A VERSATILE SPATIO-TEMPORAL CORRELATION FUNCTION FOR MOBILE FADING CHANNELS WITH NON-ISOTROPIC SCATTERING

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ABSTRACT

For the analysis and design of adaptive antenna arrays in mobile fading channels, we need a model for the spatio-temporal correlation among the array elements. In this paper we propose a general spatio-temporal correlation function, where non-isotropic scattering is modeled by von Mises distribution, an empirically-verified model for non-uniformly distributed angle of arrival. The proposed correlation function has a closed form and is suitable for both mathematical analysis and numerical calculations. The utility of the new correlation function has been demonstrated by quantifying the effect of non-isotropic scattering on the performance of two applications of the antenna arrays for multiuser multichannel detection and single-user diversity reception. Comparison of the proposed correlation model with published data in the literature shows the flexibility of the model in fitting real data.

1. INTRODUCTION

In recent years the application of adaptive antenna arrays (smart antennas) for cellular systems has received much attention [1], since they can improve the coverage, quality, and capacity of such systems by combating interference, fading, and other undesired disturbances. An adaptive array can be defined as an adaptive spatio-temporal filter, which takes advantage of both time-domain and space-domain signal characteristics. Efficient joint use of time-domain and space-domain data demands a generalization of conventional communication theory and signal processing techniques to spatial and temporal communication theory [2] and space-time signal processing techniques [3]. Needless to say, new spatio-temporal channel models have to be developed as well. Since the second-order statistics of the channel characterize the basic structure of stochastic mobile channels, we need a spatio-temporal correlation function to study the basic impact of the random channel on the performance of space-time solutions, including the adaptive antenna arrays.

In this paper we present a flexible and versatile parametric correlation function for the mobile station (MS) (similar results can be obtained for the base station (BS) as well, as we see in Section 4). We do this by generalizing the spatio-temporal correlation function in [4], originally derived for an isotropic scattering scenario where the MS receives signals from all direction with equal probability, to the non-isotropic scattering case. Note that isotropic scattering at the MS corresponds to the uniform distribution for the angle of arrival (AOA) at the MS. However, empirical results have shown that due to the structure of the mobile channel, the MS is likely to receive signals only from particular directions (see [5] and references therein). In other words, most often the MS experiences non-isotropic scattering, which results in a non-uniform distribution for the AOA at the MS. In [5] it has been shown that the application of von Mises distribution for the AOA at the MS yields an easy-to-use and closed-form expression for the temporal (or equivalently, spatial) correlation function. This correlation function has exhibited very good fit to measured data [5].

In the sequel we derive a new spatio-temporal correlation function where non-isotropic scattering is modeled by the von Mises distribution. To show the significant effect of non-isotropic scattering on the performance of smart antenna systems employing space-time data, we study the performance of an antenna array multiuser detector equipped with a channel estimator, operating in a Rayleigh fading channel. As a simpler example where only space data are employed, we also investigate the impact of non-isotropic scattering on a multi-element receiver working as a maximal ratio combiner (MRC) in a Rayleigh fading channel. In both examples we show how the proposed spatio-temporal correlation function helps us in quantifying the effect of the fading channel on the performance of antenna arrays, in the realistic scenario of non-isotropic scattering. The paper concludes with a comparison of the proposed correlation model with the published correlation data, collected by a BS-mounted array.

2. A NEW CORRELATION FUNCTION

Consider a linear uniformly-spaced antenna array shown in [4, Fig. 2], mounted on a MS. Let $r_m(t)$ denote the complex envelope at the mth element from left. Then the normalized correlation function between the complex envelopes of the mth and the nth antenna elements, defined by $\tilde{\phi}_{mn}(\tau) = E[r_m(t) r_n^*(t+\tau)] / E[|r_m(t)|^2]$, can be derived from [4]:

$$\tilde{\phi}_{mn}(\tau) = E\left[\exp\{j2\pi f_d \tau \cos(\Theta - \alpha) + j(m-n)2\pi(d/\lambda)\cos\Theta\}\right], \quad (1)$$

where E denotes mathematical expectation, $j = \sqrt{-1}$, f_d is the maximum Doppler frequency, Θ stands for the AOA, α represents the direction of the motion of the MS with respect to the horizontal axis counterclockwise, d is the spacing between any two adjacent antenna elements, and λ is the wavelength. Now we consider the von Mises probability density function (PDF) for the random variable Θ:

$$p_\Theta(\theta) = \frac{\exp\{\kappa \cos(\theta - \theta_p)\}}{2\pi I_0(\kappa)}, \quad \theta \in [-\pi, \pi), \quad (2)$$

where $I_0(.)$ is the zero-order modified Bessel function, $\theta_p \in [-\pi, \pi)$ accounts for the mean direction of AOA, and $\kappa \ge 0$ controls the width of the AOA distribution [5]. For $\kappa = 0$ (isotropic scattering) we have $p_\Theta(\theta) = 1/(2\pi)$, while for $\kappa = \infty$

