Array-based Linear Modulation Classifier with Two-Stage CFO Estimation in Fading Channels

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***Abstract*— Likelihood ratio test (LRT) -based linear modulation classifier is sensitive to unknown parameters, such as carrier frequency offset (CFO), phase shift, etc. An antenna array-based quasi-hybrid likelihood ratio test (qHLRT) approach is proposed to cope with the problem. A non-maximum likelihood (ML) estimator is employed to reduce the computational burden of multivariate maximization. A two-stage CFO estimation scheme is also proposed to increase the accuracy of CFO estimation. To combat channel fading, maximal ratio combining (MRC) technique is applied for CFO estimation as well as the computation of the likelihood functions. The Cramer-Rao lower bound (CRLB) of the proposed CFO estimation method is derived. It is shown that with nonlinear least-squares (NLS) algorithm and method-of-moment (MoM) algorithm to estimate phase and amplitude respectively, our scheme offers an effective and practical solution to recognize linear modulation formats in fading channels.**

I. Introduction

Modulation classification (MC) is an important subject in both commercial and military communication applications, such as software defined radio and electronic warfare. One class of major methods for blind MC, likelihood ratio test approaches, have been extensively studied for decades [1] [2]. The basic idea of the LRT methods is to formulate MC problem as a multiple composite hypothesis testing problem. Under the hypothesis H_i, the i-th modulation is assigned to the incoming signal. LRT methods choose H_i for which the likelihood function (LF) is maximized, assuming that the a priori probabilities of all hypotheses are equal. Unfortunately, [3] shows that using the LRT method in practical applications is restricted by non-cooperative conditions, such as the high sensitivity to unknown parameters. In computing the LF, the unknown parameters can be treated either as random variables (RV) or unknown deterministics, which leads to three likelihood techniques to solve the problem: (a) the average likelihood ratio test (ALRT), where the unknown quantities are treated as RVs with probability density functions (pdf) known and the LF is computed by averaging over them; (b) the generalized likelihood ratio test (GLRT), where the unknown parameters are treated as unknown deterministics. In this case, a logical procedure is to estimate the unknown parameters assuming certain hypothesis is true and use these estimates in the LRT as if they were correct. If ML estimates are used, the test is called GLRT; and (c) the hybrid likelihood ratio test (HLRT), where only the pdfs of several parameters are known, and ML estimates are used for the rest.

Since practically the exact pdf of the unknown parameters are hard to obtain, computation of the LF by averaging over these parameters is quite difficult [2], [4]–[6], whereas ML estimation of them suffers from high computational complexity as well [7].

Such implementation problems motivated us to propose the array-based quasi hybrid likelihood ratio test (qHLRT) classifier, which depicts two major advantages. First, it takes advantage of non-ML parameter estimates to reduce the computational complexity. Second it utilizes the spatial diversity provided by antenna arrays. Furthermore, a two-stage CFO estimator is proposed for this classifier. The amplitude, CFO and phase offset of each antenna branch are estimated separately. Then MRC technique is used to combine the branches. After that CFO is estimated again using the output of MRC. As we will see later on, this increases the accuracy of CFO estimation, due to both array and diversity gains. Consequently, the classification performance is improved, especially in low SNR environment.

The rest of paper is organized as follows; Section II describes the system model. The methods for estimation of unknown parameters are addressed in Section III. Section IV proposed the new classifier with two-stage CFO estimation and the frequency CRLB is derived as well. Numerical results are discussed in Section V, followed by the concluding remarks delivered in Section VI.

II. Problem Formulation

A. Signal Model

Suppose the total number of modulation types to be classified is N_{mod}. As we mentioned before, the classifier chooses the i-th hypothesis H_i (the i -th modulation candidate, $i = 1, 2, \ldots, N_{\text{mod}}$) for which the LF is maximized, assuming that the a priori probabilities of all hypotheses are equal. The computation of the LF of the received signals is over the observation interval $[0, NT]$.

