Stochastic Modeling and Simulation of Multiple-Input Multiple-Output Channels: A Unified Approach

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Abstract – **In this paper we present a unified framework for the characterization and simulation of random multiple-input multiple-output (MIMO) fading channels. Depending on the presence of local scatterers around the base station and mobile station, we consider several geometrical configurations, which result in parametric expressions for key channel features such as the space-time-frequency correlations, average stay duration, etc. This approach allows one to describe and simulate the complicated random MIMO channel via several parameters, which in turn facilitates the analysis and design of multiantenna systems.**

I. Introduction

Recent fundamental information theoretic results have shown that by exploiting antenna arrays at both the base station (BS) and mobile station (MS), very high transmission rates can be achieved over wireless channels [1]. This has driven the recent surge of numerous research activities and developments on multiantenna systems. In general, such systems attempt to efficiently use the spatial channel characteristics, as well as the time- and frequency-domain characteristics (along with appropriate coding and detection schemes). Therefore, space-time-frequency multiple-input multiple-output (MIMO) channel models are essential tools for design and analysis of high speed multiantenna systems. Efficient software simulation of MIMO channels is another key issue, which allows the optimization and performance comparison of different coding/detection algorithms under realistic conditions, and obviates expensive hardware experiments.

In this paper we take a stochastic approach to model random MIMO mobile Rayleigh fading channels (extension to Rice fading channels is straightforward). The basic idea is to approximate the distribution of scatterers around the transmitter and/or the receiver by effective rings or circular rings. The simple geometry of ring provides closed-form expressions for the key characteristics of the random MIMO channel, i.e., spatial, temporal, and frequency cross-correlations among the subchannels. These cross-correlations, which determine the basic structure of the MIMO channel, include some essential parameters such as the mean angle-of-arrival (AOA) at the receiver, mean angle-of-departure (AOD) at the transmitter, angle spreads at both sides, the delay spread, etc., in compact forms. Using these parameters, one can fit the model to measured data. In addition, by playing with these parameters in software simulations, system designers can study the performance and robustness of transmission/detection schemes of interest, under a variety of channel conditions.

The rest of this paper, which is a summary of the MIMO channel modeling/simulation research at NJIT, is organized as follows. In Section II, a narrowband MIMO mobile model is discussed, which is suitable for (outdoor) macrocells and some microcells, where there is no local scatterer around the BS. The wideband extension of this model in presented in Section III. Section IV is devoted to the MIMO modeling of those environments such as indoors and some microcells where both the BS and MS experience local scattering. In Section V we look at the key dynamic characteristic of mobile MIMO channels, i.e., the average stay duration. Simulation of MIMO channels is discussed in Section VI, and the conclusions are provided in Section VII.

II. The Narrowband One-Ring Mobile MIMO Model

For those outdoor channels in urban, suburban, and rural areas, where the elevated BS does not experience local scattering, one can assume, as a first-order approximation, that the local scatterers around the MS are distributed on an "effective" ring. A 2×2 example is shown in Fig. 1, with omni-directional antennas, and the MS moves in direction γ with the speed v. Each of the four time-varying channel gains $h_{11}(t)$, $h_{12}(t)$, $h_{21}(t)$, and $h_{22}(t)$ is a superposition of many multipath components, traveled from the BS towards the local scatterers on the ring, and then received by the MS in the forward channel (of course the opposite happens in the reverse channel). Therefore, each subchannel is a complex Gaussian random process. If there is no line-of-sight (LOS), the envelope and phase of each subchannel follow the Rayleigh and uniform distributions, respectively. Obviously all these subchannels are cross-correlated both in time and space.

Under the reasonable assumptions of small angle spread at the BS and large distance between the BS and MS, plus the application of the empirically-verified generic von Mises distribution for the AOA at the MS [2], and considering single-bounce scattering only, we have obtained a closed-form expression for the MIMO spatio-temporal cross-correlation between any two subchannels [3] (the closed-from MIMO cross-spectrum is given in [4]). As an example, when AOA at the MS is uniform and the BS and MS arrays are parallel, $\alpha = \beta = 90^{o}$,we have the following spatial cross-correlations for a 2×2 channel

$$E[h_{11}(t)h_{22}^*(t)] = J_0\left(2\pi\{d+\delta\Delta\}/\lambda\right), E[h_{11}(t)h_{21}^*(t)] = J_0\left(2\pi d/\lambda\right), E[h_{11}(t)h_{12}^*(t)] = J_0\left(2\pi\delta\Delta/\lambda\right), \qquad (1)$$

where $E[.]$ is the mathematical expectation, * is the complex conjugate, $J_0(.)$ is the zero-order Bessel function of the first kind, d and δ are the elements spacing at the MS and BS, respectively, 2Δ is the maximum angle spread at the BS, and λ is the wavelength.

The MIMO space-time cross-correlation of [3] includes many previously-derived results as special cases, and is suitable for both mathematical analysis and simulation studies. The impacts of this cross-correlation on the MIMO channel capacity and some space-time coding and detection schemes are discussed in [3] and [5]. Expressions for other narrowband channel characteristics of interest of the one-ring model such as the AOA distribution at the BS, and the azimuth power spectra at the BS and MS, are also derived and will be reported in another paper.

