Estimation of the Rice factor from the I/Q Components

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Abstract — **The relative strength of the line of sight between the transmitter and receiver, measured by the Ricean K factor, is an indication of the link quality which renders its estimation important in a variety of applications in wireless communications. The estimate of K from the received signal envelope has received much attention in satellite channels, and more recently in indoor wireless scenarios. In this paper we study the estimation of the Rice factor from the in-phase and quadrature-phase components of the received signal, as opposed to the envelope. Statistical analysis and preliminary simulations reveal the potential performance improvements, compared with envelope-based approaches.**

I. Introduction and Model

In wireless communications, when there is a line of sight (LoS) between the transmitter and the receiver, the received signal can be written as the sum of a complex exponential and a narrowband Gaussian process, which are known as the 'LoS component' and the 'diffuse component', respectively. The ratio of the powers of the LoS component to the diffuse component is the Ricean factor, which measures the relative strength of the LoS, and hence is a measure of link quality. Consider the communication scenario where an unmodulated carrier is transmitted, and the receiver that is traveling with a velocity v receives the transmitted waveform through a LoS component and many multipath components. The baseband in-phase/quadrature-phase (I/Q) representation of the received signal can be expressed as:

$$x(t) = \underbrace{\sqrt{\frac{K\Omega}{K+1}}}_{A:=} e^{j(2\pi f_D \cos(\theta_0)t+\phi_0)} + \underbrace{\sqrt{\frac{\Omega}{K+1}}}_{\sigma:=} h(t) \;, \qquad (1)$$

where K is the Ricean factor, θ_0 and ϕ_0 are the angle of arrival (AoA) and phase of the LoS respectively, and are assumed to be deterministic parameters; the maximum Doppler frequency f_D is the ratio of the mobile velocity v and the wavelength; $h(t)$ is the diffuse component given by the sum of a large number of multipath components, constituting a complex Gaussian process; the correlation function of $h(t)$ can be expressed as (see e.g., [16])

$$r_h(\tau) := E[h(t)h^*(t+\tau)] = \int_{-\pi}^{\pi} p_h(\theta)\; e^{j2\pi f_D \cos(\theta)\tau} d\theta \;, \qquad (2)$$

where $E[\cdot]$ denotes expectation, * denotes conjugation, $p_h(\theta)$ is the AoA distribution of the diffuse component, which, when uniform, yields the well-known Clarke's correlation function that is expressed in terms of the zeroth order Bessel function of the first kind: $r_h(\tau) = J_0(2\pi f_D \tau)$ [16]. Without loss of generality, we are assuming $E|h(t)|^2 = 1$ which implies that the power of the diffuse component in (1) is σ^2. Similarly, the power of the LoS component is given by A^2. Notice that A and σ in (1) are defined in such a way that the ratio A^2/σ^2 yields the Ricean K factor, and the received signal power is given by $E|x(t)|^2 = A^2 + \sigma^2 = \Omega$. In fact, it is often the envelope $R(t) := |x(t)|$ that is of interest, and its marginal probability density function (pdf) can be expressed in terms of Ω and K as (see e.g., [16])

$$\begin{aligned} p_R(r) &= \frac{2(K+1)r}{\Omega} \exp\left(-K - \frac{(K+1)r^2}{\Omega}\right) \\ &\times I_0\left(2r\sqrt{\frac{K(K+1)}{\Omega}}\right) \;, \end{aligned} \qquad (3)$$

where $I_n(\cdot)$ is the n^{th} order modified Bessel function of the first kind. Notice that when $K=0$, there is no LoS component, in which case (3) reduces to the Rayleigh distribution.

The relative power of the LoS component, represented by the K factor, is a useful measure of the communication link quality. Therefore, estimation of K is important in a variety of wireless scenarios, including channel characterization, link budget calculations, adaptive modulation, and geolocation applications [7], [10].

Estimation of the Ricean factor has been tackled in quite disparate contexts. In [13], Rastogi and Holt propose a moment-based approach which utilizes the second and fourth order moments of the envelope $R(t)$, in order to estimate K from the HF radio waves (see also [8]). In [18], the maximum likelihood estimator (MLE) is derived, and is shown to require a cumbersome inversion of a nonlinear function of K. In the same reference, a simpler estimator that utilizes the first and second order moments of the received envelope, which also requires the inversion of a nonlinear function of K is proposed (this estimator was later rediscovered in [12]). In [1], two moment-based estimators are compared using asymptotic analysis and simulations. The distribution fitting approaches for estimating K, proposed in [7], are robust, but are not suited for online implementation due to their complexity and hence might be more useful for *testing* whether the measured envelope is Ricean distributed, rather than estimating K. An expectation-maximization approach to finding the MLE for a multidimensional Ricean distribution is proposed in [9], but still not easy to calculate and use in a communication scenario. In [6], the moment-based approach that uses the second and fourth moments of the envelope (originally mentioned in [13]) is derived from a different perspective, assuming the LoS component in (1) is time-invariant. A dynamic approach based on the received uncoded bit error rate (BER) is recently proposed in [17].

In most of these references, the received data is assumed to be independent and identically distributed (iid). Neither the effect of correlation nor the influence of the LoS AoA θ_0 on the performance has been addressed in the literature. Performance analysis of the aforementioned estimators have mainly relied on simulations, and mostly estimation from the envelope $R(t)$ has been investigated. Moreover, the potential performance improvements attainable by using the I/Q components rather than the envelope have not been fully addressed. We will fill these gaps in this paper. In what follows, we will investigate the estimation of K from the I/Q components of the received signal given in complex baseband form, which are the real and imaginary parts of $x(t)$ (1). The I/Q components are available in applications where a coherent estimate of the channel is necessary. We will show that using the I/Q components (which contain envelope *and* phase information of the channel) improves the estimator performance, especially for small values of K. Moreover, we will see that, unlike the estimators that utilize the envelope, the performance analysis of the estimators that utilize the I/Q components and the corresponding CRB can be computed even when the samples of $x(t)$ are *correlated.* We will derive the rate of convergence of the estimators that utilize the I/Q components, which will require a novel approach because the samples of (2) are not absolutely summable.

