{K+1}} h_l(t) + \sqrt{\frac{KP_s}{K+1}} \exp\{-j2\pi f_D t \cos\alpha_0\} \\ & \times \exp\{j2\pi(l-1)(d/\lambda)\cos\alpha_0 + j\varphi_0\}. \end{aligned} \tag{2}$$

In eq. (2), $h_l(t)$ is a zero-mean unit-variance complex Gaussian process, $P_s = E[|s_l(t)|^2]$ is the average signal power, and the Ricean factor K is the ratio of the LOS power to the diffuse power. We assume equal receive power P_s and Ricean factor K at each element. In the LOS component we have $f_D = \nu/\lambda = \nu f_c/c$ as the maximum Doppler frequency in Hz, ν is the MS speed, λ is the wavelength, f_c is the carrier frequency, and c is the speed of light. Furthermore, $j^2 = -1$, and α_0 and φ_0 stand for the angle-of-arrival (AOA) and the phase of the LOS component at the BS, respectively. The L noise components are independent, with the same power P_n.

With von Mises distribution for the AOA at the BS, the space-time crosscorrelation function between the received signals at the a-th and b-th elements, defined by $C_s((b-a)\Delta, \tau) = E[s_a(t)s_b^*(t+\tau)]$ such that $\Delta = d/\lambda$, is derived

as [6]

$$C_s((b-a)\Delta,\tau) = \frac{KP_s}{K+1}\exp\{j(u+v)\cos\alpha_0\} + \frac{P_s}{K+1} \times \frac{I_0\left(\sqrt{\kappa^2-u^2-v^2-2uv+j2\kappa(u+v)\cos\alpha}\right)}{I_0(\kappa)}, \quad (3)$$

where $u = 2\pi f_D\tau$, $v = 2\pi(a-b)\Delta$ with $1 \le a \le b \le L$, $I_0(.)$ is the zero-order modified Bessel function of the first kind, $\alpha \in [-\pi,\pi)$ is the mean AOA of the diffuse component at the BS, and $\kappa \ge 0$ controls the width of the diffuse component AOA.

According to the experiments conducted at different locations and frequencies [7] [8] [9] [10], the angle spread at the BS is generally small for macrocells in urban, suburban, and rural areas, most often less than $30°$, which corresponds to $\kappa \ge 14.6$, and in some cases very small, say, less than $10°$, which translates into $\kappa \ge 131.3$ [6] [11]. In such heavily nonisotropic scattering environments, the distribution of the diffuse AOA at the BS, $p(\phi)$, can be accurately approximated by a Gaussian distribution with mean α and variance $1/\kappa$ [6]. Due to the small angle spread at the BS, it is reasonable to assume $\alpha_0 = \alpha$ [12].

III. The Space-Time Speed Estimator

A. The Peak in the Power Spectrum

The power spectral density (PSD) of $h_l(t)$ for an arbitrary AOA distribution $p(\phi)$, in the two-dimensional plane, with a unit-gain isotropic receive antenna can be expressed as [2]

$$S_{h_l}(f) = \frac{P_s}{\sqrt{f_D^2-f^2}}\left[p(\phi)+p(-\phi)\right], \quad (4)$$

where $\phi = \cos^{-1}(f/f_D)$. Assuming a Gaussian distribution with mean α and variance $1/\kappa$ for $p(\phi)$, we observe $p(\phi)/p(-\phi) = \exp(2\kappa\phi\alpha)$. Clearly, as $\kappa \to \infty$ and for a given f, $S_{h_l}(f) \approx P_s p(\phi)/\sqrt{f_D^2-f^2}$ when α and ϕ have the same sign, and $S_{h_l}(f) \approx P_s p(-\phi)/\sqrt{f_D^2-f^2}$, when α and ϕ have opposite signs. Without loss of generality, we choose positive α and ϕ, which yields

$$S_{h_l}(f) \approx \frac{P_s}{\sqrt{f_D^2-f^2}}p(\phi) = P_s\sqrt{\frac{\kappa}{2\pi(f_D^2-f^2)}}\exp\left\{-\frac{\kappa\left(\cos^{-1}(f/f_D)-\alpha\right)^2}{2}\right\}. \quad (5)$$

