Average Outage Duration of Interference-Limited Wireless Communication Systems*

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Abstract— **This paper relies on a characteristic function based approach to compute the level crossing rate and average outage duration of interference-limited wireless communication systems. The analytical results are quite general and are applicable to systems in which users are subject to non-identically distributed Rician fading. These results are confirmed by Monte Carlo simulations and some numerical examples are discussed and interpreted.**

I. Introduction

Wireless communication networks are mainly limited by mutual interference among users. Outage probability has been traditionally the most commonly used performance measure of such networks [1], [2], [3], [4]. However, in certain communication systems, such as adaptive transmission schemes [5], [6], [7], the outage probability does not provide enough information for the overall system design and configuration. In that case, in addition to outage probability, wireless communication design engineers are also interested in other performance measures, such as average outage duration (AOD), defined as the average period of time for which the signal-to-interference ratio (SIR) random process stays below a predetermined threshold level.

While special attention has been paid to the impact of the fading statistics of the desired and interfering users on the system outage probability (see for example [1], [2], [3], [4] and references therein), closed-form on the AOD of cellular mobile radio systems are still limited to systems subject to Rayleigh type of fading [1], [8] or to identically distributed interferers [9]. This paper relies on a characteristic function (CF)-based approach to develop an analytical framework to compute the AOD of interference-limited wireless communication systems operating in generalized Rician fading environment and not necessarily identically distributed interferers.

The remainder of this paper is organized as follows. Next section presents the system and channel models. Section III reviews the CF-based approach for level crossing rate (LCR) evaluation and then applies it for the computation of the AOD. Finally, Section IV offers some numerical examples along with their discussion and interpretation.

II. System and Channel Models

Let $r(t)$ denote the sum of the received complex signal of the desired user, $r_D(t)$ with the signals of

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the interfering users, $r_n(t)$, $(n=1,\cdots,N_I)$

$$\begin{aligned} r(t) &= r_D(t) + \textstyle\sum_{n=1}^{N_I} r_n(t) \\ &= \alpha_D(t)e^{j\theta_D(t)} + \textstyle\sum_{n=1}^{N_I} \alpha_n(t)e^{j\theta_n(t)}, \end{aligned} \quad (1)$$

where N_I denotes the number of interfering users. Because of the slow-fading assumption, the fading amplitudes $\alpha_l(t)$ and the angles of arrival $\theta_l(t)$ (in the horizontal plane) are all constant (time-independent) random variables over a symbol interval [10]. Here subscript l can be D for the desired user and n for the interfering users. In (1), $\alpha_l(t)$ are assumed to be independent random processes following Rayleigh- and/or Rice-distributed fading with average fading power $\Omega_l = E[\alpha_l^2] = E[s_l]$, where $E[\cdot]$ denotes mathematical expectation and $s_l = \alpha_l^2$. Thus the instantaneous SIR, Λ, is given by

$$\Lambda(t) = \frac{s_D(t)}{\sum_{n=1}^{N_I} s_n(t)}. \quad (2)$$

An outage is declared if the instantaneous SIR Λ, falls below a certain specified threshold, λ_{th}, i.e.,

$$\Lambda = \frac{s_D}{\sum_{n=1}^{N_I} s_n} \leq \lambda_{\text{th}}, \quad (3)$$

or equivalently

$$\beta = s_D - \lambda_{\text{th}} \sum_{n=1}^{N_I} s_n \leq 0. \quad (4)$$

III. Evaluation of the Average Outage Duration of Interference-Limited Systems

A. CF-Based Approach

For a random process $X(t)$ and its derivative $\dot{X}(t)$ the LCR, $N_X(x_{\text{th}})$ for a given threshold, x_{th}, can be obtained from the joint CF of X and $\dot{X}$, $\Phi_{X\dot{X}}(\omega_1,\omega_2) = E[\exp(j\omega_1 X + j\omega_2 \dot{X})]$, as in [11]

$$\begin{aligned} N_X(x_{\text{th}}) &= -\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_2}\frac{d\Phi_{X\dot{X}}(\omega_1,\omega_2)}{d\omega_2} e^{-j\omega_1 x_{\text{th}}} d\omega_1 d\omega_2 \\ &= -\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\Phi_{X\dot{X}}(\omega_1,\omega_2) - \Phi_X(\omega_1)}{\omega_2^2} e^{-j\omega_1 x_{\text{th}}} d\omega_1 d\omega_2. \end{aligned} \quad (5)$$

