A Level Crossing-Based Algorithm for the Identification of DSB and SSB Modulated Spherically Invariant Processes

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Abstract -- **Modulation classification is an inevitable stage in intelligent communication systems. In this paper a fast method for discrimination between double sideband (DSB) and single sideband (SSB) modulations is presented. The method is based on differences in characteristics of the envelope level crossing rates of DSB and SSB signals. A general non-Gaussian spherically invariant random process is used for modeling the baseband signal, and additive noise is modeled by a Gaussian random process.**

I. Introduction

Modulation identification is an intermediate step between signal detection and demodulation, and plays a key role in various communication applications such as spectrum monitoring and management, surveillance and control of broadcasting activities, and electronic warfare. Due to the high signal density, increasing activity in the frequency spectrum, and short signal duration, it is vital to develop fast algorithms for efficient modulation classification in real time.

In order to classify various analog and digital modulations, different methods are proposed ([1]-[21]). However, a fast, easy-to-implement, and noise-immune method for discrimination between double sideband (DSB) and single sideband (SSB) modulated random signals cannot be found in the literature. In what follows, an efficient discrimination method is proposed which is based upon the characteristics of envelope level crossing rates in DSB and SSB signals.

II. Signal and Noise Models

We assume that $X(t)$, the baseband random signal, is a spherically invariant random process (SIRP). SIRP's are of interest due to the fact that they allow one to relax the assumption of Gaussianity, while keeping its useful characteristics ([22]-[27]). They have been used for accurate modeling of various non-Gaussian phenomena, such as speech signals [22], radar clutter [28], impulsive noise [29], and so on.

According to [25] and [23], a SIRP is a Gaussian random process, multiplied by an independent, positive valued random variable. So, for $X(t)$, a wide sense stationary (WSS) baseband zero-mean SIRP we have:

$$X(t) = AI(t), \tag{1}$$

where A is a positive random variable with probability density function (PDF) $p_A(a)$, and $I(t)$ is a WSS, baseband, and zero-mean Gaussian random process. Note that A and $I(t)$ are independent of each other.

We also assume that $N(t)$, the noise, is a WSS, bandpass, and zero-mean Gaussian random process, with a flat power spectrum over any frequency band. With f_m as the carrier frequency, and according to the Rice's representation [15], $N(t)$ can be written as:

$$N(t) = N_c(t)\cos 2\pi f_m t - N_s(t)\sin 2\pi f_m t, \tag{2}$$

where $N_c(t)$ and $N_s(t)$ are independent, WSS, baseband, and zero-mean Gaussian random processes, with the same autocorrelation function. It is reasonable to assume that $N(t)$ is independent of A and $I(t)$.

In the presence of noise, DSB and SSB modulation of $X(t)$ produce:

$$\tilde{Y}(t) = X(t)\cos 2\pi f_m t + N(t), \tag{3}$$

$$\tilde{Z}(t) = X(t)\cos 2\pi f_m t \mp \hat{X}(t)\sin 2\pi f_m t + N(t). \tag{4}$$

In (4), minus/plus signs refer to upper/lower sideband SSB, and $\hat{X}(t)$ is the Hilbert transform of $X(t)$.

III. Envelope Level Crossing Rate in DSB

After combining (1), (2), and (3), $\tilde{Y}(t)$ can be written as:

$$\tilde{Y}(t)=\left[AI(t)+N_c(t)\right]\cos 2\pi f_m t-N_s(t)\sin 2\pi f_m t\,, \quad (5)$$

where:

$$Y(t)=\sqrt{\left[AI(t)+N_c(t)\right]^2+N_s(t)^2}\,,$$
$$\tan\Theta(t)=N_s(t)\big/\left[AI(t)+N_c(t)\right]. \quad (6)$$

Conditioned on *A*, the univariate PDF of $Y(t)$ is a Hoyt PDF [30]. In order to obtain the level crossing rate of $Y(t)$, it is necessary to determine $p_{YY'}(y,y')$, the joint PDF of $Y(t)$ and $Y'(t)$ for a fixed *t*.

III.1 Derivation of $p_{YY'}(y,y'|A)$

Conditioned on *A*, the random vector $\mathbf{V_{rect}}=\left[M_c\ M_c'\ N_s\ N_s'\right]^T$, with $M_c(t)=AI(t)+N_c(t)$, is a Gaussian vector with zero mean-vector and the following covariance-matrix:

$$\mathbf{K}=\begin{bmatrix} A^2d_0+b_0 & 0 & 0 & 0\\ 0 & A^2d_2+b_2 & 0 & 0\\ 0 & 0 & b_0 & 0\\ 0 & 0 & 0 & b_2\end{bmatrix}, \quad (7)$$

such that:

