On the Effect of Correlated Fading on Several Space-Time Coding and Detection Schemes

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Abstract— **Antenna spacings, the angle spread and the Rice factor significantly affect the correlation of fading coefficients in a multi-antenna system. In this paper we study the effect of these parameters on the performance of different types of detection schemes in a BLAST architecture. We show that the detection method based on lattice decoding is more resistant to the correlated fading than other types of detection which work based on successive interference cancellation. We notice that highly correlated fades significantly degrade the performance of detection techniques based on successive interference cancellation whereas they slightly affect the lattice decoding algorithm. We also study the effects of correlated fades on some space-time block codes. We show that space-time block codes from orthogonal design are more resistant to correlation than other high rate linear space-time block codes.**

I. Introduction and System Model

The recent increase in the popularity of multi-antenna systems is due to the large increase in capacity when using antenna arrays at the transmitter and receiver sides over wireless channels [1], [2]. The transmit and receive diversity obtained by space-time (ST) coding techniques in order to combat the wireless channel impairments [3], [4], [5], [6] are other advantageous features of multi-antenna systems. Consider the system of M transmit and N receive antennas in Figure 1, where at the transmitter side we send at each symbol period M independent information symbols $s_1, \ldots, s_M$. We assume that s_j comes from a QAM or a PAM constellation and is sent by the jth transmit antenna, $j = 1 \ldots M$. Notice that all the transmit antennas use the same constellation. We further assume that the frequency flat Rice fading channel is quasi-static over a burst of length T, and then changes randomly every T symbols. In an uncoded multi-antenna system, or Vertical Bell Labs LAyer Space Time (V-BLAST) architecture [3], and after matched filtering and sampling at the symbol rate one has, over the nth receive antenna

$$x_n = \sum_{m=1}^{M} h_{nm} s_m + w_n, \tag{1}$$

where h_{nm} is the fading coefficient between the transmit antenna m and the receive antenna n. The fading coefficients are complex Gaussian random variables with nonzero mean, variance 0.5 per real dimension, $E[|h_{nm}|^2] = 1$, $n = 1 \ldots N$, $m = 1 \ldots M$, and the cross-correlation matrix $\mathbf{R}$. The samples w_n's are assumed to be independent zero mean complex Gaussian random variables with variance σ^2 per real dimension. Over N receivers we have

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{w}. \tag{2}$$

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When using a linear space-time block code $\mathbf{B}$ [5], [6], [4], [7], the baseband received signal over T symbol periods and N receiver antennas can be written as an $N \times T$ matrix as follows

$$\mathbf{X} = \mathbf{H}\mathbf{B} + \mathbf{W}, \tag{3}$$

where $\mathbf{W}$ is an $N \times T$ matrix representing the additive noise matrix and assumed to be white and Gaussian in this paper. In the early papers on multi-antenna system [1]-[6], one assumes the ideal statistical channel model of independent fading which requires sufficient spacing among antennas and a rich scattering environment. These assumptions are not always realistic. Recent works on channel modeling take into account the correlation between fading and are closer to the real channel [8], [9], [10]. In this paper we consider the effect of narrow band correlated fading modeled as in [10] on the detection schemes for V-BLAST and on some ST codes presented in [5]-[7].

II. The channel correlation model:

Consider the wireless connection scheme depicted in Figure 2, where the transmit antenna array which is not surrounded by local scatterers, observes the receiver primarily through a particular direction with a narrow beamwidth Δ (not larger than 10^o for macrocells in urban, suburban, and rural areas). On the other hand, we assume that due to the presence of many surrounding local scatterers, the stationary receive array receives signal from all direction uniformly. Let λ denote the wavelength, while δ and d represent the adjacent antenna spacings at the transmitter and the receiver, respectively. With $\alpha = \beta = 90^o$ (parallel arrays) and K as the Rice factor, defined by the ratio of the line-of-sight (LOS) power to the diffuse power, the cross-correlation between any two channel coefficients can be written as [10]

$$\begin{aligned} \rho_{lp,mq} &= E[h_{lp} h_{mq}^*] = \frac{1}{K+1} J_0\left(\frac{2\pi d}{\lambda} + \Delta \frac{2\pi\delta}{\lambda}\right) \\ &+ \frac{K}{K+1}, \end{aligned} \tag{4}$$

where $J_0(\cdot)$ is the zero order Bessel function of the first kind. Note that $\rho_{lp,mq} \longrightarrow 1$ as $K \longrightarrow \infty$, which implies fully correlated fading.

