A qHLRT Modulation Classifier with Antenna Array and Two-Stage CFO Estimation in Fading Channels

Hong Li*, Yeheskel Bar-Ness*, Ali Abdi*, and Wei Su†

*Center for Wireless Communication and Signal Processing Research
ECE Department, New Jersey Institute of Technology, Newark, NJ 07102 USA
Email: {hl8, yeheskel.barness, ali.abdi}@njit.edu

†U.S. Army RDECOM CERDEC, Fort Monmouth, NJ 07703 USA

***Abstract*— A likelihood ratio test (LRT) -based modulation classifier is sensitive to unknown parameters, such as carrier frequency offset (CFO), phase shift, etc. To better handle this problem, a robust antenna array -based quasi-hybrid likelihood ratio test (qHLRT) approach is proposed in this paper. A non-maximum likelihood (ML) estimator is employed to reduce the computational burden of multivariate maximization. A double CFO estimation scheme is also proposed, which increases the accuracy of CFO estimation. To deal with channel fading, maximal ratio combining approach is applied for CFO estimation as well as the computation of the likelihood functions. It is shown that when implementing with the nonlinear least-squares (NLS) phase parameters estimator and the method-of-moment (MoM) amplitude estimator, our scheme offers an effective way to recognize linear modulation formats with unknown parameters in fading channels.**

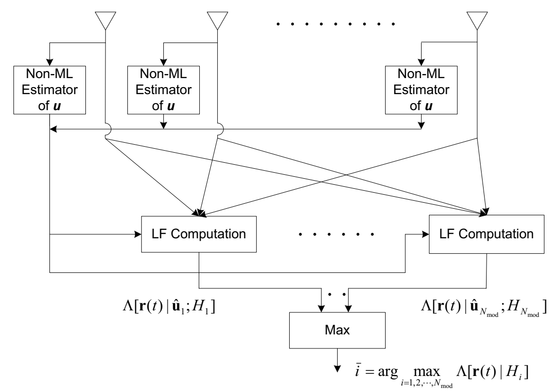


Fig. 1. Array -based qHLRT modulation classifier.

I. Introduction

Likelihood ratio test approaches for blind modulation recognition have been extensively studied since early nineties [1] [2]. However, using this method in practical applications is restricted by non-cooperative conditions, such as the high sensitivity to unknown parameters [3]. Moreover, normally in practice the probability density function (pdf) of the unknown parameters is not available. Even further, averaging over these parameters to obtain the average likelihood function is quite difficult [2], [4]–[6], whereas ML estimation of them suffers from high computational complexity as well [7].

Such implementation problems stimulated us to propose the array-based quasi hybrid likelihood ratio test (qHLRT) classifier. It utilizes the spatial diversity provided by antenna arrays, and takes advantage of non-maximum likelihood (ML) parameter estimates to reduce the computational complexity. Furthermore, a double CFO estimator is proposed for this classifier. First the CFO, phase and amplitude of each branch are estimated separately. Then the branches are combined using the MRC technique. After that CFO is estimated again using the output of MRC. As we see in the sequel, this increases the accuracy of CFO estimation, due to both array and diversity gains. Consequently, the classification performance is improved, especially in low SNR environment.

II. System Model

Consider a receiver consisting of L antenna branches with the system model is shown in Fig. 1. The likelihood-based approach for modulation classification requires the computation of the likelihood function of the noise-corrupted received signals $\mathbf{r}(t)$

$$\mathbf{r}(t) = \mathbf{s}(t; \mathbf{v}_{i,\text{array}}) + \mathbf{w}(t), \tag{1}$$

over the observation interval $0 \leq t \leq NT$, where $\mathbf{r}(t) = [r_1(t), r_2(t), \ldots, r_L(t)]^T$, $\mathbf{w}(t) = [w_1(t), w_2(t), \ldots, w_L(t)]^T$, $\mathbf{s}(t; \mathbf{v}_{i,\text{array}}) = [s_1(t; \mathbf{v}_{i,1}), s_2(t; \mathbf{v}_{i,2}), \ldots, s_L(t; \mathbf{v}_{i,L})]^T$, $\mathbf{v}_{i,\text{array}} = \left[\mathbf{u}_i^T,\ \{s_k^{(i)}\}_{k=1}^N\right]^T$, $\mathbf{u}_i = \left[f_e\ \{\alpha_{0,l}\}_{l=1}^L,\ \{\varphi_{0,l}\}_{l=1}^L\right]^T$ denotes the vector of unknown quantities for the i-th modulation format, which includes carrier frequency offset (CFO) f_e, channel amplitudes $\{\alpha_{0,l}\}_{l=1}^L$ and channel phase shifts $\{\varphi_{0,l}\}_{l=1}^L$. Different amplitude and phase shifts are assumed for each branch whereas the same CFO is considered for all branches. This is a reasonable assumption because all the antenna branches may share the same local oscillator.

Suppose the total number of modulation types to be classified is N_{mod}. The classifier chooses the i-th hypothesis H_i (the i-th modulation candidate, $i = 1, 2, \ldots, N_{\text{mod}}$) for which the likelihood function is maximized, assuming that the a priori probabilities of all hypotheses are equal.

