SIGNAL MODELING IN WIRELESS FADING CHANNELS USING SPHERICALLY INVARIANT PROCESSES

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ABSTRACT

In this paper we propose the application of spherically invariant random processes for joint modeling of the fluctuations of the signal envelope and phase in narrowband wireless fading channels. In this model, phase has uniform distribution (which is very common in fading channels), while the envelope can be distributed according to an arbitrary distribution law (which includes Rayleigh as a special case). The great utility of the spherically invariant random processes, as a large family of non-Gaussian random processes, lies on their many Gaussian-like properties, which make them very flexible for multivariate statistical analysis, optimal estimation, simulation, and other signal processing issues. Empirical justifications are also given, which strongly support the acceptability of the model.

1. INTRODUCTION

Wireless communication systems are subject to the fading phenomenon, which significantly deteriorates their performance. There are two types of fading: fast (multipath) fading and slow (shadow) fading [1]. Fast fading is the rapid fluctuations of the signal over short time-intervals, while slow fading refers to the gradual variations of the signal power over long time-intervals. In this paper we focus on the stochastic modeling of the fast fading in narrowband (frequency non-selective) wireless propagation channels.

In a narrowband propagation channel, the signal of interest behaves approximately like a single-frequency sinusoid with time-varying envelope and phase components. Whenever the physical channel satisfies the conditions of the central limit theorem (CLT), the propagated signal can be modeled as a Gaussian process [2], with envelope and phase described by Rayleigh and uniform processes, respectively. This Gaussian model is the first one developed several decades ago for fast fading, and still is in use. The great utility of this model comes from its Gaussian-based nature. Gaussian vectors and processes are known to be the most mathematically tractable models in statistics and random process theory, as well as communications and signal processing. However, empirical results have shown that for a large variety of wireless fading channels, the Gaussian model does not hold, and non-Gaussian models have to be developed.

In the area of communication, uniform distribution has been widely accepted for the phase, while different non-Rayleigh distributions have been considered for the envelope, for example Rice, Nakagami, Weibull, etc. [1]. Note that Rice-distributed envelope together with the uniformly-distributed phase constitute a Gaussian model, while for Nakagami and Weibull we have non-Gaussian models. For some applications in communication engineering where the envelope is of main concern, e.g. performance evaluation of various diversity systems, non-Gaussian based distributions like Nakagami and Weibull are useful. However, for those cases where the signal itself (or its inphase and quadrature components) are important, for example multiuser detection, these distributions become useless. In fact, even the univariate distribution of the inphase and quadrature components of the fading signal with Nakagami- or Weibull-distributed envelope is so complicated that makes any multivariate statistical analysis intractable. Maybe this is why the Nakagami, Weibull, and other non-Gaussian based envelope distributions, widely used in the communications literature, have not been used in the area of signal processing. A quick scan of the main signal processing publications in the past few years clearly reveals that only Rayleigh and Rice, the Gaussian-based envelope distributions, have been used as models for the fast fading. The main reason for this is the well-defined and mathematically-tractable methods which have been developed for multivariate statistical analysis of Gaussian vectors and processes. Hence, for fast fading, we need an empirically-supported non-Gaussian model with a well-defined and easy-to-use multivariate distribution.

2. SPHERICALLY INVARIANT RANDOM PROCESSES

Among the available models for non-Gaussian processes, spherically invariant random processes (SIRP's) play an important role. These processes are of interest mainly due to the fact that they allow one to relax the assumption of Gaussianity, while keeping many of its useful characteristics [3]-[8]. For example, the class of SIRP's is closed under linear transformations, the minimum-mean-squared-error estimator for SIRP's is linear, SIRP's have well-defined and mathematically-tractable multivariate probability density functions (PDF's) and characteristic functions, they can be fully characterized in terms of the autocorrelation function and a univariate non-Gaussian PDF, and simulation of SIRP's is straightforward. All of these useful properties come from a key characterization theorem [3] [5] [8] which states that a SIRP can be represented by a Gaussian process, multiplied by an independent and positive random variable. So, the wide sense stationary (WSS), bandpass zero-mean SIRP $X(t)$ can be written as:

$$X(t) = C\,I(t)\,, \tag{1}$$

where $I(t)$ is a WSS, bandpass zero-mean Gaussian process, independent of the positive random variable *C*. Hence, a SIRP is nothing but a Gaussian process with random variance, and in any application, by conditioning on *C*, the non-Gaussian process $X(t)$ can be easily treated as a Gaussian process.