III. Uncoded V-BLAST: Three Detection Schemes

We study the effect of correlations on the following three detection schemes of the V-BLAST architecture, assuming that the channel matrix $\mathbf{H}$ is available at the receiver:

1. V-BLAST optimal ordering detection [3]: The algorithm performs a multi-stage ZF or MMSE detection along with ordering the statistics of the channel matrix during the successive interference cancellation (SIC) process.

2. QR-iterative detection with optimal ordering and soft decisions [11]: The algorithm performs upper and lower QR decomposition on the channel matrix. It proceeds in two steps. First, by performing a backward SIC using the upper QR decomposition (from the last symbol to the first one), it obtains soft values on the detected symbols. Then it carries out a forward SIC while taking into account the stored soft values from the first step. The two SIC processes can be done using soft decisions and optimal ordering detection, which is the procedure taken in this paper.
3. Sphere decoder [4]: The main idea is to search for the closest constellation point to the received signal within a sphere of a given radius, centered at the received signal. The algorithm exploits the lattice structure of the multi-antenna architecture, in order to obtain near maximum likelihood (ML) performance with polynomial complexity in M [4], [12].
We have chosen these algorithms since the sphere decoder represents the near-ML solution[1] to the detection problem, whereas the V-BLAST and QR with optimal ordering detection cover two main classes of SIC detectors, namely, multi-stage SIC with ZF or MMSE detection, and iterative SIC with soft decisions [13].

IV. Space-times block codes

We study the effect of correlated fades on two types of ST block codes:
1. ST block codes from orthogonal design (OD) [5], [14]: On one hand, these codes achieve the maximum transmit diversity and coding gain and have linear ML decoding algorithms due to their OD; on the other hand, and also due to their OD, they incur a considerable loss in the multi-antenna channel capacity that increases with the number of receivers [16], [17]. For example, for $M = 2$, the Alamouti ST code from OD is given by [5]

$$\mathbf{B} = \boldsymbol{\mathcal{G}}_2 \quad \triangleq \quad \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix}, \tag{5}$$

where $s_1,\ s_2$ are complex information symbols (PSK, QAM), and s^* denotes the complex conjugate of s.
2. High rates linear ST block codes build on algebraic number fields [18], [4], [7]: We distinguish between two classes in this type of codes. First the diagonal algebraic ST (DAST) codes [7] which transmit at 1 symbol per channel use and achieve the maximum transmit diversity for any number of transmitters. Given a rotation $\mathbf{M}$ in dimension M which has a good minimum product distance [15], the DAST code in dimension M is given by

$$\mathbf{B} = \boldsymbol{\Xi}_M \quad = \quad \boldsymbol{\mathcal{H}}\text{diag}(u_1, \ldots, u_M), \tag{6}$$

where $\mathbf{u} = (u_1, \ldots, u_M)^T = \mathbf{M}\mathbf{s}$ with $\mathbf{s}$ the information symbol vector. The matrix $\boldsymbol{\mathcal{H}}$ is set to the Hadamard transform in dimension M if $M = 1, 2$ or if M equals a multiple of 4, and $\boldsymbol{\mathcal{H}} = \mathbf{I}_M$ for other values of M [7]. Second, the "full rate" ST codes which transmit at M symbols per channel use and are constructed to take into account all the degrees of freedom available in the multi-antenna channel[2] [18], [4]. Over M transmitters, the "full rate" ST code is denoted by $\boldsymbol{\Theta}_M$.

[1] At moderate and high SNR the sphere decoder reaches the ML performance with a moderate complexity [12].

[2] The ST codes proposed in [16] can be considered as part of this category and our results in Section V can easily cover them as well.