(extremely non-isotropic scattering) we obtain $p_\Theta(\theta)=\delta(\theta-\theta_p)$, where $\delta(.)$ is the Dirac delta function. By calculating the expectation in (1) according to (2) we obtain:

$$I_0(\kappa)\tilde{\phi}_{mn}(\tau)= \quad (3)$$
$$I_0\left(\sqrt{\kappa^2-x^2-y^2-2xy\cos\alpha+j2\kappa[x\cos(\alpha-\theta_p)+y\cos\theta_p]}\right),$$

where $x=2\pi f_d\tau$ and $y=2\pi(m-n)d/\lambda$. With $\kappa=0$, (3) reduces to Lee's spatio-temporal correlation function $J_0(\sqrt{x^2+y^2+2xy\cos\alpha})$ in [4, Eqs. (42)-(43)] for isotropic scattering, where $J_0(.)$ is the zero-order Bessel function. For $m=n=1$ (single antenna), Lee's result further simplifies to Clarke's classic temporal correlation function $J_0(x)$ [6, p. 40, Eq. (2.20)]. For a single antenna experiencing non-isotropic scattering and $\alpha=0$, (3) reduces to the temporal correlation function $I_0(\sqrt{\kappa^2-x^2+j2\kappa x\cos\theta_p})/I_0(\kappa)$ derived in [5, Eq. (2)] (this correlation function has shown very good fit to measured data [5]).

In comparison with the existing spatial correlation functions for antenna arrays [7], our proposed model in (3) has the main advantage that it includes both space and time dimensions in a single mathematically-tractable closed-form expression, flexible for fitting to array data, studying the performance of various array-based techniques [8] for different applications in fading channels with the realistic assumption of non-isotropic scattering, optimizing array configurations [9], etc..

3. TWO ARRAY APPLICATIONS

In this section we use the proposed model in (3) for two array-based applications. In the first one we need a spatio-temporal correlation function, while for the second one a spatial-only correlation function is needed. In array applications, the need for a spatio-temporal correlation function also appears in conjunction with such important fading characteristics as level crossing rate and average fade duration [4] [10], which due to space limitations we do not address here.

3.1 Efficiency of Two Multiuser Multichannel Array Detectors

For code division multiple access (CDMA) signals, recently two array-based multiuser detection schemes with imperfect estimates of the fading channel were investigated in [11]: the decision-directed detector (with more complexity) which is optimum, and the decorrelating detector (with less complexity) which is suboptimum. In terms of the asymptotic efficiency, it has been proven that the decision-directed detector is superior. However, the decorrelating detector is simpler to implement. So, it is of interest to determine how much these two detectors are different in terms of asymptotic efficiency. Here, by a simple example [12, p. 107 and p. 117] we show that the answer strongly depends on the mode of scattering, which affects the correlation function of the complex envelope in the fading channel.