The SIRP's have been successfully used for modeling radar clutter [9], speech [10], and impulsive noise [11]. Due to the attractive Gaussian-like properties of SIRP's, they have been recently employed in a variety of applications where the underlying random process is non-Gaussian, for example, analysis of two correlation estimation methods [12], vector quantization [13] and rate-distortion function [14] for speech signals, signal detection [15] [16], simulation of speech [10] and radar clutter [17], performance analysis of LMS and NLMS [18], acoustic echo cancellation [19], analyzing the behavior of array processing algorithms [20], speech separation [21], etc. In this paper, we show that according to real data, SIRP's can model the fast fading in wireless channels very well.

3. SIRP'S AND FAST FADING

Using the Rice's representation, $I(t)$ in (1) can be written as:

$$I(t) = I_c(t)\cos 2\pi f_m t - I_s(t)\sin 2\pi f_m t\ , \tag{2}$$

where f_m is a representative midband frequency, and $I_c(t)$ and $I_s(t)$ are two joint WSS, lowpass, and zero-mean Gaussian processes. In polar coordinates we have:

$$I(t) = R(t)\cos[2\pi f_m t + \Theta(t)]\ , \tag{3}$$

in which $R(t)$ and $\Theta(t)$ are the envelope and phase of $I(t)$, respectively, defined by:

$$R(t) = [I_c^2(t) + I_s^2(t)]^{1/2}\ , \qquad \tan\Theta(t) = I_s(t)/I_c(t)\ . \tag{4}$$

After replacing $I(t)$ in (1) with (3), $X(t)$ can be written as:

$$X(t) = CR(t)\cos[2\pi f_m t + \Theta(t)]\ . \tag{5}$$

which shows that $CR(t)$ and $\Theta(t)$ are the envelope and phase of $X(t)$, respectively.

Comparison of (3) and (5) reveals that the phases of the Gaussian process $I(t)$ and the SIRP $X(t)$ are the same uniform process, defined in (4). However, the envelope of the Gaussian process is Rayleigh-distributed, while the SIRP envelope can be distributed according to a very large class of non-Rayleigh distributions, because *C* is an arbitrary positive random variable. Interestingly, empirical measurements of narrowband mobile communication channels at different frequencies have shown that while the phase $\Theta(t)$ and the phase derivative $\Theta'(t)$ comply with the properties of a uniform process (uniform distribution for the phase and *t* distribution with two degrees of freedom for the phase derivative), the envelope is not Rayleigh-distributed [22] [23]. These observations partially justify the applicability of SIRP's for fast fading. However, as a reliable justification, we need to carry out a goodness-of-fit test for the spherical invariance of measured data.

4. TESTS FOR SPHERICAL INVARIANCE

For a given process $Y(t)$, let $\mathbf{Y} = [Y(t_1)\ Y(t_2)\ \ldots\ Y(t_p)]^T$, where $t_1 < t_2 < \ldots < t_p$ and p is a positive integer. Without loss of generality, assume $E(\mathbf{Y}) = \mathbf{0}$ and $\mathrm{Cov}(\mathbf{Y}) = \mathbf{I}$, where $\mathbf{0}$ is the $p\times 1$ zero vector and $\mathbf{I}$ is the $p\times p$ identity matrix. If $Y(t)$ is SIRP, then $\mathbf{Y}$ must be a spherically invariant (spherically symmetric) random vector. By definition, $\mathbf{Y}$ is spherically invariant if for every $p\times p$ orthogonal matrix $\mathbf{\Gamma}$, $\mathbf{\Gamma Y}$ and $\mathbf{Y}$ have the same distribution [24]. According to a theorem in [24], the multivariate PDF of $\mathbf{Y}$, if exists, must be proportional to $g_p(\mathbf{y}^T\mathbf{y})$, for some nonnegative function $g_p(.)$ of a scalar variable.