V. Simulations

We have simulated an uncoded V-BLAST architecture and ST block codes with M transmit and N receive antennas, using 4- and 16-QAM constellations with normalized average energy per symbol $\bar{E}_s = 1$. The random channel matrix with the correlation properties generated as in [10] is fixed over 100 symbol periods. Figure 3 shows the performance of the three detection algorithms with this set of parameters: $K = 0$, $d/\lambda = 1$, $\delta/\lambda = 10$, and $\Delta = 1^o$, 10^o. Note that the small angle spread, $\Delta = 1^o$, degrades the performance of the sphere decoder by ≈ 1 dB, while it degrades the V-BLAST optimal ordering and the QR optimal ordering by about 3 and 4 dB respectively at an error rate of $3 10^{-2}$. Figure 4 presents the effect of LOS with $K = 5\,\text{dB},\ 0$, $d/\lambda = 1$, $\delta/\lambda = 10$, and $\Delta = 10^o$ on V-BLAST architecture. We observe a degradation of ≈ 3.5 dB for the sphere decoder and ≈ 6 for the SIC-based algorithms at an error rate of $3 10^{-2}$. Note that the Rice factor increase implies more degradation on performance than the angle spread since it yields more correlation. This is also noticed on the effects of K and Δ on the multi-antenna capacity [1], [2] as shown in Figure 6. For example at 8 bits per channel use (V-BLAST with 4-QAM and $M = N = 4$), one notes that the small spread angle $\Delta = 1^o$ gives a degradation of 1 dB, whereas a Rice factor of $K = 5$ dB causes a degradation of 3.5 dB in performance. Note that this is the same degradation observed at the output of the sphere decoder in Figures 3 and 4, since the latter gives the optimal solution of the detection of the received signal in (2). Finally we show in Figure 5 the joint effect of decreasing the angle spread Δ and the Rice factor K. We note that while the highly correlated fading coefficients can make the SIC-type based algorithms collapse, the sphere decoder still gives relatively good performance in the considered range of SNR.

Figure 7 shows the effects of K and D on the OD code of rate $1/2$ symbol per channel use [14], and the DAST code $\boldsymbol{\Xi}_4$ [7] with four receivers at 2 bits per channel use. The DAST code $\boldsymbol{\Xi}_4$ outperforms the OD code while achieving the same diversity order because it achieves a higher fraction of the channel capacity [16]; however, the gain in performance[3] decreases from about 4 dB for $K = 0, \Delta = 10^o$ to about 2.3 dB for $K = 0, \Delta = 1^o$, and to about 2 dB for $K = 5\,\text{dB}, \Delta = 10^o$. This shows that the OD code is more resistant to the correlation among fading coefficients than the DAST code which is expected since the former exploits less degrees of freedom of the channel (rate $1/2$), hence it is "less dependent" on the channel's degrees of freedom than the DAST code. The latter observation is emphasized in the case of "full rate" codes as shown in Figure 8, where we present the performance of the Alamouti code [5], and the code $\boldsymbol{\Theta}_2$ [18, Chapter 4], [4] at 4 bits per channel use with $N = 2$ receivers. One notes that the code $\boldsymbol{\Theta}_2$ outperforms the Alamouti code under nearly uncorrelated fades ($K = 0, \Delta = 10^o$), whereas the Alamouti becomes better under highly correlated fading. Again, the explanation is that the ST code $\boldsymbol{\Theta}_2$ exploits all the degrees of freedom available by the channel (over $M = 2$ transmitters and $T = 2$ symbol periods one sends $M \times T = 4$ symbols [18]); hence when restricting the degrees of freedom by intro-

[3] Since both ST codes achieve the same diversity order (the same slope of the error rate curves), their difference in coding gain is the same for moderate and high SNR, which is noticed in the parallel simulated error curves in Figure 7.

ducing correlations among the fades, one affects $\mathbf{\Theta}_2$ more than the Alamouti code which exploits only 2 degrees of freedom of the channel [16].

The Doppler effect, as well as considering the correlation model at the transmitter when using algebraic ST codes in order boost their performance under correlated fading assumption as in [19], will be addressed in the journal version of this paper.

VI. Conclusion

In this paper, we have studied the effect of the angle spread Δ and the LOS parameter K, which introduce correlated fading, on the performance of different types of detection schemes in the uncoded V-BLAST architecture as well as on some space-time block codes. We have shown that for the uncoded system, successive interference cancellation algorithms are sensitive to the variations of these physical parameters, whereas the lattice decoding is more robust to the same variations of these parameters. We have noted that the effect of decreasing the angle spread Δ is less "harmful" than the effect of increasing the LOS parameter K with respect to the achieved capacity or the performance of the detection schemes. For the space-time block codes considered, we have observed that the codes from orthogonal design, which exploit less degrees of freedom offered by the multi-antenna channel than other high rate codes, are more robust to the decrease in the channel's degrees of freedom, induced by correlation, than high rate linear codes.