II. CRB for I/Q Data

In this section we will derive the CRB for the variance of K estimators that use the I/Q data in (1). The resulting CRB will be a lower bound on the variance of any unbiased estimator for K obtained not only from the I/Q components in (1), but also from the envelope. This is because any estimator that can be constructed from the envelope can be constructed from (1). We will see that unlike the derivation of the envelope CRB in [18], we will not need to make the restrictive iid assumption when deriving the I/Q CRB because of the tractability of the multivariate Gaussian pdf (as opposed to the difficulty of expressing and working with the multivariate Ricean pdf that emerges with correlated envelope samples). CRBs for fading parameters for a differently parameterized radio transmission channel can also be found in [4].

Let us define the T_s-spaced samples of (1) as $x[n] := x(nT_s)$, $\omega_0 := 2\pi f_D \cos(\theta_0) T_s$ and $h[n] := h(nT_s)$. We may then express the sampled I/Q signal in terms of σ and K as follows:

$$x[n] = \sqrt{K}\sigma e^{j(\omega_0 n + \phi_0)} + \sigma h[n] \, . \tag{4}$$

Suppose that we have a data record of N samples from (4). We would like to compute the CRB for the parameter K. We emphasize that the I/Q-CRB in this section is different from the envelope-CRB, because the data from which they are derived are different. We will calculate the I/Q-CRB for two cases:

(i) All the parameters (except K) are known;

(ii) All parameters in $\boldsymbol{\zeta} := [K \ \sigma \ \omega_0 \ \phi_0 \ \omega_D]$ are unknown and the channel correlations are given by $r_h[k] := r_h(kT_s) = J_0(\omega_D k)$, where $\omega_D = 2\pi f_D T_s$.

The first case, which appears rather unrealistic, is important because it provides a lower bound on *any* estimator of K whether any other parameter is known or not, and whether the envelope or the I/Q components are used. Another reason for considering case (i) is the resulting simplicity of the bound. We shall consider case (i) first, and then we will address case (ii).

Let $\mathbf{x} := [x[0] \ldots x[N-1]]^T$ be a length-N vector that contains the available sampled I/Q components, and $\mathbf{e}(\omega_0, \phi_0) := [\exp(j\omega_0 0 + j\phi_0) \ldots \exp(j\omega_0 (N-1) + j\phi_0)]^T$. Then the mean and covariance matrix of $\mathbf{x}$ are given by $\boldsymbol{\mu}(\boldsymbol{\zeta}) := E[\mathbf{x}] = \sigma\sqrt{K}\mathbf{e}(\omega_0, \phi_0)$, and $\boldsymbol{\Gamma}(\boldsymbol{\zeta}) := E[(\mathbf{x} - \boldsymbol{\mu}(\boldsymbol{\zeta}))(\mathbf{x} - \boldsymbol{\mu}(\boldsymbol{\zeta}))^H] = \sigma^2 \mathbf{R}(\omega_D)$, where $\mathbf{R}(\omega_D)$ denotes the normalized [1] covariance matrix of the I/Q components, and ^{H} denotes Hermittian. When all parameters in $\boldsymbol{\zeta}$ except K are known (case (i)), using (22) in Appendix I, it is easy to show that

$$\mathrm{CRB}_{\mathrm{IQ}}(K) = \frac{2K}{\mathbf{e}^H(\omega_0, \phi_0) \ \mathbf{R}^{-1}(\omega_D) \ \mathbf{e}(\omega_0, \phi_0)} \, , \tag{5}$$

which reduces to the simple $2K/N$ when $x[n]$'s are independent (i.e., when $\mathbf{R}(\omega_D) = \mathbf{I}$).

Some remarks are now in order:

Remark 1: Note that if the correlations satisfy $\sum_k |r_h[k]| < \infty$, then we can use the standard results for the asymptotic forms of Toeplitz matrices (see e.g., [5]) to conclude that the asymptotic CRB defined as $\lim_{N\to\infty} N\mathrm{CRB}_{\mathrm{IQ}}(K)$ converges to $2KS_h(\omega_0)$, for $S_h(\omega_0) \neq 0$, where $S_h(\omega)$ is the spectrum of $h[n]$. The absolute summability of the covariances hold, for example, when $h(t)$ is modeled as an autoregressive process. We see that, in this case, if $S_h(\omega_0)$ is large, the CRB increases. However, the correlation function we have adopted in (2), which is a more accurate correlation model for wireless communications, is not absolutely summable in general, so we cannot easily establish the link between $S_h(\omega)$ and the asymptotic CRB in case (i), as we did when $\sum_k |r_h[k]| < \infty$ holds.

Remark 2: The CRB in (5) considers only K as an unknown parameter, which yields a bound that is smaller than the CRB when other parameters of the model in (4) are unknown. Hence, (5) provides a lower bound to the variance of *any* unbiased estimator of K, regardless of whether it is constructed from the envelope or the I/Q components of the signal, or whether any of the other parameters are known.