Consider a receiver consisting of L antenna branches with the system model as shown in Fig. 1. Let $\mathbf{r}(t)$ denotes the

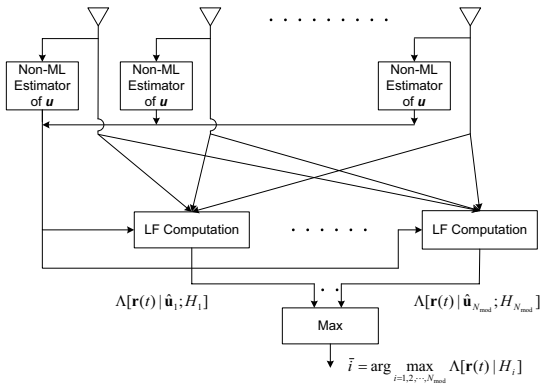


Fig. 1. Array -based qHLRT modulation classifier.

complex envelop of the noise-corrupted received signals

$$\mathbf{r}(t) = \mathbf{s}(t; \mathbf{v}_{i,\text{array}}) + \mathbf{w}(t), 0 \le t \le NT, \tag{1}$$

where $\mathbf{r}(t) = [r_1(t), r_2(t), \ldots, r_L(t)]^T$, and $\mathbf{s}(t; \mathbf{v}_{i,\text{array}}) = [s_1(t; \mathbf{v}_{i,1}), s_2(t; \mathbf{v}_{i,2}), \ldots, s_L(t; \mathbf{v}_{i,L})]^T$. The vector of unknown quantities for the i-th modulation format is denoted by $\mathbf{v}_{i,\text{array}} = \left[\mathbf{u}_i^T,\ \{s_k^{(i)}\}_{k=1}^N\right]^T$. $\mathbf{u}_i = \left[f_e,\ \{\alpha_{0,l}\}_{l=1}^L,\ \{\varphi_{0,l}\}_{l=1}^L\right]^T$ denotes the unknown parameters, which includes carrier frequency offset (CFO) f_e, channel amplitudes $\{\alpha_{0,l}\}_{l=1}^L$ and channel phase shifts $\{\varphi_{0,l}\}_{l=1}^L$. The CFO is assumed to be a small fraction of the symbol rate, say less than 10%. Different amplitude and phase shifts are assumed for each branch whereas the same CFO is considered for all branches. This is a reasonable assumption because all the antenna branches may share the same local oscillator. $\{s_k^{(i)}\}_{k=1}^N$ denote unknown data symbols.

Also, $s_l(t; \mathbf{v}_{i,l})$ stands for the complex envelope of the signal for the i-th modulation at the l-th branch,

$$s_l(t; \mathbf{v}_{i,l}) = \alpha_{0,l} e^{j2\pi f_e t + j\varphi_{0,l}} \sum_{k=1}^{N} s_k^{(i)}(t) g(t - (k-1)T),$$

where $g(t)$ is the root raised cosine pulse shape at the transmitter and all the receiver branches. Finally, $\mathbf{w}(t) = [w_1(t), w_2(t), \ldots, w_L(t)]^T$, where $w_l(t)$ is the complex additive white Gaussian noise (AWGN) with two-sided power spectral density N_0.

B. General Procedure of qHLRT

For the i-th hypothesis, the likelihood function of $\mathbf{r}(t)$, conditioned on the unknown vector $\mathbf{v}_{i,\text{array}}$, is given by [8]

$$\Lambda\left[\mathbf{r}(t)|\mathbf{v}_{i,\text{array}}; H_i\right] = \prod_{l=1}^{L} \exp\left\{ \frac{2}{N_0} \Re\left[\int_0^{NT} r_l(t) s_l^*(t; \mathbf{v}_{i,l}) dt\right] - \frac{1}{N_0} \int_0^{NT} \left|s_l(t; \mathbf{v}_{i,l})\right|^2 dt \right\}, \tag{2}$$

where $\Re$ denotes the calculation of the real part of a complex value.

Due to the unknown quantities in $\mathbf{v}_{i,\text{array}}$, the modulation classification becomes a multiple *composite* hypothesis testing problem. In our proposed qHLRT method, f_e, $\{\alpha_{0,l}\}_{l=1}^L$ and $\{\varphi_{0,l}\}_{l=1}^L$ are considered as unknown deterministics to be estimated, whereas $\{s_k^{(i)}\}_{k=1}^N$ are treated as independent and identically distributed (iid) random variables, averaged over using the conditional pdf of the i-th hypothesis $p(\mathbf{s}_i|H_i)$. Overall, three steps are involved in calculating the qHLRT likelihood function for the i-th modulation.