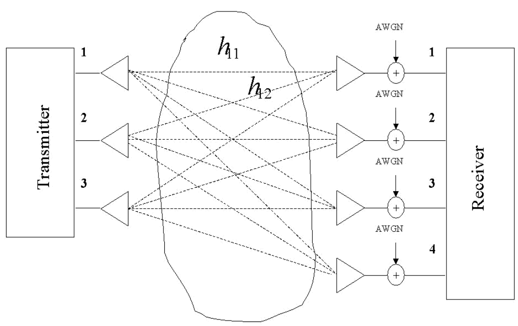

Fig. 1. Multi-antenna system with $M = 3$ transmit and $N = 4$ receive antennas.

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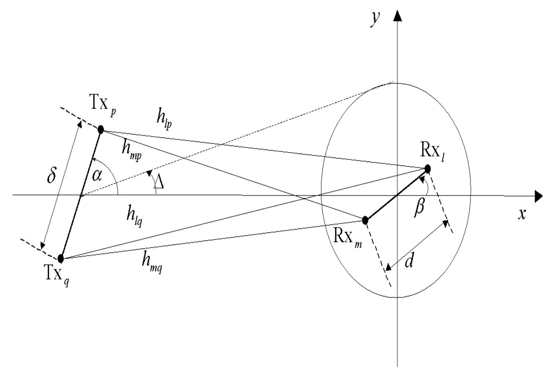

Fig. 2. Geometrical configuration of a 2×2 channel with local scatterers surrounding the stationary receiver.

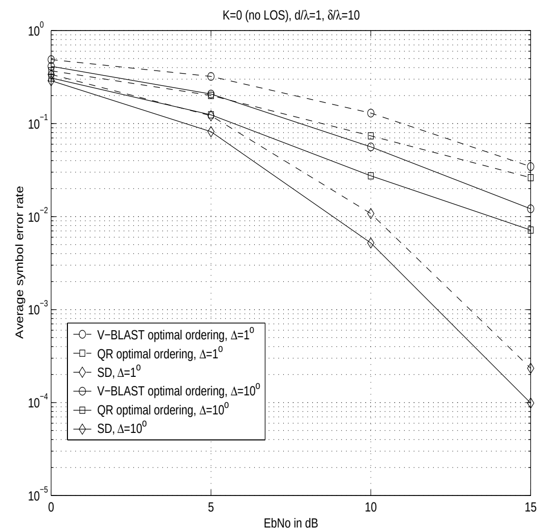

Fig. 3. The effect of the angle spread Δ on the error rate, $M = N = 4$.

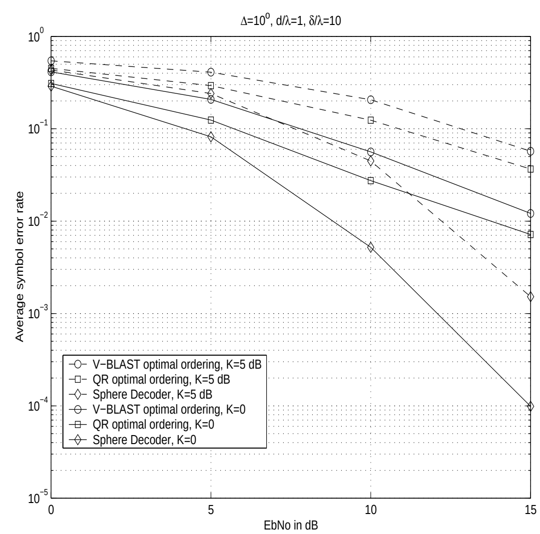

Fig. 4. The effect of the Rice factor K on the error rate, $M = N = 4$.