By taking the derivative of (5) with respect to f and setting it to zero, with the assumption of $f \ne \pm f_D$ and some simplifications, we obtain

$$\frac{1}{\kappa} + \left(\left(\frac{f_D}{f}\right)^2 - 1\right)^{\frac{1}{2}}\left(\cos^{-1}\left(\frac{f}{f_D}\right)-\alpha\right) = 0. \quad (6)$$

When $\kappa \gg 1$, $f = f_D\cos\alpha$ is the root of eq. (6). It also can be shown that the second derivative of (5) with respect to f at $f = f_D\cos\alpha$ is negative.

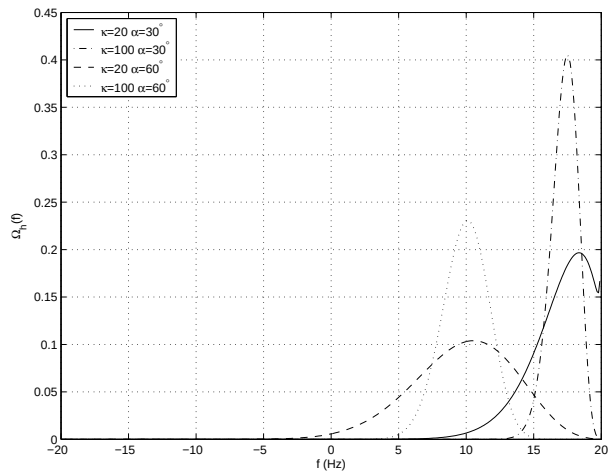

Fig. 1. The power spectral density $S_{h_l}(f)$

To show that the peak at $f = f_D\cos\alpha$ includes most of the signal power P_s as $\kappa \to \infty$, using the method of Laplace [13] one can obtain the power centered around $f = f_D\cos\alpha$, over a sufficiently small bandwidth ε

$$P_{f_D\cos\alpha} = \int_{f_D\cos\alpha-\varepsilon/2}^{f_D\cos\alpha+\varepsilon/2} S_{h_l}(f)df \approx P_s\,\mathrm{erf}(U\sqrt{\kappa/2}), \quad (7)$$

where $\mathrm{erf}(x)$ is the error function, and $U = \left|\cos^{-1}(\cos\alpha + f_D^{-1}\varepsilon/2) - \alpha\right|$. Since U is finite positive number, we have $P_{f_D\cos\alpha} \approx P_s$ as $\kappa \to \infty$. So, for large κ, the power P_s is mostly concentrated at $f = f_D\cos\alpha$.

To further validate this observation, we have plotted PSDs $S_{h_l}(f)$ in (4) for different κ and α, as shown in Fig. 1, with $f_D = 20$ Hz and $P_s = 1$.

B. The New Speed Estimator

For any given subchannel $z_l(t)$, we can use the same technique proposed in [1], to estimate $f_D\cos\alpha$ by $\hat{f}_{D,\alpha}^l$ at the l-th branch

$$\hat{f}_{D,\alpha}^l = f_s \times \left|\arg\max_{\mu_k}\hat{S}_{z_l}(\mu_k)\right|, \quad (8)$$

where $f_s = N/T$ and $\hat{S}_{z_l}(\mu_k)$ is the estimate of PSD of the N-sample discrete-time version of $z_l(t)$ with duration T, i.e., $\{z_l[n] = z_l(nT)\}_{n=0}^{N-1}$, given by

$$\hat{S}_{z_l}(\mu_k) = \frac{1}{N}\left|\sum_{n=0}^{N-1} z_l[n]e^{-j2\pi\mu_k n}\right|^2. \quad (9)$$

Furthermore, $\mu_k = k/N$, $k = 1-N/2, ..., N/2$, N even.