Assume that the MS has a two-element antenna ($M=2$), and there are two mobile users ($K=2$) according to the configuration shown in Figs. 1 and 2 ($\theta_{p,1}=0$, $\theta_{p,2}=\pi$). In Fig. 1 we have $\kappa_1=\kappa_2=0$, where the MS receives scattered plain waves from all directions with equal probability, while in Fig. 2, where $\kappa_1=\kappa_2=10$, the MS receives directional waves from two specific directions (the beamwidth in each direction is equal to $\mathrm{BW}=2/\sqrt{\kappa}\approx 36^\circ$ [5]). Suppose the first user is the desired user, while the second one is the interfering user. The MS moves from left to right ($\alpha=0$) and the users travel at speeds such that the desired user has the maximum Doppler frequency $f_{d,1}=0.1$ Hz, while the interfering user has the maximum Doppler frequency $f_{d,2}=0.05$ Hz. Assume the correlation coefficient between the users' signature waveforms is $\rho_{12}=0.5$, and the MS uses only the past two values ($I=2$) of matched filter outputs and bit decisions for fading estimation and bit detection in the presence of Rayleigh fading and zero-mean additive white Gaussian noise with variance σ^2. Suppose both users have (equal) unit power. Let us define the signal-to-noise ratio (SNR) as $\gamma=1/\sigma^2$. For $d=0.3\lambda$ and λ, the asymptotic efficiency of the desired users, η_1, calculated using the equations given in [12], is plotted in Figs. 3 and 4 versus SNR, assuming $\kappa_1=\kappa_2=0$ and $\kappa_1=\kappa_2=10$. According to both figures, as κ increases (more directional reception), the efficiency of both detectors increases significantly (which is good news). However, the difference between the detectors efficiencies increase as well, which implies that choosing the decorrelating detector, due to its lower complexity, introduces a significant loss in efficiency when we have non-isotropic scattering. Hence, we need to develop new suboptimum low-complexity detectors with efficiencies comparable with the optimum detector, in channels with directional reception.

3.2 Average Bit Error Rate of a Single-User Multichannel Array Detector

Assume that in Figs. 1 and 2, we have user one only ($K=1$), and $\theta_p=0$. Moreover, both the MS and the user are stationary ($f_d=0$). The user sends data using binary phase shift keying (BPSK) modulation scheme, and the MS is equipped with a two-branch ($M=2$) maximal ratio combiner (MRC). The average bit error rate (BER) in this case is given by [13, Eq. (12)]:

$$P_b(\gamma)=\frac{1}{4}\left\{2-\frac{1}{\rho}\left[(1+\rho)\sqrt{\frac{\gamma(1+\rho)}{1+\gamma(1+\rho)}}-(1-\rho)\sqrt{\frac{\gamma(1-\rho)}{1+\gamma(1-\rho)}}\right]\right\}, \quad (4)$$

where $\rho=|\tilde{\phi}_{12}(0)|$. In Figs. 5 and 6 we have plotted $P_b(\gamma)$ versus γ for $d=0.3\lambda$ and λ, respectively. As we expect, the average BER increases as κ increases, because it results in more correlation between the branches. Of course, a larger d can reduce the amount of correlation between branches, resulting in smaller average BER (compare Figs. 5 and 6).

4. COMPARISON WITH DATA

Although the application of antenna arrays in both MS and BS is advantageous, in this section we focus on BS since the application of arrays at the BS is more common (practical constraints usually restrict the use of an array of antennas at a MS). For statistical characterization of narrow histograms of the AOA of waves impinging the BS [14] [15] (which gives rise to the non-uniform distribution of power versus the azimuth angle [16]), three different PDF's are used so far in the literature: cosine [17], Gaussian [18], and truncated uniform [19]. All these

PDF's are considered primarily for studying the effect of non-uniformly distributed AOA on the spatial correlation among the array elements at a BS. With appropriate choice of parameters, these three PDF's can resemble visually the narrow histograms of the AOA at the BS (although the truncated uniform PDF is less likely to do that because the empirical histograms are usually bell-shaped [14] [15] and decay to zero not as abruptly as a truncated uniform PDF). So, mathematical convenience seems to be the main concern in choosing a PDF for the AOA, among empirically-acceptable candidates. From this point of view, none of these three PDF's are able to provide a simple closed-form solution (in terms of known mathematical functions) for the correlation between the complex envelopes of the array elements (which is a basic quantity in array-related studies). For the Gaussian PDF only approximate results can be found [18] [20], and for the truncated uniform PDF, closed-form results can be derived only for inline and broadside cases [21] (the cosine PDF is less likely to yield a closed-form answer because of the special integral that has to be solved). On the other hand, as we see in the sequel, von Mises PDF yields a simple and compact expression, given in (5), which is basically the same as (3). This makes the von Mises PDF a very suitable model.

Comparison of the Gaussian PDF with the histograms of AOA data has shown reasonable agreement [15] [22]. This is a good empirical support for the von Mises PDF because for large κ, the PDF in (2) resembles a small-variance Gaussian PDF with mean θ_p and standard deviation $1/\sqrt{\kappa}$ [23, p. 60]. In fact, for any beamwidth (angle spread) smaller than 40^o (which correspond to $\kappa > 8.2$ according to the definition of beamwidth as $\mathrm{BW} = 2/\sqrt{\kappa}$ in [5]), the plots of Gaussian and von Mises PDF are indistinguishable (two typical standard deviations for the Gaussian PDF are 15^o [22] and 6^o [15], which correspond to $\kappa = 14.6$ and $\kappa = 91.2$, respectively). However, recall that von Mises PDF is able to provide a general and closed-form solution for the space-time correlation between the complex envelopes of the array elements, while Gaussian PDF cannot.