Remark 3: Unlike the envelope CRB which goes to infinity as K gets smaller, the I/Q CRB in (5) goes to zero (see also Fig. 1). This is also the case for the CRB for the more realistic case (ii), where $\boldsymbol{\zeta}$ is unknown, which is derived in Appendix I (see (24)). In Fig. 1, we show the CRB for the envelope data model, CRB for case (i) in (5) where the parameters are known, and the CRB for case (ii) in (24) where the parameters are unknown. We observe that CRBs for the I/Q data become smaller as K gets smaller for both cases (i) and (ii), which is also proved in the Appendix I for both cases. This reinforces our intuition that the additional phase information in the I/Q data, which is not present in the envelope data, offers potential improvements in estimator performance, particularly for small K. This insight motivates us to search for estimators of K from the I/Q components. We now propose such an estimator.

III. Estimation from the I/Q components

In pursuit of finding an estimator for K from the I/Q components, the first thing that comes to mind is the MLE con-

[1] so that it has ones on the main diagonal

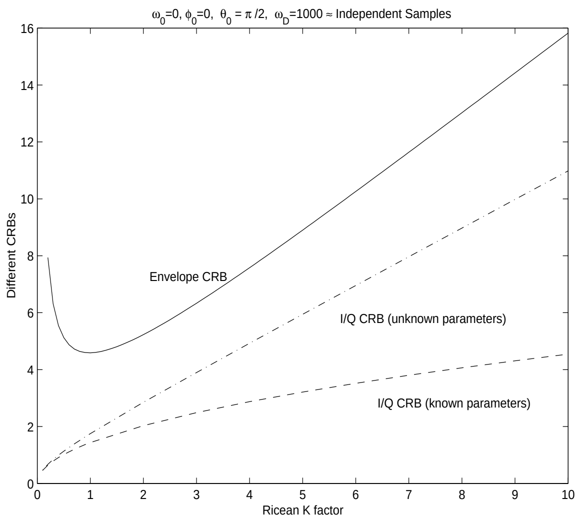

Figure 1: Different CRBs for K

structed from the I/Q data (which is different from the MLE constructed from the envelope data). But an MLE from the I/Q data will have to involve joint estimation of $K, \sigma, \omega_0, \phi_0$, and ω_D, which requires a multidimensional search and hence is not practical. An alternative to the MLE is a nonlinear least-squares approach of [15], which was proposed to estimate A, ω_0, σ, and ϕ_0, but not K. In what follows we provide a yet simpler approach.

Let $\hat{\omega}_0 = \operatorname{argmax}_\omega |N^{-1} \sum_{n=0}^{N-1} x[n] e^{-j\omega n}|$. Consider now the following statistics of $x[n]$:

$$X_1(N) := \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\hat{\omega}_0 n} , \qquad (6)$$

$$X_2(N) := \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 = \hat{\mu}_2 . \qquad (7)$$

Let us assume, for the moment, that $\hat{\omega}_0 \approx \omega_0$. Recalling the definition of $x[n] = Ae^{j(\omega_0 n + \phi_0)} + \sigma h[n]$, and substituting it in (6), we arrive at $X_1(N) = Ae^{j\phi_0} + N^{-1}\sigma \sum_{n=0}^{N-1} h[n] e^{-j\omega_0 n}$. Since as seen from (11), $N^{-1} \sum_{n=0}^{N-1} h[n] e^{-j\omega_0 n}$ goes to zero in the mean-squared sense as N increases, we see that for sufficiently large N, $|X_1(N)|^2 \approx A^2$. On the other hand, $\hat{\mu}_2$ in (7) converges to $\Omega = A^2 + \sigma^2$. This prompts us to propose the following estimator for K that approximates A^2/σ^2:

$$\hat{K}(N) = \frac{|X_1(N)|^2}{X_2(N) - |X_1(N)|^2} \qquad (8)$$

Notice that (8) can be implemented with low-complexity FFT processing. We will now derive the AsV of (8).

IV. Asymptotic Variance of $\hat{K}(N)$

In this section, we calculate the asymptotic variance of $\hat{K}(N)$ for the case when the samples $x[n]$ are independent, and also when they are correlated. To simplify the analysis we will assume that $\hat{\omega}_0 \approx \omega_0$. Hence, our calculations will yield the AsV when ω_0 is known perfectly, which is a lower bound to the AsV of (8). We will begin with the case where the samples $x[n]$ are independent. In order to derive the AsV of the estimators proposed in this paper, we will be using the following well-known result which is obtained by slightly adapting [11, Theorem 3.16] to our problem.

Theorem: *Suppose the two statistics $X_1(N)$ and $X_2(N)$ converge to x_1 and x_2 respectively in the mean squared sense, and the estimator of interest is given as a function of these statistics: $\hat{K}(N) = g(X_1(N), X_2(N))$. Let $d(N)$ denote the rate of convergence of the vector $[X_1(N) X_2(N)]$, so that $d(N)[(X_1(N) - x_1) \ \ (X_2(N) - x_2)]$ converges to a random vector with mean zero, and covariance matrix whose (l, k) element is given by*

$$c_{lk} := \lim_{N\to\infty} d^2(N) \operatorname{cov}(X_l(N), X_k(N)) , \quad l, k = 1, 2 . \qquad (9)$$

Then the scaled estimator $d(N)(g(X_1(N), X_2(N)) - g(x_1, x_2))$ converges in distribution to a random variable with zero mean and variance given by

$$\operatorname{AsV} = \sum_{l=1}^{2} \sum_{k=1}^{2} c_{lk} \frac{\partial g(a_1, a_2)}{\partial a_l} \frac{\partial g(a_1, a_2)}{\partial a_k} \Big|_{a_1 = x_1, a_2 = x_2} . \qquad (10)$$