According to the above discussion, if $Y(t)$ is SIRP, then the bivariate PDF of $Y(t)$ is of the form $g_2(y_1^2 + y_2^2)$, for some nonnegative function $g_2(.)$. As a result, the contours of the bivariate PDF of $Y(t)$ are circles. This property has been used in [7] [10] for verifying the applicability of SIRP's for speech modeling, by visual inspection of the two-dimensional histograms of speech signals and their contours. This heuristic method, although informative, does not show the spherical invariance in higher dimensions. To overcome this problem, we have used two types of quantile-quantile plots, *t*-plot and β-plot, derived based on some robust statistics, for graphical testing the spherical invariance of a random vector with any dimension [25].

5. SPHERICAL INVARIANCE OF FAST FADING DATA

Here we give a brief summary of the data used. For the details, the interested reader should refer to [23]. The data was collected in a suburban housing development in Greenville, Texas, and an urban area among the buildings of the campus of the University of Texas at Arlington. The UHF transmitter generated a 910.25 MHz carrier at a nominal power of 0.2 W (the wavelength was 0.33 m). The distance between the transmitter and the mobile receiver was approximately 1 km. The speed of the mobile was fixed at 6.7 m/s. The collected data are twelve sets of narrowband inphase and quadrature components, taken at twelve different locations. The length of each record is approximately 47 m or 7 s. The analog signals were digitized with the sampling frequency 35156.25 Hz. The number of samples for each record was approximately 250000. The two successive samples were separated by 28 μs (0.2 mm in space). The antennas at the transmitter and the mobile receiver were both quarter wave vertical antennas, with omnidirectional patterns.

We model the received signal at the mobile station as:

$$\begin{aligned} S(t) &= B(t)A(t)\cos[2\pi f_m t + \Phi(t)] \\ &= B(t)[D_c(t)\cos 2\pi f_m t - D_s(t)\sin 2\pi f_m t], \end{aligned} \tag{6}$$

where $B(t)A(t)$ is the envelope, with $B(t)$ as the slow fading component and $A(t)$ as the fast fading component. In the above formula, f_m is the carrier frequency, $\Phi(t)$ is the phase, and $D_c(t) = A(t)\cos\Phi(t)$ and $D_s(t) = A(t)\sin\Phi(t)$ are the inphase and quadrature components of the fast fading fluctuations. Note that any of the twelve data sets contains the two signals $B(t)D_c(t)$ and $B(t)D_s(t)$. Using the method described in [26], $B(t)$ was estimated for each of the twelve sets. Now we want to see whether the fast fading signal $Z(t) = S(t)/B(t)$ can be modeled as a SIRP for each set.

At first, we compared, for all signal sets, the empirical cumulative distribution function (CDF) of

$\Phi(t) = \tan^{-1}[D_s(t) / D_c(t)]$ with the uniform distribution and of $\Phi'(t)$ against the t distribution with two degrees of freedom [2]. For the seven data sets #0011, 0013, 0014, 0015, 0017, 0018, and 0022, empirical and theoretical CDF's were very close. These observations, in agreement with [22] [23], show that the seven data sets have the potential to be SIRP. Note that the empirical CDF of $A(t) = [D_c^2(t) + D_s^2(t)]^{1/2}$ was significantly far from the Rayleigh distribution for the sets #0013, 0014, 0015, 0017, 0018, and 0022. As a typical example, the empirical CDF's of the phase, phase derivative, and the envelope of set #0013 have been plotted in Fig. 1, together with the CDF's of uniform, t with two degrees of freedom, and Rayleigh distributions, respectively.