Using exactly the same notation as [17], it is straightforward to show that for the linear uniformly-spaced antenna array at the BS in [17, Fig. 6] we have:

$$I_0(\kappa)\tilde{\phi}_{mn}(\tau) = I_0\left(\sqrt{\kappa^2 - x^2 - y^2 + 2xy\cos\gamma + j2\kappa[x\cos(\gamma-\alpha) - y\cos\alpha]}\right), \quad (5)$$

provided that AOA has a von Mises PDF with the mean direction $\alpha \in [-\pi, \pi)$ and the width control parameter $\kappa \geq 0$. All of the parameters in (5) are the same as (3), except for γ in (5) which represents the direction of the motion of the MS with respect to the horizontal axis counterclockwise, in place of α in (3) (the γ here should not be confused with the SNR symbol γ, used in Section 3). The two sign changes in (5), in comparison with (3), come from different ways of numbering the array elements: in [4, Fig. 2], the elements are numbered from left to right, while elements numbering in [17, Fig. 6] is from right to left.

Now we compare our correlation model with the data published in [17], where the data are spatial cross-correlations between the square of the envelopes of a two element array, mounted on a BS. We do this by considering two models for the AOA PDF at the BS: the simple model with $p_\Theta(\theta) = \exp\{\kappa\cos(\theta-\alpha)\}/2\pi I_0(\kappa)$, and the composite model with $p_\Theta(\theta) = \zeta\exp\{\kappa\cos(\theta-\alpha)\}/2\pi I_0(\kappa) + (1-\zeta)/2\pi$, where $0 \leq \zeta \leq 1$ indicates the amount of directional reception. The composite PDF reduces to the von Mises PDF for $\zeta = 1$, and simplifies to the uniform PDF for $\zeta = 0$. Consequently, the associated spatial correlation functions for a two element array at a BS can be written as:

$$\tilde{\phi}_{12}(0) = I_0\left(\sqrt{\kappa^2 - 4\pi^2(d/\lambda)^2 + j4\pi\kappa(d/\lambda)\cos\alpha}\right)\Big/ I_0(\kappa), \quad (6)$$

$$\tilde{\phi}_{12}(0) = \zeta\, I_0\left(\sqrt{\kappa^2 - 4\pi^2(d/\lambda)^2 + j4\pi\kappa(d/\lambda)\cos\alpha}\right)\Big/ I_0(\kappa) + (1-\zeta)J_0(2\pi d/\lambda). \quad (7)$$

Figs. 7-8 show Lee's correlation data, plotted together with $|\phi_{12}(0)|^2$ calculated according to (6) and (7) for both models. For a given α (known a priori for each data set), the unknown κ for the simple model and the unknown pair (κ, ζ) for the composite model are estimated by the nonlinear least squares method (implemented via a systematic numerical search technique). Based on these figures (and many others not shown due to space limitations), the von Mises PDF is able to account for the variations of the correlation versus antenna spacing with reasonable accuracy (compare our correlation plots with those drawn in [17] assuming the cosine PDF and [21] using the truncated uniform PDF, both for the same data sets. Interestingly, the correlation plots in [17] can also be considered as curves obtained based on a Gaussian PDF, because for small BW, the cosine PDF can be approximated by a Gaussian PDF [21]). Note that in Fig. 7 both models are similar ($\zeta = 0.98$), while in Fig. 8 the composite model shows a much better fit ($\zeta = 0.74$). In general the composite model was able to improve the fits obtained by the simple model, which is not surprising because it has the additional parameter ζ. This is in agreement with the noise-like signal introduced in [17].