Let $d(N) = \sqrt{N}$, $g(a_1, a_2) = |a_1|^2/(a_2 - |a_1|^2)$ (c.f. (8)), $x_1 = Ae^{j\phi_0}$ and $x_2 = \Omega$ (because (6) converges to $Ae^{j\phi_0}$ and (7) converges to Ω). To calculate the c_{lk} in (9) we will use the following covariance expressions which are straightforward to show [19]:

$$\operatorname{var}(X_1) = \sigma^2 \frac{1}{N} \sum_{k=-N+1}^{N-1} \left(1 - \frac{|k|}{N}\right) r_h[k] e^{-j\omega_0 k} \qquad (11)$$

$$\begin{aligned} \operatorname{var}(X_2) &= \frac{\Omega^2}{(K+1)^2} \frac{1}{N} \sum_{k=-N+1}^{N-1} \left(1 - \frac{|k|}{N}\right) \\ &\times \left[|r_h[k]|^2 + 2K \operatorname{Re}\left(r_h[k] \ e^{-j\omega_0 k}\right)\right] \end{aligned} \qquad (12)$$

$$\operatorname{cov}(X_1, X_2) = Ae^{j\phi_0} \sigma^2 \frac{1}{N} \sum_{k=-N+1}^{N-1} \left(1 - \frac{|k|}{N}\right) r_h[k] e^{j\omega_0 k} . \qquad (13)$$

where $r_h[k] := r_h(kT_s)$, and $\operatorname{Re}(\cdot)$ denotes real part, and we have dropped the N dependence on the left hand side for convenience. Using (11)-(13) with $r_h[k] = \delta[k]$ [2], we obtain $c_{11} = \sigma^2$, $c_{12} = c_{21}^* = Ae^{j\phi_0}\sigma^2$, and $c_{22} = \Omega^2(2K+1)/(K+1)^2$. Differentiating $g(a_1, a_2)$ we get $\partial g(a_1, a_2)/\partial a_1 = Ae^{-j\phi_0}\Omega/\sigma^2$, $\partial g(a_1, a_2)/\partial a_2 = -A^2/\sigma^4$ and substituting in (10), and simplifying, we finally obtain

$$\operatorname{AsV}(\hat{K}) = K(K^2 + K + 1) . \qquad (14)$$

We observe that this simple polynomial function of K suggests that the estimator should be more accurate for smaller values of K.

The more realistic, but challenging problem is calculating the asymptotic variance when the samples $x[n]$ are correlated. If the correlations of $x[n]$ are well-behaved enough for $\sum_k |r_h[k]| < \infty$ to hold (as seen in exponentially-decaying ARMA, or Gaussian shaped correlation functions) then (11)-(13) are $\mathcal{O}(N^{-1})$, so the rate of convergence is the same as the iid case [3]. We will, however, adopt the model in (2) for the correlation function, which is *not* absolutely summable,

[2] Krönecker's delta function $\delta[k]$ is used because the samples are assumed uncorrelated.

[3] We will use the standard notation $F(N) = \mathcal{O}(G(N))$ to mean that $F(N)/G(N)$ is a bounded sequence.

but has the merits of being motivated by physical considerations. Moreover, (2) does not constrain our results to isotropic scattering, i.e, $p_h(\theta)$ does not have to be equal to $1/(2\pi)$ in $-\pi \leq \theta < \pi$, which allows for directional receptions, and generalizes Clarke's model [16]. To our knowledge, this is the first time that (2) is used in performance analysis of estimators in wireless communications.

Let us first characterize the asymptotic behavior of $r_h(\tau)$, which will be central in the calculation of the asymptotic variance. Using the method of stationary phase, it can be shown that (2) can be expressed as [3]:

$$\begin{aligned} r_h(\tau) &= (f_D\tau)^{-1/2}\, p_h(0)\, e^{j(2\pi f_D\tau - \frac{\pi}{4})} \\ &+ (f_D\tau)^{-1/2}\, p_h(\pi)\, e^{-j(2\pi f_D\tau - \frac{\pi}{4})} + \mathcal{O}(\tau^{-1}). \quad (15) \end{aligned}$$

Notice that when $p_h(\theta)$ is uniform, $r_h(\tau) \sim (f_D\tau)^{-1/2}\cos(2\pi f_D\tau - \pi/4)$, which is a well-known asymptotic expansion of $J_0(\cdot)$. Hence, we reach the interesting conclusion that under some regularity conditions on $p_h(\theta)$, for large enough τ, the correlation function is the sum of a sinusoid whose envelope goes to zero as $\tau^{-1/2}$ and an error term which goes to zero faster as τ^{-1}. In our AsV derivation for $\hat{K}_{\mathrm{IQ}}$ in (8), we will need the correlation function for the *sampled* I/Q components. Recalling that $\omega_D = 2\pi f_D T_s$, the sampled correlation function can be expressed, using (15) as

$$\begin{aligned} r_h[k] &= \left(\frac{2\pi}{\omega_D k}\right)^{1/2} p_h(0)\, e^{j(\omega_D k - \frac{\pi}{4})} \quad (16) \\ &+ \left(\frac{2\pi}{\omega_D k}\right)^{1/2} p_h(\pi)\, e^{-j(\omega_D k - \frac{\pi}{4})} + \mathcal{O}(k^{-1}). \quad (17) \end{aligned}$$

Notice that the correlation in (17) are not absolutely summable. The slow-decaying nature of the correlation function in (17) results in a slower convergence rate for $X_1(N)$ and $X_2(N)$ as compared to the independent case. First let us determine how fast the variances of $X_1(N)$ and $X_2(N)$ in (6) and (7) go to zero. We will then invoke the Theorem in this section to determine the rate of convergence of $\hat{K}(N)$ which is the goal in this subsection.