As a formal assessment of the suitability of SIRP model for a data set with the corresponding inphase and quadrature components, we have to test the spherical invariance of all the vectors composed of the samples of the inphase and quadrature components [9]. In our work, we have considered the vector $[D_c(t_1) \ldots D_c(t_p) \; D_s(t_1) \ldots D_s(t_p)]^T$ for several choices of p, where the samples are equally spaced in time such that for any i and k, $t_i - t_k = \varepsilon / f_d$ with f_d as the maximum Doppler shift and ε as a given positive number (for our data $f_d = 20\,\text{Hz}$). The spherical invariance of the vector, for each data set and several choices of p and ε, has been tested using the t-plot and the β-plot described in [25]. For the set #0013, these two pairs of plots have been shown in Figs. 2 and 3, for $p = 2, 5$, respectively, assuming $\varepsilon = 0.005$. A straight line of the empirical data on each plot shows the acceptability of the hypothesis of spherical invariance of the underlying vector. According to Figs. 2 and 3, and similar figures for other data sets, we can verify the applicability of the SIRP model for the fast fading phenomenon in wireless channels.

The interesting fact regarding the application of SIRP's for fast fading is the great flexibility that this model provides for describing the statistics of the envelope, which is of great importance in communication engineering. As the reader may have noticed, we have not restricted the envelope to follow a specific distribution, and our results are independent of the envelope statistics. Results concerning the appropriate distribution for the envelope are given in a forthcoming paper.

6. CONCLUSION

In this paper and based on the measurements of narrowband wireless propagation channels, it has been shown that spherically invariant random processes can be used for modeling the fast fading signal fluctuations. This general model is capable of carrying information about the envelope, phase, and their time derivatives together, with uniform distribution for the phase (which is very common in fading channels) and no restriction on the envelope distribution. The main advantage of non-Gaussian SIRP's is their Gaussian-based structure, which admits them to retain most of the attractive properties of Gaussian processes, such as mathematically-tractable multivariate statistical analysis, linear solution for minimum-mean-squared-error estimation problems, straightforward simulation, … These characteristics make the SIRP model an ideal non-Gaussian statistical tool for developing and/or analysis of signal processing algorithms for wireless fading channels.

ACKNOELEDGEMENT

The work of the first and the third authors have been supported in part by the National Science Foundation, under the Wireless Initiative Program, Grant #9979443.

7. REFERENCES

[1] J. D. Parsons, *The Mobile Radio Propagation Channel*. New York: Wiley, 1992.

[2] W. C. Jakes, Jr., Ed., *Microwave Mobile Communications*. New York: Wiley, 1974.

[3] K. Yao, "A representation theorem and its applications to spherically-invariant random processes," *IEEE Trans. Inform. Theory*, vol. 19, pp. 600-608, 1973.

[4] K. Chu, "Estimation and decision for linear systems with elliptical random processes," *IEEE Trans. Automat. Contr.*, vol. 18, pp. 499-505, 1973.

[5] G. L. Wise and N. C. Gallagher, Jr., "On spherically invariant random processes," *IEEE Trans. Inform. Theory*, vol. 24, pp. 118-120, 1978.

[6] S. T. Huang and S. Cambanis, "Spherically invariant processes: Their nonlinear structures, discrimination, and estimation," *J. Multivar. Anal.*, vol. 9, pp. 59-83, 1979.

[7] H. Brehm, "Description of spherically invariant random processes by means of *G*-functions," in *Combinatorial Theory*. Lecture Notes in Mathematics, vol. 969, D. Jungnickel and K. Vedder, Eds, Berlin: Springer, 1982, pp. 39-73.

[8] B. Picinbono, *Random Signals and Systems*. Englewood Cliffs, NJ: Prentice Hall, 1993.