5. CONCLUSION

Space-time processing using antenna arrays over wireless mobile fading channels offer several advantages in cellular systems, such as mitigating fading, intersymbol interference, cochannel interference, etc.. Efficient joint use of both space and time dimensions demands for spatio-temporal channel models. As a basic channel model, we need a two dimensional spatio-temporal correlation function among the random signals sensed by the array elements, to characterize the second order dependence structure of the random channel in both space and time. In this paper we have proposed a flexible spatio-temporal correlation function for propagation scenarios with non-isotropic scattering (signal reception from specific directions). The non-uniform distribution for the angle of arrival, which characterizes the non-isotropic scattering, is modeled by von Mises PDF which has previously shown to be successful in describing the measured data. The proposed spatio-temporal correlation function is general enough to include important special cases such as Lee's spatio-temporal correlation function and Clarke's temporal correlation function, both derived for isotropic scattering. Moreover, its compact mathematical form facilitates analytical manipulations of array-based techniques and results in terms of closed-form expressions for such important fading parameters as spectral moments (successive derivatives of the correlation function). Based on two case studies (multiuser detection and diversity reception) and using the new spatio-temporal correlation function, we have shown that non-isotropic scattering (typical of many mobile channel scenarios) has a significant impact on the performance of array processors, and should be taken into account in the analysis and design of adaptive antenna arrays for mobile fading channels.

Theoretically, the new correlation function is applicable to both MS and BS. However, since practical restrictions limit the use of multiple antennas at a MS, the proposed correlation function seems to be of much more use in a BS. Therefore, the empirical justification of the new correlation function is demonstrated by comparison with published data collected at a BS.

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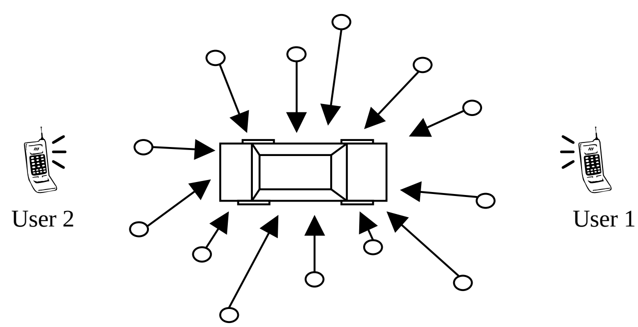

Figure 1. Isotropic scattering in an open area (circles are scatterers).

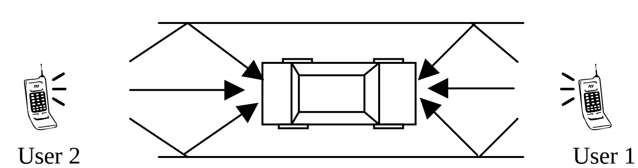

Figure 2. Non-isotropic scattering in a narrow street.

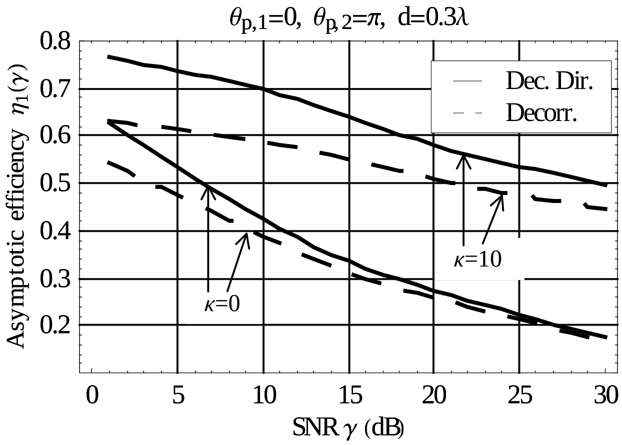

Figure 3. Asymptotic efficiency of two multiuser array detectors.

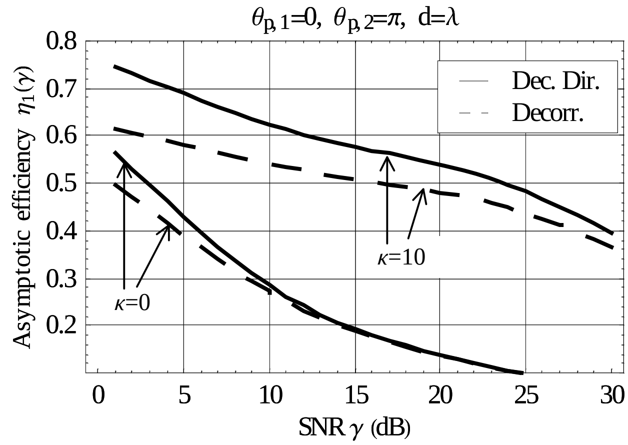

Figure 4. Asymptotic efficiency of two multiuser array detectors.

