Nonuniform Distribution of Phases in Indoor Multipath Channels and Its Effect on the Statistics of the Spatial Envelope Fading

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EXTENDED SUMMARY

When a signal propagates through a multipath propagation environment, it breaks into several multipath components; and at the receiver, the superposition of these multipath components is observed. In general, the phase of each component, with respect to an arbitrary but fixed reference, depends on its path length. It changes by 2π as its path length changes by a wavelength (30 cm at 1 GHz). Considering the geometry of paths in an indoor radio propagation channel, moderate changes (on the order of meters) in the position of the receiver result in great changes for the phases. When one considers a large ensemble of points in space, therefore, it is reasonable to expect uniform distribution for the phases. For small sampling distances, however, great deviations from uniformity occur. As a result, statistics of the envelope, when the receiver moves over very short distances, deviate from that predicted based on the standard assumption of uniform phase distributions. It is the purpose of this paper to explore the envelope statistics assuming nonuniform phase distributions.

To the best of our knowledge, only few authors have paid attention to the nonuniform distribution of phases, based on two different points of view. In the first approach, geometrically-based methods are developed in [1]-[3] for simulating the envelope; while based on the second approach, [4]-[6] have derived the univariate envelope probability density function (PDF) theoretically. The first approach does not provide closed-form expressions for the first and second order statistics of the envelope, which can significantly facilitate performance prediction and system design tasks. Therefore, we take the second approach outlined in [5], which in comparison to [4] and [6] is more appropriate for application to the indoor multipath fading channel. In what follows, we address average bit error rate (BER) calculation and level crossing rate (LCR) for the Beckmann-I [5] envelope PDF.

Let the number of multipath components be a large constant. Let the multipath components be independent of each other, and the phase of each multipath component have a symmetric distribution about zero, independent of the corresponding amplitude of that multipath component. Then based on the central limit theorem, in-phase, $I_c(t)$, and

quadrature, $I_s(t)$, components of the received signal become independent Gaussian random variables such that $E[I_c(t)] = \mu_c \neq E[I_s(t)] = 0$, $Var[I_c(t)] = \sigma_c^2 \neq Var[I_s(t)] = \sigma_s^2$, where E and Var denote expectation and variance, respectively. Under these assumption, the Beckmann-I PDF for the envelope $R(t) = \sqrt{I_c^2(t) + I_s^2(t)}$ can be written as [5]:

$$p_R(r) = \frac{r}{2\pi\sigma_c\sigma_s}\int_0^{2\pi} \exp\left[-\frac{(r\cos\gamma - \mu_c)^2}{2\sigma_c^2} - \frac{r^2\sin^2\gamma}{2\sigma_s^2}\right] d\gamma \,. \tag{1}$$

This PDF can also be expressed as an infinite series containing Bessel function products [5] or Legendre function [4].

Average BER calculations for the Beckmann-I PDF may not yield simple and/or closed-form formulas, because of its complicated integral form. Nevertheless, we have shown that the moment generating function (MGF) of R^2, the power, defined by $\phi_{R^2}(z) = E[\exp(-zR^2)]$, has a simple form:

$$\phi_{R^2}(z) = \frac{1}{\sqrt{(1+2\sigma_c^2 z)(1+2\sigma_s^2 z)}} \exp\left(-\frac{\mu_c^2 z}{1+2\sigma_c^2 z}\right). \tag{2}$$

As discussed in [7], we can get closed-form expressions and numerically efficient solutions for the average BER of different modulation schemes using the MGF. For example, average BER of binary DPSK is given by $0.5\,\phi_{R^2}(E_b/N_0)$.

Regarding the second order statistics of Beckmann-I PDF, we have derived its LCR as follows:

$$LCR\{R,\ell\} = \frac{2\sigma_c'\sigma_s'\ell}{(2\pi)^{3/2}\sigma_c\sigma_s}\int_0^{2\pi}\left(\frac{\sin^2\theta}{\sigma_c'^2} + \frac{\cos^2\theta}{\sigma_s'^2}\right)^{1/2} \exp\left[-\frac{(\ell\cos\theta - \mu_c)^2}{2\sigma_c^2} - \frac{\ell^2\sin^2\theta}{2\sigma_s^2}\right] d\theta \,, \tag{3}$$

where ℓ is the level, and $\sigma_c'^2 = Var[I_c'(t)] \neq \sigma_s'^2 = Var[I_s'(t)]$, with $I_c'(t)$ and $I_s'(t)$ as time derivatives of $I_c(t)$ and $I_s(t)$.

It should be pointed out that Beckmann-I PDF in (1) and the new results of this contribution in (2) and (3) include well-known results as special cases. For $\mu_c = 0$ and $\sigma_c^2 = \sigma_s^2$ we get Rayleigh statistics. $\mu_c \neq 0$ and $\sigma_c^2 = \sigma_s^2$ result in Rice statistics. Hoyt [5] statistics, not deeply studied in the literature, comes out when $\mu_c = 0$ and $\sigma_c^2 \neq \sigma_s^2$.

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