$$d_0=E\left[I(t)^2\right]=2\int_0^{B_I}G_I(f)df\,,$$

$$d_2=E\left[I'(t)^2\right]=8\pi^2\int_0^{B_I}f^2G_I(f)df\,,$$

$$b_0=E\left[N_c(t)^2\right]=E\left[N_s(t)^2\right]=2B_IN_0\,,$$

$$b_2=E\left[N_c'(t)^2\right]=E\left[N_s'(t)^2\right]=8\pi^2B_I^{\,3}N_0/3, \quad (8)$$

where $G_I(f)$ is the power spectrum of $I(t)$, symmetric around $f=0$ and non-zero only for $|f|<B_I$, and the constant N_0 is the power spectrum of $N_c(t)$ and $N_s(t)$ for $|f|<B_I$. By introducing the random vector $\mathbf{V_{polar}}=\left[Y\ Y'\ \Theta\ \Theta'\right]^T$ and through the transformation $\mathbf{V_{rect}}\rightarrow\mathbf{V_{polar}}$ we get:

$$p_{YY'}(y,y'|A)=\int_0^{2\pi}p_{YY'\Theta}(y,y',\theta|A)\,d\theta\,, \quad (9)$$

$$p_{YY'\Theta}(y,y',\theta|A)=\exp\left(\frac{Q_2Q_4y'^2}{-2\det(\mathbf{K})\left(Q_4\cos^2\theta+Q_2\sin^2\theta\right)}\right)$$
$$\exp\left(\frac{\left(Q_1\cos^2\theta+Q_3\sin^2\theta\right)y^2}{-2\det(\mathbf{K})}\right)\frac{\sqrt{2\pi}\,y}{4\pi^2\sqrt{Q_4\cos^2\theta+Q_2\sin^2\theta}}, \quad (10)$$

where:

$$Q_1=b_0b_2(A^2d_2+b_2),$$
$$Q_2=b_0b_2(A^2d_0+b_0),$$
$$Q_3=b_2(A^2d_0+b_0)(A^2d_2+b_2),$$
$$Q_4=b_0(A^2d_0+b_0)(A^2d_2+b_2). \quad (11)$$

III.2 Conditional level crossing rate of $Y(t)$

Using (9) and according to the Rice's formula for the ℓ-crossing rate of $Y(t)$ [31] we obtain:

$$E\left[C_\ell\{Y(t)\}\big|A\right]=\frac{2\sqrt{2\pi}}{\pi^2}\sqrt{\frac{A^2d_2+b_2}{b_0(A^2d_0+b_0)}}\,\ell\exp\left(\frac{-\ell^2}{2(A^2d_0+b_0)}\right)$$
$$\times\int_0^{\pi/2}\sqrt{1-\lambda_2^{\,2}\sin^2\theta}\,\exp\left(\frac{-\lambda_0^{\,2}\ell^2\sin^2\theta}{2b_0}\right)d\theta\,, \quad (12)$$

in which:

$$\lambda_0^{\,2}=\frac{A^2d_0}{A^2d_0+b_0}<1,\quad \lambda_2^{\,2}=\frac{A^2d_2}{A^2d_2+b_2}<1.$$

IV. Envelope Level Crossing Rate in SSB

After combining (1), (2), and (4), $\tilde{Z}(t)$ can be written as:

$$\begin{aligned}\tilde{Z}(t)&=\left[AI(t)+N_c(t)\right]\cos 2\pi f_m t\\ &\quad-\left[\pm A\hat{I}(t)+N_s(t)\right]\sin 2\pi f_m t\\ &=Z(t)\cos\left[2\pi f_m t+\Psi(t)\right],\end{aligned} \quad (13)$$

where:

$$Z(t)=\sqrt{\left[AI(t)+N_c(t)\right]^2+\left[\pm A\hat{I}(t)+N_s(t)\right]^2}\,,$$

$$\tan\Psi(t)=\left[\pm A\hat{I}(t)+N_s(t)\right]\Big/\left[AI(t)+N_c(t)\right]. \quad (14)$$

Conditioned on *A*, the univariate PDF of $Z(t)$ is a Rayleigh PDF. In order to obtain the level crossing rate of $Z(t)$, it is necessary to determine $p_{ZZ'}(z,z')$, the joint PDF of $Z(t)$ and $Z'(t)$ for a fixed *t*.