III. The Wideband Mobile MIMO Model with a Single Circular Ring

Since the one-ring model of Fig. 1 does not reflect some wideband characteristics of the channels considered in Section II, we proposed a mobile MIMO model with a single circular ring of scatterers around the MS [6]. A 2×2 example is shown in Fig. 2, where $2\Delta_2$ is the maximum angle spread at the BS, and $T_{lp}(f,t)$ is the time-varying frequency response between the pth transmitter and the lth receiver in the forward channel. If the number of scatterers is large enough, each $T_{lp}(f,t)$ is a two-dimensional complex Gaussian process. Note that $T_{lp}(f,t)$ is the Fourier transform of $h_{lp}(\tau,t)$, the time-varying channel impulse response, with respect to τ. Under the same assumptions as those behind the one-ring model of the previous section, this model provides a closed-form expression for the MIMO space-time-frequency cross-correlation between any two subchannels. As an example, for parallel arrays, $\alpha = \beta = 90^{o}$, with uniform AOA around the MS, we have this space-frequency cross-correlation between $T_{11}(0,t)$ and $T_{22}(f,t)$ in a 2×2 channel

$$\begin{aligned} E[T_{11}(0,t)T_{22}^*(f,t)] &= 2(\Delta_2^2 - \Delta_1^2)^{-1}\exp[j2\pi(f/f_c)(D/\lambda)] \\ &\quad \times \int_{\Delta_1}^{\Delta_2} \Delta J_0\left(2\pi\left\{d^2 + D^2(f/f_c)^2\Delta^2 + \delta^2\Delta^2 + 2d\,\delta\Delta\right\}^{1/2}/\lambda\right)d\Delta, \end{aligned} \qquad (2)$$

where Δ_1 is shown in Fig. 2, $j^2 = -1$, D is the distance between the BS and MS, and f_c is the carrier frequency. As expected, (2) simplifies to (1), $J_0\left(2\pi\{d+\delta\Delta_2\}/\lambda\right)$, if $f = 0$ and $\Delta_1 \to \Delta_2$.

Expressions for some channel functions of interest in the single circular ring model such as the AOA distribution at the BS, the azimuth power spectra at the BS and MS, the distribution of time-of-arrival (TOA), and the delay power spectrum (also known as power delay profile) are all derived and compared with published measured data. The TOA distribution and the delay power spectrum results are presented in [6], and the rest will be reported in another paper.

IV. THE NARROWBAND TWO-RING MOBILE MIMO MODEL

So far the focus has been on those cases where only the MS was surrounded by local scatterers. However, for indoor channels and some microcells, there are scatterers local to BS as well. To model this, we add an "effective" ring of scatterers to the BS, as shown in Fig. 3 for a 2×2 example. Under some assumptions and including only single-bounce scatterings, the simple geometry of ring allows a closed form expression for the MIMO spatio-temporal cross-correlation between any two subchannels [7]. As an example, in the forward channel, with uniform AOD and AOA distributions at the BS and MS, respectively, together with parallel arrays ($\alpha = \beta = 90^{\circ}$), we have the following spatial cross-correlation between $h_{11}(t)$ and $h_{22}(t)$, in a 2×2 channel

$$E[h_{11}(t)h_{22}^{*}(t)] = \eta J_0\left(2\pi\{d+\delta\Delta\}/\lambda\right) + (1-\eta)J_0\left(2\pi\{d\Delta'+\delta\}/\lambda\right), \tag{3}$$

where $0 \le \eta \le 1$ represents the contribution of the MS ring, and Δ and Δ' are shown in Fig. 3. For $\eta = 1$, (3) simplifies to $J_0\left(2\pi\{d+\delta\Delta\}/\lambda\right)$ in (1), as expected.

Comparison of the two-ring model with indoor data, collected at Brigham Young University, in terms of the spatial cross-correlations and also the MIMO channel capacity, are reported in [7].

V. DYNAMIC CHARACTERISTICS OF MOBILE MIMO CHANNELS

In mobile communications, the average stay duration (ASD), which tells us how long a signal stays within a region (for example, between two thresholds), is an important feature of the time-varying random channel. Of course ASD is a close relative of the average fade duration (AFD) and the level crossing rate (LCR), both widely used in mobile communications. To calculate the ASD for an *M*-transmit *N*-receive multiantenna fading channel, the *joint* dynamic behavior of *MN* correlated random processes should be studied. This requires a multidimensional approach, as opposed to the scalar LCR and AFD problems in mobile communications [8].

Consider a time-varying narrowband $M \times N$ MIMO channel, composed of *MN* complex zero-mean cross-correlated Gaussian processes. Suppose at $t = t_0$, all the subchannel gains $h_{lp}(t)$'s are observed. Now we want to determine for how long, on average, the maximum absolute deviation of all the *MN* processes from their observed values at $t = t_0$, does not exceed a certain bound ε. This is equivalent to the average stay time of a vector Gaussian process, consisting of 2*MN* real processes, within a hypercube with equal sides of length 2ε. For $M = N = 1$, a single-input single-output mobile channel, the idea is shown in Fig. 3 of [8].