Using (17) and (11) we show in Appendix II the following: [4]

$$\mathrm{var}(X_1(N)) = \begin{cases} \mathcal{O}(N^{-1}) & \text{if } |\omega_0| < \omega_D \\ \mathcal{O}(N^{-\frac{1}{2}}) & \text{if } |\omega_0| = \omega_D \end{cases}, \quad (18)$$

and similarly, using (17) and (13), we show in Appendix II that

$$\mathrm{var}(X_2(N)) = \begin{cases} \mathcal{O}(N^{-1}\log(N)) & \text{if } |\omega_0| < \omega_D \\ \mathcal{O}(N^{-\frac{1}{2}}) & \text{if } |\omega_0| = \omega_D \end{cases}. \quad (19)$$

Comparing (11) and (13), it is apparent that $\mathrm{var}(X_1(N))$ and $\mathrm{cov}(X_1(N), X_2(N))$ converge at the same rate given by (18).

Recall that when the samples are independent (or more generally when $\sum_k |r_h[k]| < \infty$), the convergence rate of the variances of both $X_1(N)$ and $X_2(N)$ is $d^{-2}(N) = N^{-1}$. It is interesting that, for correlation functions of the form in (2), the rate of convergence of both $X_1(N)$ and $X_2(N)$ depends on whether $|\omega_0| = \omega_D$. Physically, $|\omega_0| = \omega_D$ when $\theta_0 = 0$ in (1), i.e., the LoS is in the same direction as the mobile. So, when $|\omega_0| < \omega_D$, $X_1(N)$ converges faster than $X_2(N)$, and when $|\omega_0| = \omega_D$, the variances of $X_1(N)$ and $X_2(N)$ converge to zero at the same rate. Now we are ready to invoke the Theorem.

The Theorem requires that both $X_1(N)$ and $X_2(N)$ should converge when scaled by the same sequence. So, when $|\omega_0| < \omega_D$, the scaling sequence should be the slower of the $X_1(N)$ and $X_2(N)$, so that the faster one will converge to zero (if we were to scale with the rate of the faster one, the slower statistic would go to infinity). Hence, when $|\omega_0| < \omega_D$, we choose $d(N)$ so that $d^{-2}(N) = N^{-1}\log(N)$. To calculate the AsV, we need to substitute in (10), $d(N) = \sqrt{N/\log(N)}$, the partial derivatives of $g(a_1, a_2)$, which are given right before (14), and c_{lk} which, using (9), (11)-(13), (18) and (19), are given by $c_{11} = c_{12} = c_{21} = 0$, and $c_{22} = [\Omega^2/(K+1)^2]\ C_1$, where C_1 and the resulting AsV for $|\omega_0| < \omega_D$ are given by

$$\mathrm{AsV}(\hat{K}) = K^2 \underbrace{\left[\lim_{N\to\infty} \frac{1}{\log(N)} \sum_{k=-N+1}^{N-1} \left(1 - \frac{|k|}{N}\right) |r_h[k]|^2\right]}_{=:C_1}, \quad (20)$$

and the limit C_1, which is independent of K, can be shown to exist using (17). When $|\omega_0| = \omega_D$ we have that both $X_1(N)$ and $X_2(N)$ converge at the same rate. So we select $d^{-2}(N) = N^{-1/2}$ and substitute in (10) $d(N) = \sqrt{N^{1/2}}$, the partial derivatives, and c_{lk}, which, using (9) and (11-19) turn out to be $c_{11} = \Omega C_2$, $c_{12} = c_{21}^* = Ae^{j\phi_0}\sigma^2 C_2$, and $c_{22} = \Omega^2 2K/(K+1)^2\ C_2$, where C_2 and the resulting AsV are given by

$$\mathrm{AsV}(\hat{K}) = (K^3 + K)\ C_2 \quad (21)$$

where $C_2 := \left[\lim_{N\to\infty} N^{-1/2} \sum_{k=-N+1}^{N-1} \left(1 - \frac{|k|}{N}\right)\ r_h[k]\ e^{j\omega_0 k}\right]$ is independent of K.

Hence, loosely speaking, we can say that when the data are independent the AsV of $\hat{K}(N)$ is proportional to $K(K^2 + K + 1)$ when the estimator is scaled by $d(N) = \sqrt{N}$; when the data are correlated with correlation function given in (2), we have the case $|\omega_0| < \omega_D$: the AsV of $\hat{K}(N)$ is proportional to K^2 when the estimator is scaled by $d(N) = \sqrt{N/\log(N)}$; and the case $|\omega_0| = \omega_D$: the AsV of $\hat{K}(N)$ is proportional to $K^3 + K$ when the estimator is scaled by $d(N) = \sqrt{N^{1/2}}$.

Some conclusions that we can draw from this analysis are as follows. Regardless of whether the data samples are correlated or not, the $\hat{K}$ becomes more accurate if K is small. In fact, our motivation for pursuing the estimation of K from the I/Q components was precisely this reason: while all unbiased estimators of K from the envelope yield an unbounded variance as K gets smaller, the accuracy of $\hat{K}$ increases with smaller K. It is also important to notice that the value of ω_0 makes a difference in the performance, so much so that it makes a difference in the *rate* of convergence. In fact even for finite N, motivated by Remark 1 following (5), values of ω_0 for which $S_h(\omega_0)$ is small, yield better performance. If we adopt the isotropic scattering model, corresponding to a uniform $p_h(\theta)$, $S_h(\omega) = \sigma_h^2\pi^{-1}[1 - (\omega/\omega_D)^2]^{-1/2}, |\omega| < \omega_D$. Since $S_h(\omega_D)$ is infinite, when $|\omega_0| = \omega_D$ the performance is worse as compared to when $|\omega_0| < \omega_D$. This spectrum has a minimum at 0, hence, $\omega_0 = 0$ (implying $\theta_0 = \pi/2$) seems to be the best AoA for the LoS component as far as the performance of $\hat{K}_{\mathrm{IQ}}$ is concerned. Physically, this is a LoS that is perpendicular

[4] Notice that since $\omega_0 = \omega_D\cos(\theta_0)$, $|\omega_0| \leq \omega_D$ always holds, so we do not consider the case $\omega_0 > \omega_D$.

to the direction of the mobile yielding a time-invariant LoS component.