IV.1 Derivation of $p_{ZZ'}(z,z'|A)$

Since Hilbert transform is a linear transform, $\hat{I}(t)$ is also a WSS, baseband, and zero-mean Gaussian random process, like $I(t)$. Conditioned on *A*, the random vector $\mathbf{W_{rect}}=\left[M_c\ M_c'\ M_s\ M_s'\right]^T$, with $M_s(t)=\pm A\hat{I}(t)+N_s(t)$, is a Gaussian vector with zero mean-vector and the following covariance-matrix:

$$\mathbf{L}=\begin{bmatrix} A^2d_0+b_0 & 0 & 0 & \pm A^2d_1 \\ 0 & A^2d_2+b_2 & \mp A^2d_1 & 0 \\ 0 & \mp A^2d_1 & A^2d_0+b_0 & 0 \\ \pm A^2d_1 & 0 & 0 & A^2d_2+b_2 \end{bmatrix}, \quad (15)$$

with:

$$d_1=E\left[I(t)\hat{I}'(t)\right]=4\pi\int_0^{B_I} f\,G_I(f)\,df\ . \quad (16)$$

Since **L** in (15) and the covariance-matrix in [32] have the same structure, we immediately obtain [32]:

$$p_{ZZ'}(z,z'|A)=\frac{z}{\sqrt{2\pi(A^2d_0+b_0)R}}\exp\left(\frac{z^2}{-2(A^2d_0+b_0)R}\right)$$
$$\times\exp\left(\frac{(A^2d_0+b_0)z'^2}{-2R^2}\right), \quad (17)$$

where:

$$R=(A^2d_0+b_0)(A^2d_2+b_2)-A^4d_1^{\,2}\ . \quad (18)$$

IV.2 Conditional level crossing rate of $Z(t)$

Again due to the similarity between **L** in (15) and the covariance-matrix in [32], it can be shown that [32]:

$$E\left[C_\ell\{Z(t)\}\middle|A\right]=\frac{\sqrt{2}}{\sqrt{\pi}}\frac{1}{A^2d_0+b_0}\ell\exp\left(\frac{-\ell^2}{2(A^2d_0+b_0)}\right)$$
$$\times\sqrt{\frac{(A^2d_0+b_0)(A^2d_2+b_2)-A^4d_1^{\,2}}{A^2d_0+b_0}}\ .(19)$$

V. Numerical Results

Let $X(t)$ be a speech signal with Laplace PDF, so A must have a Rayleigh PDF [22]. We consider $p_A(a)=a\exp(-a^2/2)$. For noise we take $N_0=1$. If signal is band limited such that $B_I=1$, then according to (8), $b_0=2$ and $b_2=26.3$. We have modeled $G_I(f)$ by a beta distribution $\beta(a,b;f)$ with parameters a and b [33], because it conveniently generates different left and right skewed shapes. For $G_I(f)=\beta(1,1.5;f)$ we have $d_0=2$, $d_1=5.03$, $d_2=18.1$, while for $G_I(f)=\beta(1,0.6;f)$ we get $d_0=2$, $d_1=7.85$, $d_2=38$. In the following tables, maximum level crossing rates of $Y(t)$, $Z(t)$, and the levels ℓ_m at which these maxima occur are listed. Note that $\mathrm{SNR}=10\log_{10}(2d_0/b_0)$ dB, because $E[A^2]=2$.

VI. Discussion and Conclusion

According to Table I and Table II, and the results for several other power spectra (not listed here), we observe that for a fixed SNR and a given modulation type, ℓ_m is independent of the shape of the power spectrum, indicating the robustness of ℓ_m. Hence, ℓ_m can serve as a good and reliable criterion for distinguishing DSB from SSB. It is interesting to note that variations of ℓ_m in terms of SNR is less for DSB. Maximum envelope level crossing rates also show good discrimination ability. As SNR increases, the resolution power of both ℓ_m and maximum envelope level crossing rate increase. By taking advantage of both these two features, one can efficiently recognize the underlying modulation type, even when SNR is not large.

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Table I Position and magnitude of the peak of the level crossing for the DSB envelope $Y(t)$ and the SSB envelope $Z(t)$, assuming the power spectrum $\beta(1,1.5; f)$

$G_I(f)$	SNR (dB)	$\max_{\ell} E[C_\ell\{Y(t)\}]$	ℓ_m	$\max_{\ell} E[C_\ell\{Z(t)\}]$	ℓ_m
$\beta(1,1.5; f)$	3	1.66	1.7	1.33	2
$5\times\beta(1,1.5; f)$	10	1.65	2.1	0.93	3
$20\times\beta(1,1.5; f)$	16	1.72	2.7	0.71	5.7

Table II Position and magnitude of the peak of the level crossing for the DSB envelope $Y(t)$ and the SSB envelope $Z(t)$, assuming the power spectrum $\beta(1, 0.6; f)$

$G_I(f)$	SNR (dB)	$\max_{\ell} E[C_\ell\{Y(t)\}]$	ℓ_m	$\max_{\ell} E[C_\ell\{Z(t)\}]$	ℓ_m
$\beta(1, 0.6; f)$	3	1.93	1.8	1.49	2
$5\times\beta(1, 0.6; f)$	10	2.16	2.2	1.08	3
$20\times\beta(1, 0.6; f)$	16	2.39	2.7	0.82	5.7