To calculate the ASD in a narrowband MIMO channel, one needs a closed-from expression for the space-time cross-correlation among the subchannels. Depending on the propagation environment, the space-time cross-correlations of the ring models, discussed in the previous sections, can be used. As an example, here we consider a simple case where the spatial cross-correlation between the subchannels in negligible (large element spacings in both arrays), and the temporal correlation of each subchannel is $J_0(2\pi f_D \tau)$, where f_D is the maximum Doppler frequency. Then one can show that [8]

$$\text{ASD}=[\Phi(\theta+\varepsilon)-\Phi(\theta-\varepsilon)]\exp[(\theta^2+\varepsilon^2)/2]\big/[2\sqrt{2}MNf_D\cosh(\theta\varepsilon)],\ \ \Phi(y)=(2\pi)^{-1/2}\int_{-\infty}^{y} e^{-z^2/2}dz\,, \tag{4}$$

where cosh(.) is the hyperbolic cosine and θ, a real number, is the common value of all the subchannels at $t = t_0$, i.e., $h_{lp}(t_0) = \theta + j\theta$, for all *l*'s and *p*'s. In Fig. 4, we have plotted (4) when $M = N$, with respect to θ. Note that the ASD decreases as the number of antennas increases. Further details and several applications can be found in [8].

VI. SIMULATION OF MOBILE MIMO CHANNELS

When the number of scatterers is not small, one can reasonably model each subchannel of a MIMO mobile fading channel by a complex Gaussian random process. Of course these processes are cross-correlated, as discussed in previous sections. For performance analysis of multiantenna systems, fast and efficient software simulation of MIMO channels is of high importance.

In [4], a comprehensive study of the computational complexity of several stochastic simulation techniques for cross-correlated complex Gaussian processes is carried out. We have shown that the spectral representation technique is the fastest one, and offers the same simulation accuracy as other methods. The one-ring model of Section II is used for the numerical examples in [4]. The spectral-based Matlab© MIMO simulator is also available on http://web.njit.edu/~abdi.

VII. CONCLUSIONS

By summarizing some recent research in our group, we have shown that the simple geometry of ring and circular ring provides a unified transparent structure for stochastic modeling and simulation of MIMO channels in a variety of propagation environments. One of the most important MIMO characteristics is the space-time-frequency cross-correlations among the subchannels, which are expressed in terms of several key channels parameters. MIMO average stay duration, a measure of the time variations of the channel, is also discussed. Fast simulation of MIMO channels is addressed as well. Further details and comparison with measured data can be found in the cited references.

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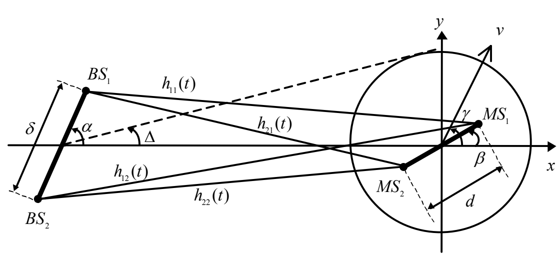

Fig. 1. The 2×2 narrowband one-ring MIMO mobile model with MS local scatterers.

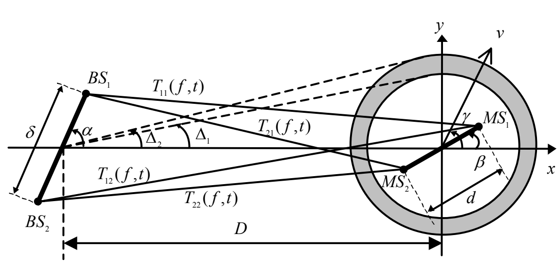

Fig. 2. The 2×2 wideband MIMO mobile model with a circular ring of MS scatterers.

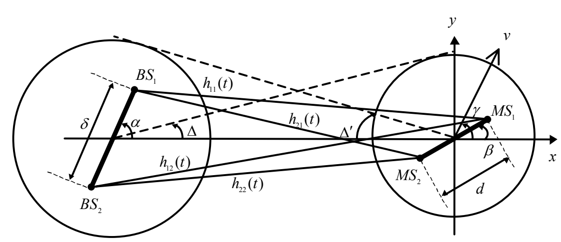

Fig. 3. The 2×2 narrowband two-ring MIMO mobile model with BS and MS local scatterers.

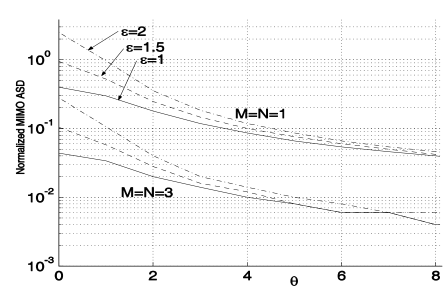

Fig. 4. Normalized MIMO ASD with the same number of transmit/ receive antennas.