[9] E. Conte and M. Longo, "Characterization of radar clutter as a spherically invariant random process," *IEE Proc. F, Commun., Radar, Signal Processing*, vol. 134, pp. 191-197, 1987.

[10] H. Brehm and W. Stammler, "Description and generation of spherically invariant speech-model signals," *Signal Processing*, vol. 12, pp. 119-141, 1987.

[11] M. P. Shinde and S. N. Gupta, "A model of HF impulsive atmospheric noise," *IEEE Trans. Electromagn. Compat.*, vol. 16, pp. 71-75, 1974.

[12] H. Brehm and W. Stammler, "Application of relay- and polarity-correlation to the analysis of speech signals," in *Proc. European Signal Processing Conf.*, Erlangen, W. Germany, 1983, pp. 21-24.

[13] H. Brehm and K. Trottler, "Vector quantization of spherically invariant random processes," in *Proc. European Signal Processing Conf.*, Erlangen, W. Germany, 1983, pp. 383-386.

[14] H. Brehm and K. Trottler, "Rate distortion functions for speech-model signals", in *Proc. European Signal Processing Conf.*, the Hague, the Netherlands, 1986, pp. 353-356.

[15] E. Conte, M. Lops, and G. Ricci, "Adaptive matched filter detection in spherically invariant noise," *IEEE Signal Processing Lett.*, vol. 3, pp. 248-250, 1996.

[16] L. Izzo and M. Tanda, "Diversity reception of fading signals in spherically invariant noise," *IEE Proc. Commun.*, vol. 145, pp. 272-276, 1998.

[17] M. Rangaswamy, D. Weiner, and A. Öztürk, "Computer generation of correlated non-Gaussian radar clutter," *IEEE Trans. Aerospace Electronic Syst.*, vol. 31, pp. 106-116, 1995.
[18] M. Rupp, "The behavior of LMS and NLMS algorithms in the presence of spherically invariant processes," *IEEE Trans. Signal Processing*, vol. 41, pp. 1149-1160, 1993.
[19] P. L. De Leon II and D. M. Etter, "Experimental results of subband acoustic echo cancelers under spherically invariant random processes," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Atlanta, GA, 1996, pp. 961-964.
[20] C. D. Richmond, "PDF's, confidence regions, and relevant statistics for a class of sample covariance-based array processors," *IEEE Trans. Signal Processing*, vol. 44, pp. 1779-1793, 1996.
[21] J. P. LeBlanc and P. L. De Leon, "Speech separation by kurtosis maximization," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Seattle, WA, 1998, pp. 1029-1032.
[22] P. C. F. Eggers and J. B. Andersen, "Measurements of complex envelopes of mobile scenarios at 450 MHz," *IEEE Trans. Vehic. Technol.*, vol. 38, pp. 37-42, 1989.
[23] H. Allen Barger, "Dynamic characteristics of a narrowband land mobile communication channel," *IEEE Trans. Vehic. Technol.*, vol. 47, pp. 216-224, 1998.
[24] K. T. Fang, S. Kotz, and K. W. Ng, *Symmetric Multivariate and Related Distributions*. London: Chapman and Hall, 1990.
[25] R. Z. Li, K. T. Fang, and L. X. Zhu, "Some Q-Q probability plots to test spherical and elliptical symmetry," *J. Comput. Graph. Stat.*, vol. 6, pp. 435-450, 1997.
[26] A. Abdi and M. Kaveh, "On the utility of the gamma PDF in modeling shadow fading (slow fading)," in *Proc. IEEE Vehic. Technol. Conf.*, Houston, TX, 1999, pp. 2308-2312.