The AOD of $X(t)$, $T_X(x_{\text{th}})$, which indicates how long on average the random process $X(t)$ stays below the given threshold x_{th}, is given for a random process by [10]

$$T_X(x_{\text{th}}) = \frac{\text{Prob}[X \leq x_{\text{th}}]}{N_X(x_{\text{th}})} = \frac{\int_{-\infty}^{x_{\text{th}}} p_X(x)dx}{N(x_{\text{th}})}, \quad (6)$$

where $\text{Prob}[X \leq x_{\text{th}}]$ is the cumulative distribution function (CDF) of X evaluated at x_{th}.

B. Application to AOD Computation

To obtain the AOD of the SIR process Λ, we just need to determine the average time the process β, defined in (4), spends below zero. Since the CDF of β evaluated at zero or equivalently the outage probability of the SIR Λ has been extensively explored and several numerical techniques are available for all fading scenarios of interest [12], [13], we just need to compute the zero-crossing rate $N_\beta(0)$ of β. This can be computed from (5) by substituting zero for the threshold x_{th} and using the joint CF of β and $\dot{\beta}$. We now derive a closed-form expression for this joint CF.

Let us represent $r_l(t)$ in (1) as $r_l(t) = x_l(t) + jy_l(t)$, where $x_l(t)$ and $y_l(t)$ are the inphase and quadrature components of the lth user, respectively. The process $x_l(t)$ is a real stationary Gaussian process with mean a_l and auto-covariance function $C_{x_l x_l}(\tau)$, while $y_l(t)$ is a zero-mean real stationary Gaussian process with the same auto-covariance function, i.e. $C_{y_l y_l}(\tau) = C_{x_l x_l}(\tau)$, and both have the same variance $b_{l,0}$. In general, for an arbitrary AOA distribution, $x_l(t)$ and $y_l(t)$ are correlated processes with the cross-covariance function $C_{x_l y_l}(\tau) = -C_{y_l x_l}(\tau)$. It is easy to verify that $C_{r_l r_l}(\tau) = C_{x_l y_l}(\tau) + jC_{y_l y_l}(\tau)$ [10]. According to the statistical properties of $x_l(t)$ and $y_l(t)$, $\alpha_l(t) = |r_l(t)| = \sqrt{x_l^2(t) + y_l^2(t)}$ is a Rician process and for $a_l = 0$ it becomes a Rayleigh process. Using the inphase and quadrature expression of the desired and interference users β in (4) can be written as

$$\begin{aligned} \beta(t) &= s_D(t) - \lambda_{\text{th}} \sum_{n=1}^{N_I} s_n(t) \\ &= (x_D^2(t) + y_D^2(t)) - \lambda_{\text{th}} \sum_{n=1}^{N_I} (x_n^2(t) + y_n^2(t)), \end{aligned} \quad (7)$$

while for the derivative we have

$$\begin{aligned} \dot{\beta}(t) &= 2(x_D(t)\dot{x}_D(t) + y_D(t)\dot{y}_D(t)) \\ &- 2\lambda_{\text{th}} \sum_{n=1}^{N_I} [x_n(t)\dot{x}_n(t) + y_n(t)\dot{y}_n(t)], \end{aligned} \quad (8)$$

which for a fixed time instant $t = t_0$ can be rewritten in terms of the random variable $\beta = \beta(t_0)$ as

$$\dot{\beta} = 2(x_D\dot{x}_D + y_D\dot{y}_D) - 2\lambda_{\text{th}} \sum_{n=1}^{N_I} [x_n\dot{x}_n + y_n\dot{y}_n]. \quad (9)$$