By setting $\tau = 0$ in (3), together with the Gaussian approximation for AOA, the spatial cross-correlation function $C_z((b-a)\Delta,0) = E[z_a(t)z_b^*(t)]$, $1 \le a \le b \le L$, can be

accurately approximated by [6]

$$C_z((b-a)\Delta,0) = C_s((b-a)\Delta,0) \approx \left\{ \frac{P}{K+1} \exp\left(-\frac{v^2 \sin^2\alpha}{2\kappa}\right) + \frac{KP}{K+1} \right\} \times \exp\{jv\cos\alpha\}, \quad (10)$$

where $C_z(\cdot,0) = C_s(\cdot,0)$, $1 \le a < b \le L$, due to the independence of noise components at different branches. Now, we can estimate $\cos\alpha$ via

$$\widehat{\cos\alpha} \approx \frac{\measuredangle \hat{C}_z(\Delta,0)}{-2\pi\Delta}, \quad (11)$$

where $\measuredangle$ denotes the phase of a complex number and $\hat{C}_z(\Delta,0)$ is the estimate of $C_z(\Delta,0) = E[z_a(t)z^*_{a+1}(t)]$, $\forall\ a \in [1, L-1]$, given by

$$\hat{C}_z(\Delta,0) = \frac{1}{L-1}\sum_{l=1}^{L-1} \hat{C}^l_z(\Delta,0). \quad (12)$$

In eq. (12), $\hat{C}^l_z(\Delta,0) = N^{-1}\sum_{n=0}^{N-1} z_l(n)z^*_{l+1}(n)$, $l \in [1, L-1]$, is the l-th adjacent-antenna-pair estimate of $C_z(\Delta,0)$. Now we justify the one-lag phase estimation in (11). Note that the estimator given in (11) is equivalent to the one-lag Pulse-Pair (PP) frequency estimation [eq. (6), 14] in the presence of Doppler spread, as the spatial cross-correlation function (10) used in this paper takes the same form as the channel autocorrelation function utilized in [14]. Therefore, the performance analysis conducted for PP frequency estimation in [14] can be applied to (11) as well. Although there exists an optimum lag to minimize the variance of the PP estimation of $\cos\alpha$, it has been shown [Fig. 1, 14] that at high SNR the one-lag and optimum-lag PP estimations are comparable and their performances are close to the CRLB. Clearly, according to (10), we have infinite SNR when estimating $\cos\alpha$ using (11).

Finally, f_D can be estimated via

$$\hat{f}_D = \hat{f}_{D,\alpha}/\widehat{\cos\alpha}, \quad (13)$$

where $\hat{f}_{D,\alpha} = L^{-1}\sum_{l=1}^{L} \hat{f}^l_{D,\alpha}$, and $\hat{f}^l_{D,\alpha}$ and $\widehat{\cos\alpha}$ are given in (8) and (11), respectively.

IV. Numerical Results

In this section, we first present the performance of the proposed estimator using Monte Carlo simulation and next investigate the effect of AOA parameters κ and α, the Ricean factor K, the number of antenna elements L, and element spacing Δ. Finally we compare this new estimator with conventional speed estimation techniques. In each simulation, unless specified otherwise, 500 independent realizations of $L = 8$ space-time correlated zero-mean complex Gaussian processes for the scattering scenario $\{\kappa = 100,\ \alpha = 60°\}$ are generated using spectral method [15], with $N = 256$ complex samples per realization, over $T = 1$ second, and $\Delta = 1/2$. For each channel, the noise is Gaussian with signal-to-noise-ratio $\mathrm{SNR} = P_s/P_n = 10\,\mathrm{dB}$, and the receiver bandwidth is fixed at 101 Hz. Note that for the case of Rayleigh channel with a single antenna at the BS, $L = 1$, we have calculated the theoretical estimation error using equations (13) and (32) of [1].