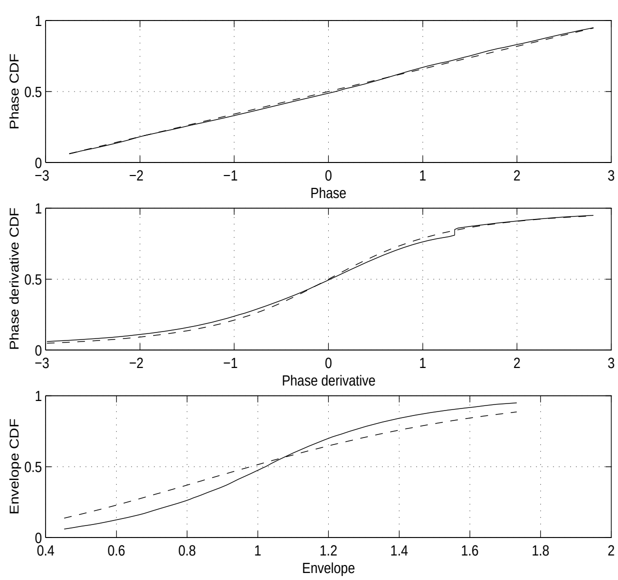

Figure 1. Solid: empirical CDF, Dashed: theoretical CDF.

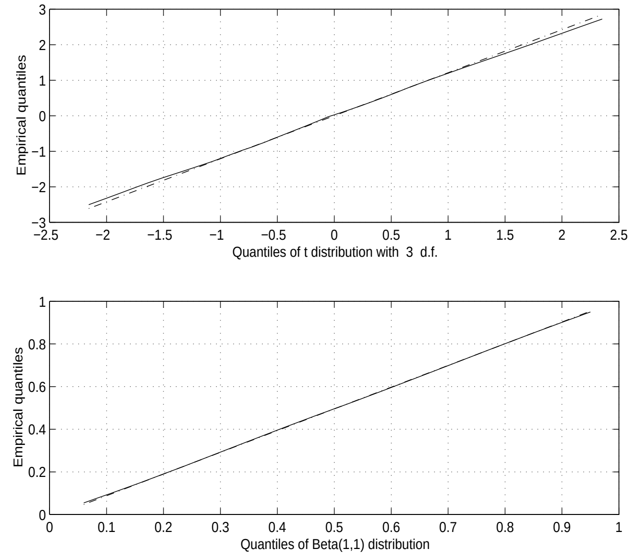

Figure 2. Testing the spherical symmetry of the 4-variate joint PDF of the inphase and quadrature components. Upper: *t*-plot, lower: β-plot, dashed curve: reference line, the distance between two adjacent samples: 0.25 ms.

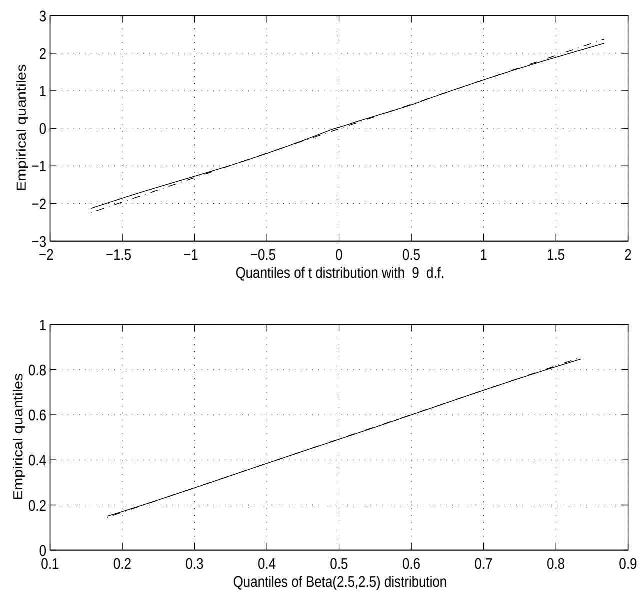

Figure 3. Testing the spherical symmetry of the10-variate joint PDF of the inphase and quadrature components. Upper: *t*-plot, lower: β-plot, dashed curve: reference line, the distance between two adjacent samples: 0.25 ms.