- **step (a)** The likelihood function, conditioned on the unknown $\mathbf{u}_i$, is averaged over $p(\mathbf{s}_i|H_i)$

$$\Lambda\left[\mathbf{r}(t)|\mathbf{u}_i; H_i\right] = \int \Lambda\left[\mathbf{r}(t)|\mathbf{v}_{i,\text{array}}; H_i\right] p(\mathbf{s}_i|H_i) d\mathbf{s}_i. \tag{3}$$

- **step (b)** Some blind algorithms are applied to estimate the unknown parameter vector $\mathbf{u}_i$, assuming H_i is true.

- **step (c)** By substituting the estimate $\hat{\mathbf{u}}_i$ into (3) we obtain

$$\Lambda\left[\mathbf{r}(t)|H_i\right] = \Lambda\left[\mathbf{r}(t)|\hat{\mathbf{u}}_i; H_i\right]. \tag{4}$$

The decision is made according to the following criterion, to choose $\bar{\imath}$ as the modulation type

$$\bar{\imath} = \arg \max_{i=1,2,\cdots,N_{\text{mod}}} \Lambda\left[\mathbf{r}(t)|H_i\right]. \tag{5}$$

C. Conditional Likelihood Function

Averaging (2) with respect to $\{s_k^{(i)}\}_{k=1}^N$ and bearing in mind that data symbols are independent from each other, yields the likelihood function conditioned only on the unknown parameters $\mathbf{u}_i$ as

$$\Lambda\left[\mathbf{r}(t)|\mathbf{u}_i; H_i\right] = \prod_{k=1}^{N} E_{s_k^{(i)}}\left[\exp\left\{ -\frac{T}{N_0}\left(\sum_{l=1}^{L} \alpha_{0,l}^2\right) \left|s_k^{(i)}\right|^2 + \frac{2}{N_0} \Re\left[\sum_{l=1}^{L} \alpha_{0,l} e^{-j\varphi_{0,l}} R_{k,l}^{(i)}\right]\right\}\right] \tag{6}$$

where we assume $f_e T \ll 1$. $R_{k,l}^{(i)} = s_k^{(i)*} e^{-j2\pi f_e (k-1)T} r_{k,l}$, and $r_{k,l} = \int_0^{NT} r_l(t) g(t-(k-1)T) dt$, which can be regarded as the output of the L matched filters at the receiver, sampled at $t = (k-1)T$, $k = 1, \ldots, N$. Note that $E_{s_k^{(i)}}[\cdot]$ in (6) is nothing but a finite summation over all the M_i possible alphabets of the i-th modulation, divided by M_i, for the k-th interval.

The summations of l from 1 to L in (6) imply that the LF calculation is based on maximal ratio combining. In other words, the antenna array -based LRT algorithm explores the spatial diversity, to improve the classification performance.

To obtain the unconditional LF, the unknown parameter vector $\mathbf{u}_i$ will be estimated via certain non-ML estimator, as described in next section.

III. Non-ML Estimation of Parameters

Since modulation recognition is a non-cooperative communication practice, only non data-aided (NDA) open-loop algo-

rithms are applicable for estimating the unknown parameters.

A. Estimation of CFO and Phase

Several NDA CFO and phase estimators have been investigated in [9]. After comparing their performance, the symbol-rate-sampling nonlinear least-squares (NLS) estimator with a monomial nonlinearity is chosen as it shows a good compromise between complexity and performance.

At the output of the receiver matched filter, with symbol rate sampling, we obtain the discrete-time data as

$$r_n = \alpha_0 s_n^{(i)} T e^{j2\pi f_e nT + j\varphi_0} + w_n, \tag{7}$$

where $w_n = \int_{(n-1)T}^{nT} w(t)dt$ with variance σ_w^2. r_n can be represented in its polar form as $r_n = \rho_n e^{j\phi_n}$. By applying a nonlinear transformation one obtains the sequence y_n as

$$y_n = G(\rho_n) e^{jD\phi_n} \tag{8}$$

where $G(\cdot)$ is a real-valued non-negative nonlinear function and D is an integer which depends on the modulation format. For CFO estimation in QAM modulation, D is set to 4, while in M-PSK modulation, we have $D = M$.