Given this set-up and under the fading independence assumption among all desired and interfering users, we have

$$\begin{aligned}\Phi_{\beta\dot\beta}(\omega_1,\omega_2) &= \Phi_{s_D\dot s_D}(\omega_1,\omega_2)\\ &\times \prod_{n=1}^{N_I}\Phi_{s_n\dot s_n}(-\lambda_{\rm th}\omega_1,-\lambda_{\rm th}\omega_2), \quad (10)\end{aligned}$$

where (based on the Turin classical result on the CF of quadratic forms in Gaussian variables [14]) $\Phi_{s_l\dot s_l}(\zeta_l\omega_1,\zeta_l\omega_2)$ is given by [11]

$$\Phi_{s_l\dot s_l}(\zeta_l\omega_1,\zeta_l\omega_2)=\frac{\exp\left(-\frac{a_l^2(2b_{l,2}\zeta_l^2\omega_2^2-j\zeta_l\omega_1)}{1+4(b_{l,0}b_{l,2}-b_{l,1}^2)\zeta_l^2\omega_2^2-j2b_{l,0}\zeta_l\omega_1}\right)}{1+4(b_{l,0}b_{l,2}-b_{l,1}^2)\zeta_l^2\omega_2^2-j2b_{l,0}\zeta_l\omega_1}, \quad (11)$$

where ζ_D=1 for the desired user and $\zeta_n=-\lambda_{\rm th}$ for the interference users. We now express (11) in terms of the Rician factor of the lth user, $K_l=a_l^2/(2b_{l,0})$, and the average power $\Omega_l=E[\alpha_l^2]=a_l^2+2b_{l,0}$, by substituting $a_l^2=K_l\Omega_l/(K_l+1)$ and $2b_{l,0}=\Omega_l/(K_l+1)$ into (11), leading to

$$\Phi_{s_l\dot s_l}(\zeta_l\omega_1,\zeta_l\omega_2) = \frac{\exp\left(-\frac{K_l[h(K_l,\Omega_l,f_{d_l},\zeta_l)-1]}{h(K_l,\Omega_l,f_{d_l},\zeta_l)}\right)}{h(K_l,\Omega_l,f_{d_l},\zeta_l)}, \quad (12)$$

where

$$h(K_l,\Omega_l,f_{d_l},\zeta_l)=1+\frac{2\pi^2 f_{d_l}^2\Omega_l^2\zeta_l^2\omega_2^2}{(K_l+1)^2}-j\frac{\Omega_l\zeta_l\omega_1}{K_l+1}. \quad (13)$$

When the nth user has a Rayleigh distribution, $a_l=0$, (12) significantly simplifies to

$$\Phi_{s_l\dot s_l}(\zeta_l\omega_1,\zeta_l\omega_2)=\frac{1}{1+2\pi^2 f_{d_l}^2\Omega_l^2\zeta_l^2\omega_2^2-j\Omega_l\zeta_l\omega_1}. \quad (14)$$

Using (12) and (14) the joint CF of β and $\dot\beta$ for any combination of the fading models of the desired and interference users can be obtained. For example, in a Rician/Rician fading model the joint CF of β and $\dot\beta$ in (10) can be written from (12) as

$$\begin{aligned}\Phi_{\beta\dot\beta}(\omega_1,\omega_2) &= \frac{\exp\left(-\frac{K_D[h(K_D,\Omega_D,f_{d_D},1)-1]}{h(K_D,\Omega_D,f_{d_D},1)}\right)}{h(K_D,\Omega_D,f_{d_D},1)}\\ &\times \prod_{n=1}^{N_I}\frac{\exp\left(-\frac{K_n[h(K_n,\Omega_n,f_{d_n},-\lambda_{\rm th})-1]}{h(K_n,\Omega_n,f_{d_n},-\lambda_{\rm th})}\right)}{h(K_n,\Omega_n,f_{d_n},-\lambda_{\rm th})}. \quad (15)\end{aligned}$$

For the Rician/Rayleigh fading model it can be written from (12) and (14)

$$\begin{aligned}\Phi_{\beta\dot\beta}(\omega_1,\omega_2) &= \frac{\exp\left(-\frac{K_D[h(K_D,\Omega_D,f_{d_D},1)]}{h(K_D,\Omega_D,f_{d_D},1)}\right)}{h(K_D,\Omega_D,f_{d_D},1)}\\ &\times \prod_{n=1}^{N_I}\frac{1}{1+2\pi^2 f_{d_n}^2\Omega_n^2\lambda_{\rm th}^2\omega_2^2+j\Omega_n\lambda_{\rm th}\omega_1}. \quad (16)\end{aligned}$$

IV. Numerical Examples

This section describes several plots showing some results for the SIR AOD. A simulator was implemented in MATLAB for the purpose of verification, and simulated points are shown in some of these plots. We used the IFFT simulation technique developed by Young and Beaulieu [15], which approximates Rayleigh- or Rice-distributed flat-fading channels with greater efficiency than older methods such as the filtered white Gaussian noise technique.