V. Simulations

To test the possible improvements attainable by using the I/Q components in estimating K as compared to envelope-based estimators, we compared the I/Q estimator of proposed herein, with the envelope-based estimator that relies on the first and second order moments which we term $\hat{K}_{1,2}$ (see also [1]). We chose $N = 1,000$ data points, and $\omega_D = 18.85$ which corresponds to a vehicle velocity of $v = 100$ km/hr, $T_s = 0.01$ sec., and a carrier frequency of 900 MHz. We observe from Fig. 2 that the estimator that relies on the I/Q components performs significantly better than $\hat{K}_{1,2}$. Moreover, the I/Q CRB provides a tight lower bound on the estimation error of the I/Q estimator particularly for small values of K.

VI. Conclusions

Motivated by the fact that the envelope CRB increases without bound as K gets smaller, we studied the estimation of K from the I/Q data. We observed that the I/Q CRB goes to zero as K gets smaller, a property also held by held by the AsV of a novel I/Q-based estimator that we proposed. The performance analysis for this estimator for *correlated* samples yielded insights into the effect of θ_0 on the estimator performance.

Appendix I: *CRB for I/Q - Based Estimators*

For I/Q data which is complex Gaussian with mean $\boldsymbol{\mu}(\boldsymbol{\zeta})$ and covariance matrix $\boldsymbol{\Gamma}(\boldsymbol{\zeta})$, the elements of the FIM are given by [20]

$$\begin{aligned}[\mathbf{J}(\boldsymbol{\zeta})]_{kl} &= 2\mathrm{Re}\left[\left(\frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\zeta_k}\right)^H \boldsymbol{\Gamma}^{-1}(\boldsymbol{\zeta})\left(\frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\zeta_l}\right)\right] \\ &+ \mathrm{tr}\left[\boldsymbol{\Gamma}^{-1}(\boldsymbol{\zeta})\frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\zeta_k}\boldsymbol{\Gamma}^{-1}(\boldsymbol{\zeta})\frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\zeta_l}\right], \qquad (22)\end{aligned}$$

where ζ_k denotes the k^{th} element of $\boldsymbol{\zeta}$, and $\mathrm{tr}(\cdot)$ denotes the trace of a matrix.

We now calculate the CRB for estimators of K that use the I/Q components assuming that all the elements in $\boldsymbol{\zeta}$ are unknown. For this, we need the partial derivatives of $\boldsymbol{\mu}(\boldsymbol{\zeta})$ and $\boldsymbol{\Gamma}(\boldsymbol{\zeta})$ with respect to ζ_k, which are given below:

$$\begin{aligned}&\frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial K} = \frac{\sigma}{2\sqrt{K}}\mathbf{e}(\omega_0,\phi_0) \qquad \frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\sigma} = \sqrt{K}\mathbf{e}(\omega_0,\phi_0)\\ &\frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\omega_0} = \sigma\sqrt{K}\mathbf{e}_2(\omega_0,\phi_0) \qquad \frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\phi_0} = j\sqrt{K}\sigma\mathbf{e}(\omega_0,\phi_0) \qquad (23)\\ &\frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\sigma} = 2\sigma\mathbf{R}(\omega_D) \qquad \frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\omega_D} = \sigma^2\mathbf{R}'(\omega_D)\end{aligned}$$

$$\frac{\partial\boldsymbol{\mu}(\boldsymbol{\zeta})}{\partial\omega_D} = \frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial K} = \frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\omega_0} = \frac{\partial\boldsymbol{\Gamma}(\boldsymbol{\zeta})}{\partial\phi_0} = \mathbf{0}$$

where $\mathbf{e}_2(\omega_0,\phi_0) := e^{j(\phi_0+\pi/2)}[0e^{j\omega_0 0}\ldots(N-1)e^{j\omega_0(N-1)}]^T$, $[\mathbf{R}'(\omega_D)]_{kl} = \partial J_0(\omega_D(k-l))/\partial\omega_D = -(k-l)J_1(\omega_D(k-l))$. Using (23) and (22), all entries of the 5×5 FIM can be computed, and the (1,1) element of the inverse FIM will be the CRB of K estimators that utilize the I/Q components when $\boldsymbol{\zeta}$ is unknown. Let $J_{11} := [\mathbf{J}(\boldsymbol{\zeta})]_{11}$ for brevity. An important point is that J_{11} is proportional to $1/K$ and hence goes to infinity as K goes to zero. Also the submatrix $[\mathbf{J}(\boldsymbol{\zeta})]_{2:5,2:5}$ consisting of the second through fifth row, and second through fifth column of $[\mathbf{J}(\boldsymbol{\zeta})]$, stays constant as K goes to zero. This can be easily verified using (23) and (22). Furthermore, let $[\mathbf{J}(\boldsymbol{\zeta})]_{1,2:5}$ and $[\mathbf{J}(\boldsymbol{\zeta})]_{2:5,1}$ be vectors consisting of the second through fifth column of the first row and second through fifth row of the first column, respectively. Then, as a consequence of the matrix inversion lemma [14, pp. 512], the (1,1) element of the inverse FIM (which is the CRB of interest) is given by

$$[\mathbf{J}^{-1}(\boldsymbol{\zeta})]_{11} = \frac{1}{J_{11}} + C \qquad (24)$$

where $C = [\mathbf{J}(\boldsymbol{\zeta})]_{1,2:5}\left(J_{11}^2[\mathbf{J}(\boldsymbol{\zeta})]_{2:5,2:5} - J_{11}[\mathbf{J}(\boldsymbol{\zeta})]_{2:5,1}[\mathbf{J}(\boldsymbol{\zeta})]_{1,2:5}\right)^{-1}$ $\times[\mathbf{J}(\boldsymbol{\zeta})]_{2:5,1}$. But as K goes to zero, J_{11} goes to infinity, and C remains bounded, which shows that $[\mathbf{J}^{-1}(\boldsymbol{\zeta})]_{11}$ in (24), which is the CRB of interest, goes to zero as K goes to zero.