The conventional Viterbi and Viterbi (V&V) [10]-like nonlinearities rely on the monomial transformations $G(\rho_n) = \rho_n^k$, $k = 0, \ldots, 4$, which are simpler to compute than the optimal matched nonlinearities presented in [11]. Define a class of processes as $y_n^{(k)}$, which are obtained via the monomial transformations

$$y_n^{(k)} = \rho_n^k e^{jD\phi_n}, \quad k = 0, 1, \ldots, 4. \tag{9}$$

Also define zero-mean processes $u_n^{(k)} = y_n^{(k)} - E\{y_n^{(k)}\}$. It turns out that $E\{y_n^{(k)}\}$ is a constant amplitude chirp signal, and hence, $y_n^{(k)} = E\{y_n^{(k)}\} + u_n^{(k)}$ can be interpreted as a constant amplitude harmonic embedded in additive noise. The class of monomial NLS estimators are given by

$$\hat{f}_e^{(k)} = \frac{1}{D} \arg \max_{|f_0| < \frac{1}{2}} \frac{1}{N} \left| \sum_{n=0}^{N-1} y_n^{(k)} e^{-j2\pi f_0 n} \right|. \tag{10}$$

The mean square error (MSE) of the NLS CFO estimate gets smaller as SNR increases.

With the estimate of the carrier frequency offset, equalization is applied to remove the CFO from the received signal. The phase offset estimation can be processed once the CFO removal is completed. Two kinds of estimate approaches can be used; one is the NLS method as aforementioned, the other is the method-of-moment approach, which is very attractive because of its easiness in implementation. However, it is shown that the method-of-moment phase estimator essentially is identical to the NLS estimator [8].

Followed the definition of equation (10), the class of monomial NLS phase estimator is given by

$$\hat{\varphi}_0^{(k)} = \frac{1}{D} \text{angle} \left\{ \sum_{n=0}^{N-1} y_n^{(k)} e^{-j2\pi D \hat{f}_e^{(k)} n} \right\}. \tag{11}$$

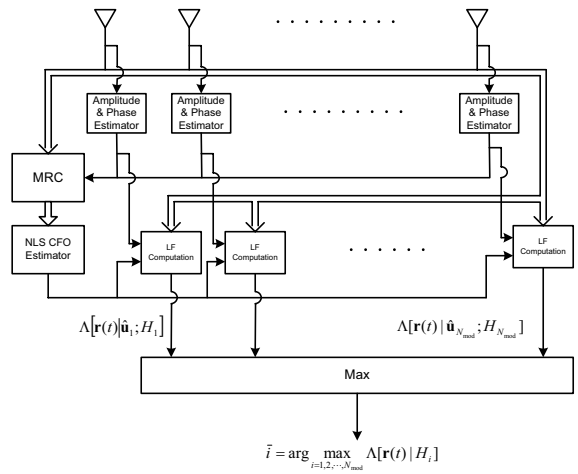


Fig. 2. Robust array-based qHLRT classifier with the two-stage CFO estimator.

In this paper, we use the 4-th order monomial transformation, i.e., $k = 4$.

B. Amplitude Estimation

Method-of-moment (MoM) is selected for amplitude estimation [12], since it does not depend on the phase parameters while the NLS amplitude estimator essentially involves dephasing for amplitude estimation [13].

Let $M_2 = E[|r_n|^2]$ and $M_4 = E[|r_n|^4]$ denote the second and fourth absolute moment of r_n, respectively. Based on (7) and the fact that the noise is independent of the signal, one can show that

$$M_2 = \alpha_0^2 E\left[\left|s_n^{(i)}\right|^2\right] T^2 + N_0 T, \tag{12}$$

$$M_4 = \alpha_0^4 E\left[\left|s_n^{(i)}\right|^4\right] T^4 + 4\alpha_0^2 E\left[\left|s_n^{(i)}\right|^2\right] N_0 T^3 + 2N_0^2 T^2. \tag{13}$$

By calculating $N_0 T$ from (12) and substituting it into (13), eventually we get the basis for estimating α_0

$$\alpha_0^4 = \frac{2M_2^2 - M_4}{\left(2E^2\left[\left|s_n^{(i)}\right|^2\right] - E[|s_n^{(i)}|^4]\right) T^4}. \tag{14}$$

Note that the estimates of the second and fourth absolute moment of r_n are $\hat{M}_2 = N^{-1}\sum_{n=0}^{N-1}|r_n|^2$ and $\hat{M}_4 = N^{-1}\sum_{n=0}^{N-1}|r_n|^4$.