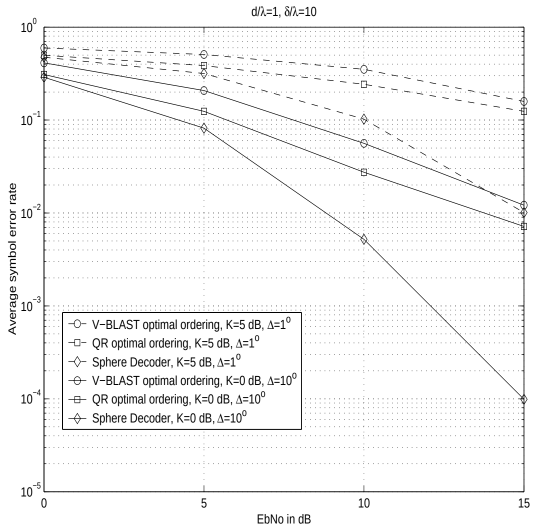

Fig. 5. The effect of the combined angle spread Δ and the Rice factor K on the error rate, $M = N = 4$.

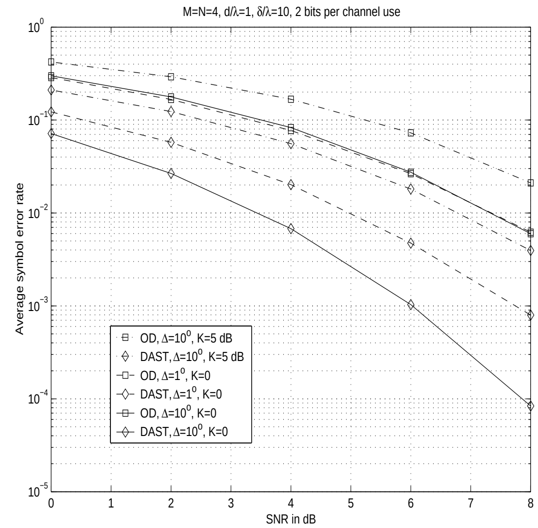

Fig. 7. The effect of the Rice factor K and the angle spread Δ on DAST and OD codes, $M = N = 4$.

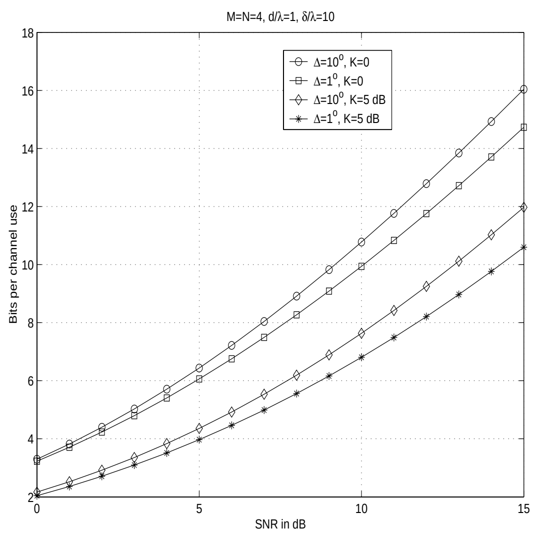

Fig. 6. The effect of the Rice factor K and the angle spread Δ on the channel capacity, $M = N = 4$.

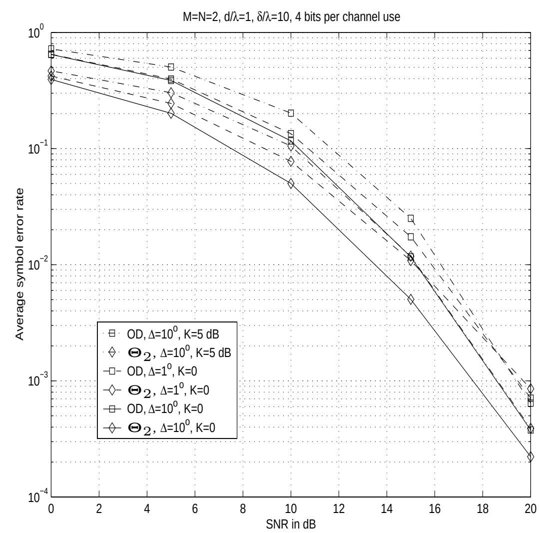

Fig. 8. The effect of the Rice factor K and the angle spread Δ on the "full rate" and the Alamouti codes, $M = N = 2$.

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