Appendix II: *Rates of Convergence*

In this appendix, we will derive (18) and (19). In order to show (18) for $|\omega_0| < \omega_D$, we need to show that $N\mathrm{var}(X_1(N))$ converges to a finite constant. To show that $N\mathrm{var}(X_1(N))$ converges, we need to establish $\sum_{k=-\infty}^{\infty}(1-|k|/N)r_h[k]e^{-j\omega_0 k} < \infty$ for which it suffices to show that $\sum_{k=-\infty}^{\infty} r_h[k]e^{-j\omega_0 k} < \infty$ because of the Cesaro summability theorem [11, pp. 411]. Substituting (17) for $r_h[k]$ we can write $r_h[k]e^{-j\omega_0 k} \approx a[k]b[k]$ where

$$\begin{aligned}b[k] &:= p_h(0)\ e^{j((\omega_D-\omega_0)k-\frac{\pi}{4})} + p_h(\pi)\ e^{-j((\omega_D+\omega_0)k-\frac{\pi}{4})},\\ a[k] &:= \left(\frac{2\pi}{\omega_D k}\right)^{1/2}, \qquad (25)\end{aligned}$$

and the approximation is due to the $\mathcal{O}(k^{-1})$ term in (17). We can now apply Drichlet's test [2, pp. 365] which states that if $a[k]$ converges monotonically to zero and the partial sums of $b[k]$ are bounded, then $\sum_k a[k]b[k]$ converges. Since these two conditions hold in our case ($|\omega_0| < \omega_D$), we conclude that $\sum_{k=-\infty}^{\infty} r_h[k]e^{-j\omega_0 k} < \infty$, which is what we needed to show.

Let us now show that (18) for $|\omega_0| = \omega_D$ holds. To do this, we need to show that

$$\begin{aligned}&\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)r_h[k]e^{-j\omega_D k}\\ \approx &\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)a[k]\ b[k] = \mathcal{O}(N^{1/2}). \qquad (26)\end{aligned}$$

where $a[k]$ and $b[k]$ are obtained by substituting $|\omega_0| = \omega_D$ in (25) and are given by $a[k] = [(2\pi)/(\omega_D k)]^{1/2}$, $b[k] = [p_h(0)\ \exp(-j\frac{\pi}{4}) + p_h(\pi)\ \exp -j((\omega_D+\omega_0)k-\frac{\pi}{4})]$, which is the sum of a constant and an exponential. But $\sum_{k=-N+1}^{N-1} a[k]b[k]$ and $N^{-1}\sum_{k=-N+1}^{N-1}|k|a[k]b[k]$ add up to (26), and they are both $\mathcal{O}(N^{1/2})$. This can be seen after substituting for $a[k]$ and $b[k]$, and using the following: [5] $\sum_{k=-N+1}^{N-1} k^{-1/2} = \mathcal{O}(N^{1/2})$, $N^{-1}\sum_{k=-N+1}^{N-1} k^{1/2} = \mathcal{O}(N^{1/2})$, and $\sum_{k=-N+1}^{N-1} k^{-1/2}e^{-j((\omega_D+\omega_0)k-\frac{\pi}{4})} = \mathcal{O}(1)$, where the first two equalities are obtained by integrating $k^{-1/2}$ and $k^{1/2}$ respectively, and the third expression is obtained using Drichlet's theorem. This establishes the equality in (26) which is what we wanted to show.

[5] Since we are interested in asymptotic expressions for large N, we are not concerned with the fact that $k^{-1/2}$ is unbounded for $k = 0$

We will now show that (19) holds. For $|\omega_0| < \omega_D$, we need to show that

$$\sum_{k=-N+1}^{N-1} (1-|k|/N)\left[|r_h[k]|^2 + 2K\mathrm{Re}\left(r_h[k]e^{-j\omega_0 k}\right)\right] = \mathcal{O}(\log(N)) \ , \quad (27)$$

which would establish that (13) is $\mathcal{O}(N^{-1}\log(N))$. We know from (18) that for $|\omega_0| < \omega_D$, $\sum_k r_h[k]e^{j\omega_0 k}$ converges; hence, we can do away with the second term in the square brackets in (27), and see that establishing (27) amounts to showing

$$\sum_{k=-N+1}^{N-1} |r_h[k]|^2 - \sum_{k=-N+1}^{N-1} \frac{|k|}{N}|r_h[k]|^2 = \mathcal{O}(\log(N)) \ . \quad (28)$$

Using (17), it is straightforward to show that the first term on the left hand side of (28) is

$$\sum_{k=-N+1}^{N-1} |r_h[k]|^2 \approx \left(\frac{2\pi}{\omega_D}\right) \sum_{k=-N+1}^{N-1} \Big(\frac{p_h^2(0)+p_h^2(\pi)}{k} + 2p_h(0)p_h(\pi)\frac{\cos\left(\omega_D k - \frac{\pi}{4}\right)}{k}\Big) \ , \quad (29)$$

which is $\mathcal{O}(\log(N)) + \mathcal{O}(1)$, where the $\mathcal{O}(\log(N))$ term is obtained by integrating k^{-1}, and the $\mathcal{O}(1)$ term is obtained by using Drichlet's theorem. But the second term on the left hand side of (28) is $\sum_{k=-N+1}^{N-1}(|k|/N)|r_h[k]|^2 = \mathcal{O}(1)$, which can be verified similar to (29). So (28) must be $\mathcal{O}(\log(N))$. This establishes what we wanted to show.