IV. Modulation Classifier with Two-Stage CFO Estimation

The proposed array-based qHLRT linear modulation classification system is depicted in Fig. 2. In order to fully make use of the spatial diversity, MRC technique is applied to assist the computation of likelihood functions for each possible modulation format. MRC is such a combiner that adds the incoming signals after phase correction and amplitude weighting for each branch. Thus, it is necessary to estimation of phase and

amplitude for each branch. At the same time, CFO has to be estimated and corrected as it causes accumulated phase drift. According to the discussion presented in the preceding section, NLS method is used for phase related parameters (CFO and phase offset) estimation, while the MoM approach is applied to estimate amplitude.

The imperfect CFO estimate leads to the progressive rotation of constellation points around the origin. The problem becomes significant as the length of the observation interval NT increases. In other words, the smearing of the constellation points along arcs is more prominent for a larger number of the observed symbols N. It makes the signal recognition more difficult. On the other hand, the calculation of likelihood function might need large number of symbols to achieve good classification performance. These contradicting requirements motivate the introduction of a two-stage CFO estimation scheme in the system.

The first stage of CFO estimation is carried out at each branch separately, together with estimation of amplitude and phase offset, so that one can acquire the parameter estimates for each branch. This paves the road for combining branches via MRC operation. The second stage of CFO estimation works with the output of MRC. It is known [8] that the mean square error of the nonlinear least square CFO estimator becomes smaller as SNR increases. Since MRC is an effective way to enhance SNR, the second NLS CFO estimate using the MRC output offers a better result, which is a critical factor for likelihood function computation and modulation classification. Furthermore, the spatial diversity of the antenna array provides the capability to combat deep fades. As a result, the overall performance of the array-based two-stage CFO estimator is improved in fading channel.

A. Frequency CRLB for BPSK with Known Phase

To evaluate the performance of the second stage CFO estimation, we derive here the frequency CRLB assuming that channel amplitudes and phase offsets are known. The goal is to deliver the idea that the estimation variance of the proposed two-stage approach is roughly reciprocally proportional to the SNR of the MRC output.

Following the same notation as (7), the received complex envelopes for the diversity branches are

$$r_{n,l} = \alpha_l s_n^{(i)} e^{j2\pi f_e n + j\varphi_l} + w_{n,l}, \tag{15}$$

where $l = 1, \cdots, L$ and $n = 1, \cdots, N$. Also we assume $T = 1$ for the presentation brevity. The output of the MRC are

$$\begin{aligned} x_n =& \sum_{l=1}^{L} \alpha_l e^{-j\varphi_l} r_{n,l} \\ =& \sum_{l=1}^{L} \alpha_l^2 s_n^{(i)} e^{j2\pi f_e n} + w'_n, \end{aligned} \tag{16}$$

where the complex fading gain is assumed known. The w'_n's are independent zero-mean complex Gaussian random variables with variance $\sigma_{w'}^2 = \sigma_w^2 \sum_l \alpha_l^2$.

The CRLB on the variance of the estimator of f_e, namely $\hat{f}_e$, for a sequence of N symbols is given by [14]

$$\text{CRLB}(\hat{f}_e) = \frac{1}{-E\left[\frac{\partial^2 \ln p(\mathbf{X}|f_e)}{\partial f_e^2}\right]}, \tag{17}$$

where $p(\mathbf{X}|f_e)$ is pdf of the samples $\mathbf{X}$, which are independent random variables. Considering that the transmit symbols are equally likely, i.e., the pdf of symbols $p_S(s_n^{(i)})$ is independent of n, $p(\mathbf{X}|f_e)$ can be written as