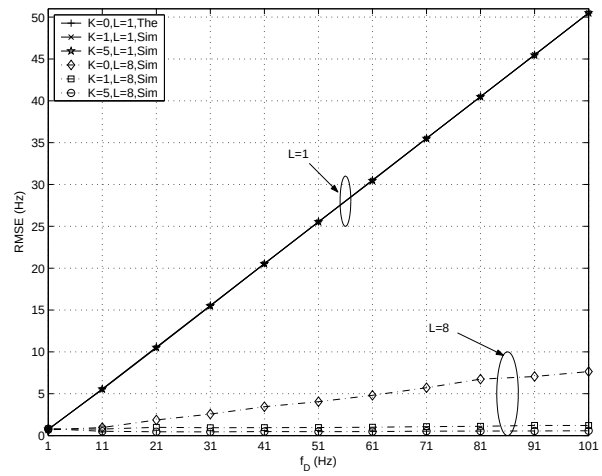

Fig. 2. Estimation error versus f_D for both Rayleigh and Ricean fading channels

Fig. 2 illustrates the estimator's root mean square error (RMSE) versus f_D, for both Rayleigh and Ricean channels. Clearly the performance of the array-based estimator is much better. Fig. 2 also shows that Ricean factor helps to decrease the estimation error because the LOS makes the peak of PSD stronger and estimate of α more accurate. The effect of κ and α on the performance, for $f_D = 41$ Hz, is shown in Fig. 3. As one can see, the estimation error decreases as κ increases or α becomes smaller. Again the array-based estimator provides much smaller estimation error. The effects of L and Δ are also investigated. Simulation results in Fig. 4 show that the performance improves significantly as L changes from one to two, because the spatial information is used. However, further increase of the number of antennas does not result in much performance gain, as the L estimates $\left\{\hat{f}^l_{D,\alpha}\right\}_{l=1}^{L}$ are highly correlated, due to the small element spacing. For example, for $f_D = 41$ Hz, $\kappa = 100$, and $\alpha = 60°$ in Rayleigh fading channel, the RMSE in Hz is 20.7, 4.2, 3.2, for $L = 1, 2, 10$, respectively. We have also observed from Fig. 5 that the array-based estimator is fairly insensitive to the choice of Δ. For example, for $f_D = 41$ Hz, $\kappa = 100$, $\alpha = 60°$, and $L = 8$, in Rayleigh fading, the RMSE changes within 0.5 Hz, as Δ varies from 0.25 to 1.

Now we compare our proposed space-time speed estimator with two widely-known methods: inphase zero crossing rate based estimator (IZCR) and inphase integration or covariance based estimator (ICOV) [2] [3]. Applying either IZCR or ICOV at each branch, the Doppler spread can be obtained by averaging over the L estimates. This results in

$$\hat{f}_{D,IZCR} = \frac{1}{L}\sum_{l=1}^{L} \hat{f}^l_{D,IZCR} = \frac{\sqrt{2}}{LT}\sum_{l=1}^{L} N_{x_l}(0,T), \quad (14)$$