Care must be taken when using filtering simulators of this type. In particular, there are at least two limiting factors: the sampling rate and the sequence length. The sampling rate must be set such that the sampling period is considerably smaller than the expected AOD. Otherwise the outages will not be distinguished. Secondly, the sequence length (i.e. number of samples simulated) should of course be as large as possible. However, in the case of the Young and Beaulieu simulator, we may quantify this fact by noting that the length of the filter itself is $k_m=Nf_mT$, where N is the number of samples being simulated, f_m is the maximum Doppler frequency in use and T is the sampling period. N must be large enough to allow a reasonably-sized filter sequence. ($k_m=50-100$ at minimum seems to suffice.) This problem is exasperated when higher sampling rates (or smaller outage times) are desired, so that SIR simulations such as those described in this section may become prohibitive on less powerful machines.

Fig. 1 and Fig. 2 show the AOD for several maximum Doppler frequencies and for several values of the number of interfering users, respectively. These two plots include simulated points which confirm the analytical results. Here, all the interferers are assumed to be traveling at the same velocity and to possess the same local average power $\Omega_n=\Omega_I$, $n=1...N_I$. The local means actually used for the simulations were $\Omega_D=5$, $\Omega_I=0.2$ in Fig. 1 and $\Omega_D=1$, $\Omega_I=0.2$ in Fig. 2. The maximum Doppler frequency is an indicator of the velocity of the mobile units. In Fig. 1, $N_I=6$ and in Fig. 2, both Doppler frequencies are 7 Hz. Using a 900 MHz carrier, 7 Hz corresponds to about 5 mph, whereas 94 Hz is roughly 70 mph. These cases can be regarded as "slow" and "fast" fading, respectively. Several important comments on these plots are in order. First, it is clear from Fig. 1 that the speed of the desired user is the dominant factor in determining the system AOD. That is, the interferers' speeds are relatively unimportant, especially for high average CIR. On the other hand, from Fig. 2 it is apparent that there is diminishing increase in the AOD as the interferers grow more numerous. The points regarding the dominance of the desired-user's speed are reinforced by Figs 3 and 4, which show the effect of variation in speed when either the desired user or the interferers are kept at constant speed. In Fig. 3 we observe a consistent and progressive effect on the AOD, whereas Fig. 4 demonstrates only small variations.

Finally in Fig. 5 and Fig. 6 we investigate the effect of the Ricean factor. In Fig. 5 it is clear that at high average CIR the distribution of the interferers has little effect, whereas in general a line-of-sight component among the interferers tends to cause higher outage times. Fig. 6 shows an ambivalent response to the desired-user Ricean factor: it causes improvement only at high average CIR.

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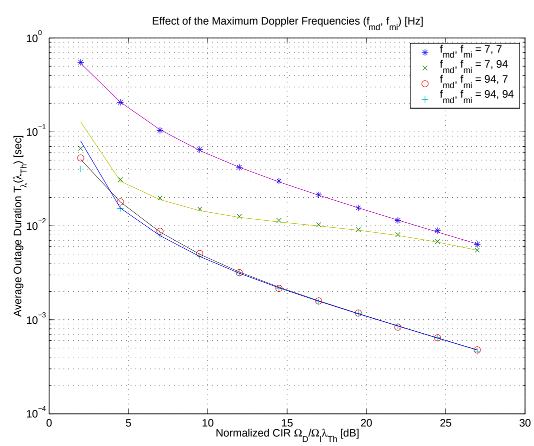

Fig. 1. Average outage duration of interference-limited systems for various scenarios of interest.