$$\begin{aligned} p(\mathbf{X}|f_e) = \prod_{n=1}^{N} \Bigg[p_S(s^{(i)}) \sum_{s^{(i)} \in \mathbf{C}} \frac{1}{2\pi\sigma_{w'}^2} \exp\Bigg(-\frac{1}{2\sigma_{w'}^2} \times \\ \left| x_n - \sum_{l=1}^{L} \alpha_l^2 s_n^{(i)} e^{j2\pi f_e n} \right|^2 \Bigg) \Bigg], \end{aligned} \tag{18}$$

where $\mathbf{C}$ denotes the constellation set of symbols $s_n^{(i)}$. Obviously, $p(\mathbf{X}|f_e)$ of modulated samples depends on the modulation type. To simplify the evaluation, BPSK modulation is used in the subsequent discussion. With BPSK assumption, $p(\mathbf{X}|f_e)$ can be further expressed as

$$\begin{aligned} p(\mathbf{X}|f_e) = \prod_{n=1}^{N} \Bigg[\frac{1}{2\pi\sigma_{w'}^2} \exp\left(-\frac{|x_n|^2 + |\alpha_M|^2}{2\sigma_{w'}^2} \right) \times \\ \cosh\left(\frac{\Re\left(x_n \alpha_M e^{-j\psi_n}\right)}{\sigma_{w'}^2} \right) \Bigg], \end{aligned} \tag{19}$$

where $\alpha_M = \sum_l \alpha_l^2$, and $\psi_n = 2\pi f_e n$.

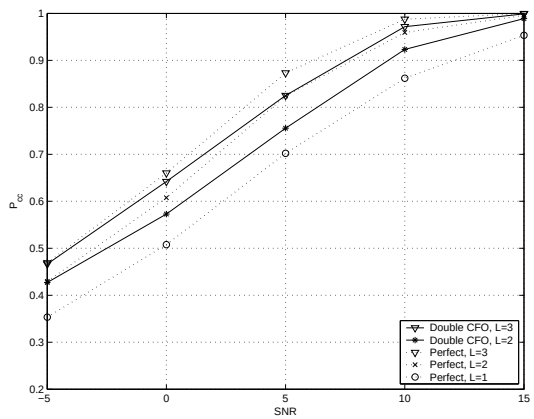
Following the notation adopted in [15], the second partial derivative is

$$\frac{\partial^2 \ln p(\mathbf{X}|f_e)}{\partial f_e^2} = \frac{\alpha_M^2}{\sigma_{w'}^2} \sum_{n=1}^{N} \left[(2\pi n)^2 \Lambda(x_n) \right], \tag{20}$$

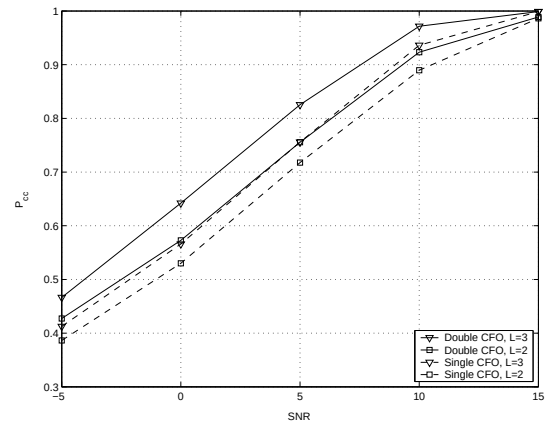
where the definition of $\Lambda(\cdot)$ was given as eqn. (9) in [15]. Using the fact that $E[\Lambda(x_n)]$ is independent of n, and $\sum_{n=1}^{N} (2\pi n)^2 = 2\pi^2 N(N+1)(2N+1)/3$, we obtain the frequency CRLB with known phase for BPSK as

$$\text{CRLB}(\hat{f}_e) = \frac{3}{2\pi^2 N(N+1)(2N+1)\frac{\alpha_M^2}{\sigma_{w'}^2}} \frac{1}{F\left(\frac{\alpha_M^2}{\sigma_{w'}^2}\right)}, \tag{21}$$