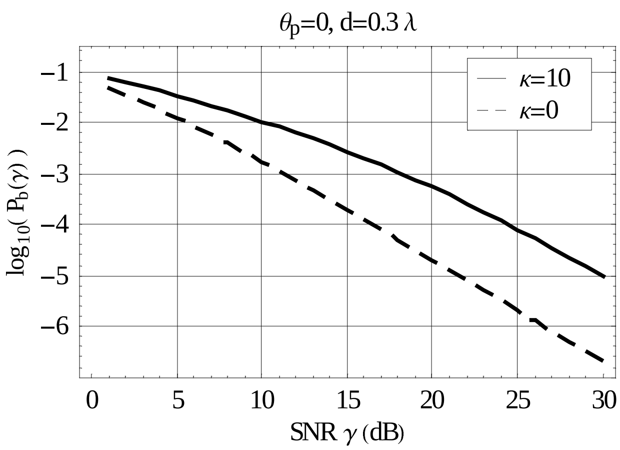

Figure 5. Bit error rate of BPSK with two-branch MRC.

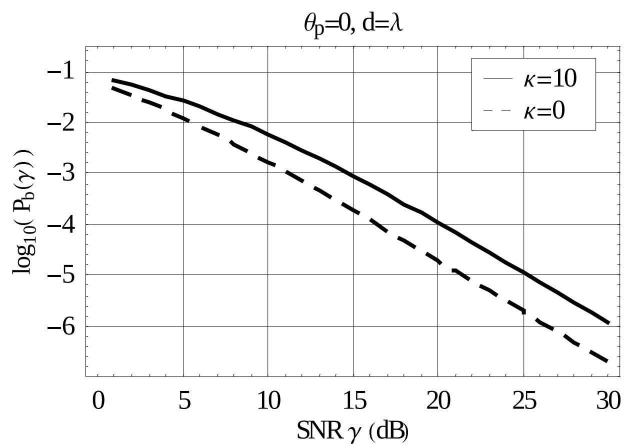

Figure 6. Bit error rate of BPSK with two-branch MRC.

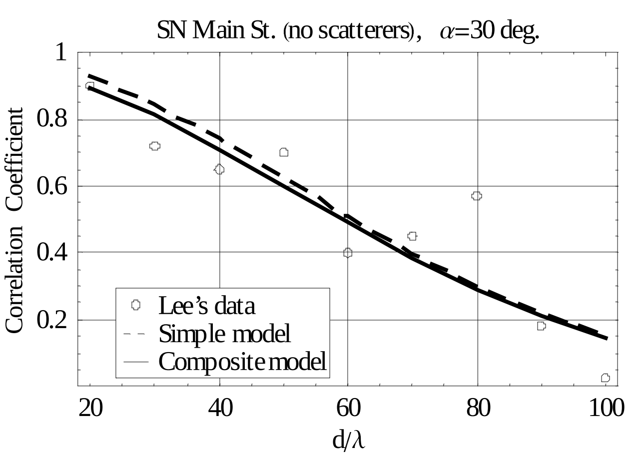

Figure 7. Correlation coefficient versus antennas spacing
Simple: $BW = 0.5^{o}$, Composite: $BW = 0.5^{o}$, $\zeta = 0.98$

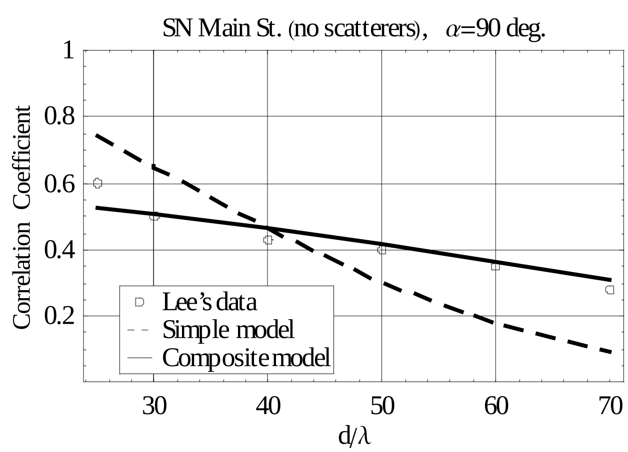

Figure 8. Correlation coefficient versus antennas spacing
Simple: $BW = 0.4^{o}$, Composite: $BW = 0.2^{o}$, $\zeta = 0.74$