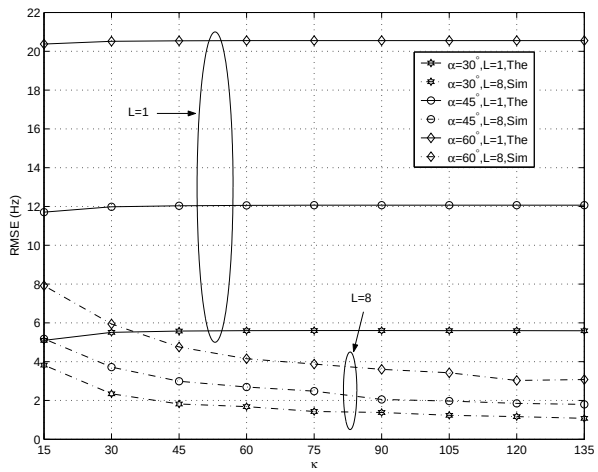

Fig. 3. Effect of κ and α in a Rayleigh fading channel

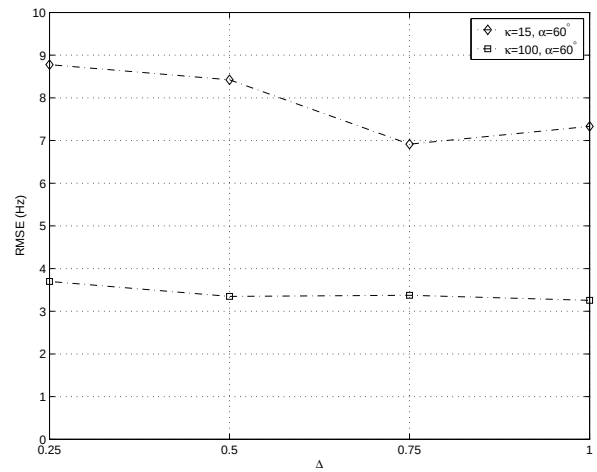

Fig. 5. Effect of the number of element spacing Δ in Rayleigh fading channel

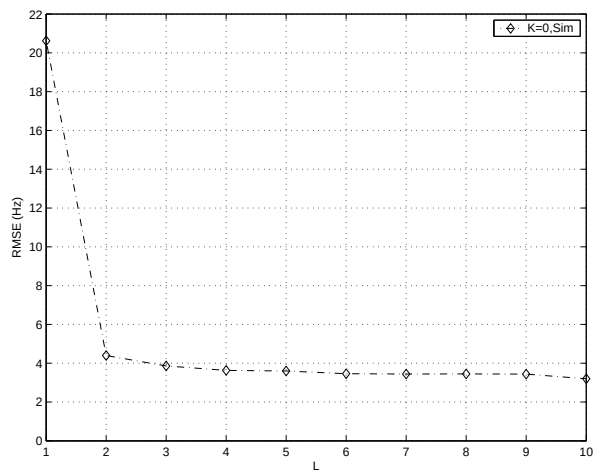

Fig. 4. Effect of the number of antenna elements L in Rayleigh fading channel

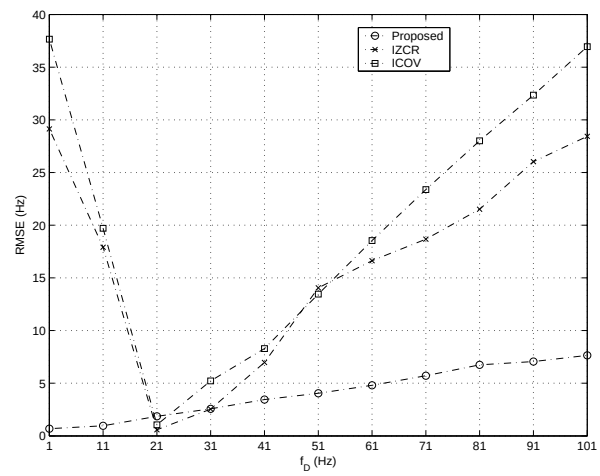

Fig. 6. Performance comparison in a Rayleigh fading channel

and

$$\hat{f}_{D,ICOV} = \frac{1}{L}\sum_{l=1}^{L}\hat{f}_{D,ICOV}^{l} = \frac{\sqrt{2}}{L}\sum_{l=1}^{L}\sqrt{\frac{\int_0^T \dot{x}_l^2(t)dt}{\int_0^T x_l^2(t)dt}}, \quad (15)$$

where $x_l(t)$ is the inphase component of $z_l(t)$, $N_{x_l}(0,T)$ stands for the number of times that process $x_l(t)$ crosses the threshold level 0 with positive slope over the time interval T, and dot denotes differentiation with respect to the time t. As shown in Fig. 6, the new speed estimator exhibits excellent performance when compared with IZCR and ICOV whose performances are significantly degraded due to the receiver noise and nonisotropic scattering.

V. Conclusion

In this paper we have proposed a space-time Doppler spread estimating method in mobile fading channels. This new method exploits both the characteristic of the Doppler spectrum of a fading channel and the spatial information provided by multiple antennas. By decoupling space and time processing, this algorithm is of low computational complexity, which makes it attractive to practical applications. In comparison with the conventional crossing-rate based method and covariance-based method, the new solution exhibits excellent performance over different channel conditions, validated by extensive Monte Carlo simulations.

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