where $F\left(\alpha_M^2/\sigma_{w'}^2\right) = -E[\Lambda(x)]$. It can be shown that $F\left(\alpha_M^2/\sigma_{w'}^2\right)$ is monotonic increasing function. In other words, the frequency CRLB decreases as $\alpha_M^2/\sigma_{w'}^2$ increases. In fact, $\alpha_M^2/\sigma_{w'}^2$ is nothing but the signal-to-noise ratio of the MRC output. Therefore, theoretically speaking the CFO estimation performance is improved by making use of the MRC outputs. However, we should note that this conclusion is based on the assumption of perfectly known channel fading gain. The unknown phase and amplitude will of course deteriorate the estimator performance. The CRLB for frequency and phase joint estimation as well as the effect from the unknown fading gain are our ongoing work.

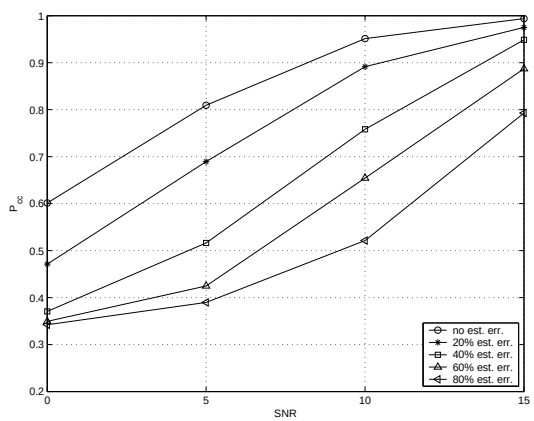


(a) Classifier with double CFO estimates vs. classifier with perfect estimates

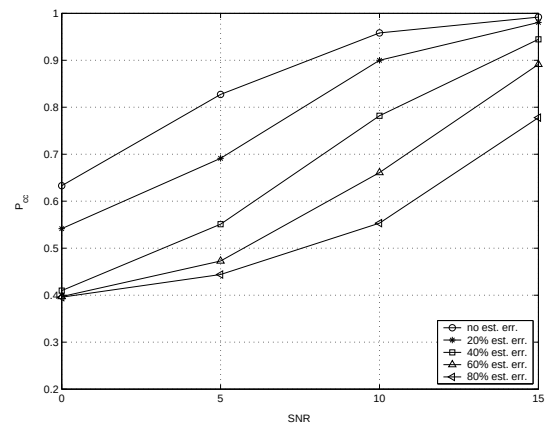


(b) Classifier with double CFO estimates vs. classifier with single CFO estimate

Fig. 3. Performance of the proposed classifier in Rayleigh fading, $f_eT = 0.05$.



(a) qHLRT with double CFO estimates. Modulation pool: 4-QAM,16-QAM and 64-QAM. L=2.



(b) Perfect estimation of unknown parameters. Modulation pool: 4-QAM,16-QAM and 64-QAM. L=2.

Fig. 4. Effect of the imperfect knowledge on noise power.

V. Simulation and Discussion

In this section the proposed array -based qHLRT classifier is examined via computer simulations. The performance of the proposed classifier is compared not only to the classifier with the perfect estimate of unknown parameters, but also to the classifier with single CFO estimation on each branch only. The impact of the imperfect knowledge on the additive noise power is also addressed. Moreover, the effect of spatial correlation among the antennas is examined.

The pool of modulation candidates to be recognized includes 4QAM, 16QAM, 64QAM, BPSK, 8PSK, and 16PSK. The roll-off factor of the raised cosine pulse shaping filter is 0.35. Normalized constellations are generated to assure fair comparison, i.e., $E[|s_k^{(i)}|^2] = 1$, for each i. The received SNR per symbol per branch is defined as $\gamma = S_0T/N_0$, where S_0 is the power of the signal, which due to the normalization is independent of the modulation type. In all the experiments, we set $T = 1$ and $S_0 = 1$, and change SNR by varying N_0. The length of data symbol sequence is $N = 500$, and every simulation result is obtained via performing 1000 Monte Carlo trials. To evaluate the performance of classifiers, we define the probability of correct classification as $P_{cc} = N_{\text{mod}}^{-1} \sum_{i=1}^{N_{\text{mod}}} P_c^{(i|i)}$, where $P_c^{(i|i')}$ is the probability that the i-th modulation is received, where in fact the i' -th modulation has been originally transmitted.