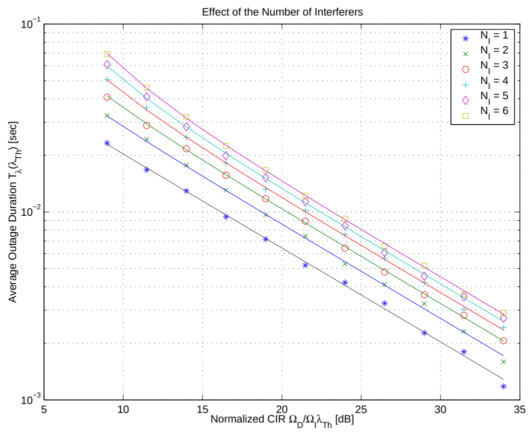

Fig. 2. Effect of the number of interferers on the average outage duration of interference-limited systems.

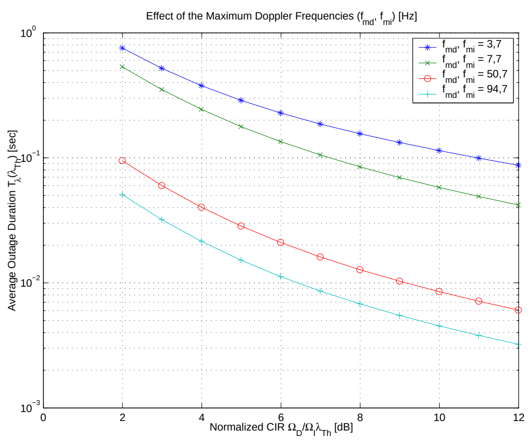

Fig. 3. Effect of the desired user speed on the average outage duration of interference-limited systems.

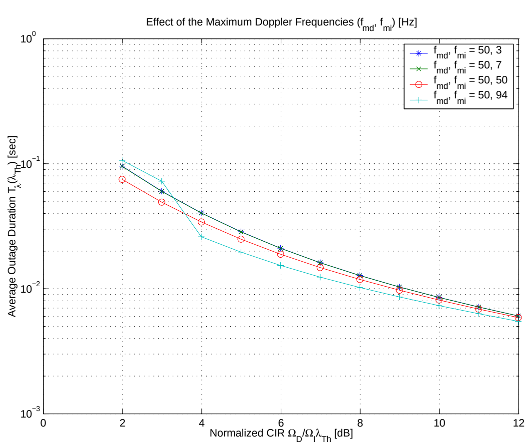

Fig. 4. Effect of the interferers' speed on the average outage duration of interference-limited systems.

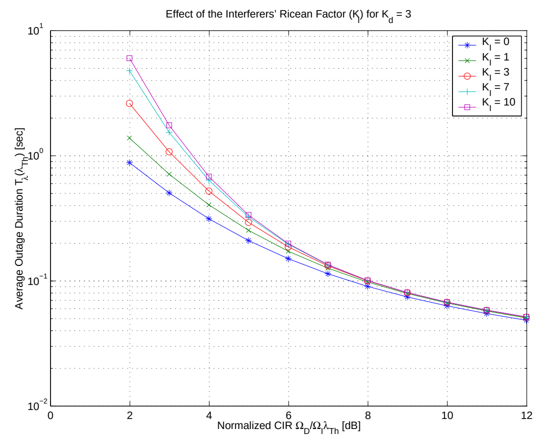

Fig. 5. Effect of the interferers' Ricean factor on the average outage duration of interference-limited systems.

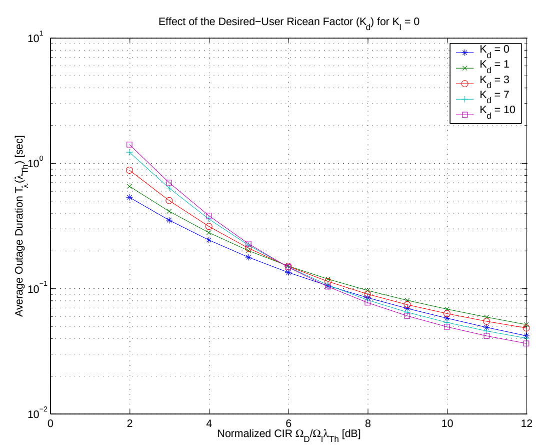

Fig. 6. Effect of the desired user's Ricean factor on the average outage duration of interference-limited systems.