Using a similar approach, it is not difficult to show that for the case $|\omega_0| = \omega_D$, (27) is given by $\mathcal{O}(\log(N)) + \mathcal{O}(N^{1/2}) = \mathcal{O}(N^{1/2})$, which completes the derivations of (18) and (19).

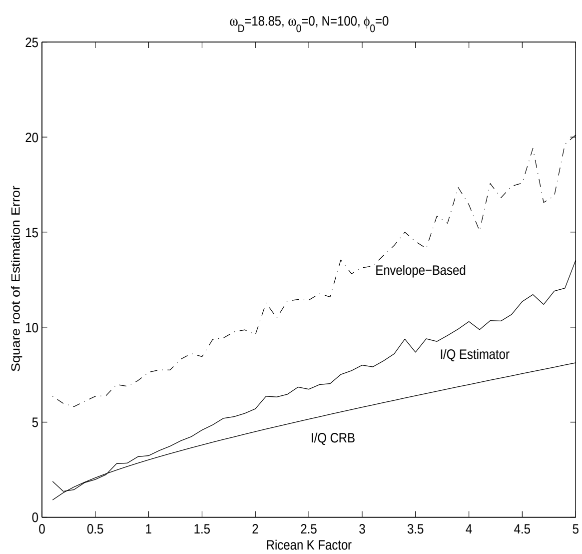

Figure 2: Comparison of envelope-based and I/Q-based estimators and CRB

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References

[1] A. Abdi, C. Tepedelenlioglu, G. B. Giannakis, and M. Kaveh, "On the estimation of the K parameter for the Rice fading distribution," *IEEE Communications Letters*, vol. 5, no. 3, pp. 92-94, 2001.

[2] T. M. Apostol, *Mathematical Analysis*, Addison Wesley, 1957.

[3] E. T. Copson, *Asymptotic Expansions*, Cambridge University Press, 1965.

[4] F. Gini, M. Luise and R. Reggiannini, "Cramer-Rao bounds in the parametric estimation of fading radiotransmission channels," *IEEE Transactions on Comm.*, vol. 46, no. 10, 1998.

[5] R. Gray, "Toeplitz and circulant matrices: A review," `http://www-ee.stanford.edu/~gray/toeplitz.html`, 2001.

[6] L. J. Greenstein, D. G. Michelson, and V. Erceg, "Moment-method estimation of the Ricean K-factor," *IEEE Communications Letters*, vol. 3, no. 6, pp. 175-176, 1999.

[7] D. Greenwood and L. Hanzo, "Characterization of mobile radio channels," in *Mobile Radio Communications*. R. Steele, Ed., Pentech, 1992, pp. 92-185.

[8] W. K. Hocking, "Reduction of the effects of non-stationarity in studies of amplitude statistics of radio wave backscatter," *Journal of Atmospheric and Terrestrial Physics*, vol. 49, no. 11/12, pp. 1119-1131, 1987.

[9] T. L. Marzetta, "EM algorithm for estimating the parameters of a multivariate complex Ricean density for polarimetric SAR," in *Proc. ICASSP*, Detroit, MI, 1995, pp. 3651-3654.

[10] K. Pahlavan, P. Krishnamurthy, and A. Beneat, "Wideband radio propagation modeling for indoor geolocation applications," *IEEE Communications Magazine*, vol. 36, no. 4, pp. 60-65, 1998.

[11] B. Porat, *Digital Processing of Random Signals*, Prentice Hall, 1994.

[12] F. van der Wijk, A. Kegel, and R. Prasad, "Assessment of a pico-cellular system using propagation measurements at 1.9 GHz for indoor wireless communications," *IEEE Transactions on Vehicular Technology*, vol. 44, no. 1, pp. 155-162, 1995.

[13] P. K. Rastogi and O. Holt, "On detecting reflections in presence of scattering from amplitude statistics with application to D region partial reflections," *Radio Science*, vol. 16, no. 6, pp. 1431-1443, 1981.

[14] T. Söderström, and P. Stoica, *System Identification*, Prentice Hall, 1989.

[15] P. Stoica, A. Jakobsson, and J. Li, "Cisoid parameter estimation in the colored noise case: asymptotic Cramer-Rao bound, maximum likelihood, and nonlinear least-squares," *IEEE Trans. on Signal Proc.*, vol. 45, no. 8, pp. 2048-2059, 1997.

[16] G. L. Stüber, *Principles of Mobile Communication*, Kluwer, 1996.

[17] M. Sumanasena, and B. Evans, "Rice factor estimation algorithm," *Electronics Letters*, vol. 37, no. 14, pp. 918-919, 2001.

[18] K. K. Talukdar and W. D. Lawing, "Estimation of the parameters of the Rice distribution," *Journal of Acoustical Society of America*, vol. 89, no. 3, pp. 1193-1197, 1991.

[19] C. Tepedelenlioğlu and G. B. Giannakis, "On velocity estimation and correlation properties of narrowband communication channels," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 4, pp. 1039-1052, 2001.

[20] A. Zeira and A. Nehorai, "Frequency domain Cramer-Rao bound for Gaussian Processes," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol 38, no. 6, pp. 1063-1066, 1990.