A. Performance Comparison

We plotted in Fig. 3(a), the P_{cc} performance of the array-based qHLRT classifier with two-stage CFO estimates in Rayleigh fading, using solid lines. All the CFO, phase offsets and amplitudes are estimated blindly. The dotted curves show the performance of the classifier with perfect estimates of all the unknown parameters. These curves can be considered as performance benchmarks.

Comparing with the curve with L=1, with all the parameters known, we note that the performance of our proposed classifier is better than any single antenna system. It is because in Rayleigh fading channels, some deep fades account for significant reception performance degradation and multiple receive

antennas take care of them. For example, for an acceptable performance ($P_{cc} \geq 0.9$), a two-antenna qHLRT system can have a 3dB gain over the ideal single antenna system, whereas with three antennas the gain increases to 5dB.

Fig. 3(b) illustrates the improvement of the classifier with two-stage CFO estimates, compared with single CFO estimates. The advantage is more significant in the low SNR regime. The proposed CFO estimation scheme does increase the accuracy of CFO estimation, without extra hardware, and consequently, the overall system performance is boosted compared to the qHLRT approach with single-stage CFO estimation.

B. Effect of the Imperfect Knowledge of Noise Power

Fig. 4 depicts the effect of the imperfect knowledge of noise power on the system performance. Still, Rayleigh fading is considered. Without loss of the generality, we use 4-QAM, 16-QAM and 64-QAM as modulation candidates to be classified. Each curve represents different estimation error percentage of the additive noise power σ_w^2. In Fig. 4(a), we observe that the imperfect noise power estimate does affect the proposed scheme. However, when compared to Fig. 4(b), which shows the impact on the classifier without unknown parameters (CFO, amplitude and phase offset), we find that such degradation occurs not only in the proposed qHLRT scheme, but also in any LRT-based algorithm. To explain the phenomenon, one may argue that the noise power is only involved in the computation of the likelihood function.

C. Effect of Correlation among the Antennas

In the derivation and simulation of the array classifier in previous Sections, the antenna elements were assumed to be far apart enough so that the branches were uncorrelated. To see the effect of correlation on the performance, in Fig. 5 we examined the ALRT with perfect estimates and the qHLRT with the double CFO setup, both with $L = 2$. The correlation between the two antennas is defined by

$$C = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \tag{22}$$

where $\rho_{11} = \rho_{22} = 1$, and $\rho_{12} = \rho_{21} = \rho$ is simulated with $0 \leq \rho \leq 1$. As expected, P_{cc} decrease when ρ increases. The performance degradation seems to be less at high SNRs. Nevertheless, our array classifier which assumes uncorrelated branches, appear to be reasonably robust to some possible correlations that may exist between the branches.

VI. Conclusion

A quasi-hybrid likelihood ratio test approach, combined with antenna arrays, is employed to classify linear modulation signals with unknown parameters. To increase the accuracy of CFO estimation a two-stage CFO estimation scheme is proposed. The maximal ratio combining technique, applied before the second CFO estimator as well as to the computation of likelihood functions, provides an efficient means to handle channel fading. As shown in the simulation results, our scheme offers an effective and practical solution to recognize linear modulation formats in fading channels.

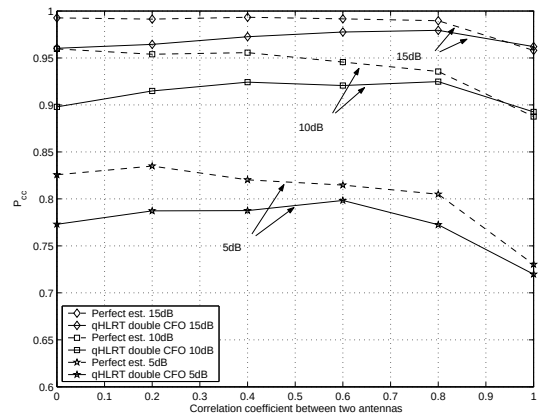


Fig. 5. Effect of the correlation among the antennas, modulation pool: 4-QAM,16-QAM and 64-QAM, L=2.

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