With complex additive Gaussian white noise (AWGN) $\mathbf{w}(t)$ and for the i-th hypothesis, the likelihood function of $\mathbf{r}(t)$, conditioned on the unknown vector $\mathbf{v}_{i,\text{array}}$, is given by [8]

$$\Lambda\left[\mathbf{r}(t)|\mathbf{v}_{i,\text{array}}; H_i\right] = \prod_{l=1}^{L} \exp\left\{ \frac{2}{N_0} \Re\left[\int_0^{NT} r_l(t) s_l^*(t; \mathbf{v}_{i,l}) dt \right] - \frac{1}{N_0} \int_0^{NT} \left| s_l(t; \mathbf{v}_{i,l}) \right|^2 dt \right\}, \quad (2)$$

where $s_l(t; \mathbf{v}_{i,l}) = \alpha_{0,l} e^{j2\pi f_e t + j\varphi_{0,l}} \sum_{k=1}^{N} s_k^{(i)}(t) g(t - (k-1)T)$, which stands for the complex envelope of the signal for the i-th modulation at the l-th branch, $\{s_k^{(i)}\}_{k=1}^N$ denote unknown data symbols, N_0 is the two-sided power spectral density of AWGN. $g(t)$ is the root raised cosine pulse shape at the transmitter and all the receiver branches, the convolution $g(t) \otimes g(-t)$ is Nyquist, i.e.,

$$\int_{-\infty}^{\infty} g(t) g(t - kT) dt = \begin{cases} T & k = 0 \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

$\Re$ denotes the calculation of the real part of a complex value.

Due to the unknown quantities in $\mathbf{v}_{i,\text{array}}$, the modulation classification becomes a multiple *composite* hypothesis testing problem. In our proposed qHLRT method, f_e, $\{\alpha_{0,l}\}_{l=1}^L$ and $\{\varphi_{0,l}\}_{l=1}^L$ are considered as unknown deterministics to be estimated, whereas $\{s_k^{(i)}\}_{k=1}^N$ are treated as independent and identically distributed (iid) random variables, averaged over using the conditional pdf of the i-th hypothesis $p(\mathbf{s}_i|H_i)$. Overall, three steps are involved in calculating the qHLRT likelihood function for the i-th modulation.

- **step (a)** The likelihood function, conditioned on the unknown $\mathbf{u}_i$, is averaged over $p(\mathbf{s}_i|H_i)$

$$\Lambda\left[\mathbf{r}(t)|\mathbf{u}_i; H_i\right] = \int \Lambda\left[\mathbf{r}(t)|\mathbf{v}_{i,\text{array}}; H_i\right] p(\mathbf{s}_i|H_i) d\mathbf{s}_i. \quad (4)$$

- **step (b)** Some blind algorithms are applied to estimate the unknown parameter vector $\mathbf{u}_i$, assuming H_i is true.
- **step (c)** By substituting the estimate $\hat{\mathbf{u}}_i$ into (4) we obtain

$$\Lambda\left[\mathbf{r}(t)|H_i\right] = \Lambda\left[\mathbf{r}(t)|\hat{\mathbf{u}}_i; H_i\right]. \quad (5)$$

The decision is made according to the following criterion, to choose $\bar{\imath}$ as the modulation type

$$\bar{\imath} = \arg \max_{i=1,2,\cdots,N_{\text{mod}}} \Lambda\left[\mathbf{r}(t)|H_i\right]. \quad (6)$$

III. Conditional Likelihood Function

Averaging (2) with respect to $\{s_k^{(i)}\}_{k=1}^N$ and bearing in mind the independence of data symbols, yields the likelihood function conditioned only on the unknown parameters $\mathbf{u}_i$ as

$$\Lambda\left[\mathbf{r}(t)|\mathbf{u}_i; H_i\right] = \prod_{k=1}^{N} E_{s_k^{(i)}} \left[\exp\left\{ -\frac{T}{N_0} \left(\sum_{l=1}^{L} \alpha_{0,l}^2 \right) \left| s_k^{(i)} \right|^2 + \frac{2}{N_0} \Re\Big[\sum_{l=1}^{L} \alpha_{0,l} e^{-j\varphi_{0,l}} R_{k,l}^{(i)} \Big] \right\} \right] \quad (7)$$

where by assuming $f_e T \ll 1$, $R_{k,l}^{(i)} = s_k^{(i)*} e^{-j2\pi f_e (k-1)T} r_{k,l}$, $r_{k,l} = \int_0^{NT} r_l(t) g(t - (k-1)T) dt$, which can be regarded as the output of the L matched filters at the receiver, sampled at $t = (k-1)T$, $k = 1, \ldots, N$. Note that $E_{s_k^{(i)}}[\cdot]$ in (7) is nothing but a finite summation over all the M_i possible alphabets of the i-th modulation, divided by M_i, for the k-th interval.

The summations of l from 1 to L in (7) indicate that the computation of the likelihood function of the antenna array is based on maximal ratio combining. In other words, the antenna array -based LRT algorithm explores the spatial diversity, to improve the classification performance.

In the discussed qHLRT scheme, the unknown parameter vecter $\mathbf{u}_i$ will be estimated via certain non-ML estimator, as described in next section. The criterion applied for decision on the modulation type is given in (6).

IV. Non-ML Estimation of Parameters

Since modulation recognition is a non-cooperative communication task, only non data-aided (NDA) open-loop algorithms are applicable for estimating the unknown parameters. Several NDA CFO and phase estimators are studied in [9]. After comparing their performance, the symbol-rate-sampling nonlinear least-squares (NLS) estimator with a monomial nonlinearity is chosen as it shows a good compromise between complexity and performance. The mean square error (MSE) of the NLS CFO estimate gets smaller as SNR increases. Also note that the NLS estimates of the phase and amplitude of each branch are decoupled.

Method-of-moment (MoM) is selected for amplitude estimation [10], since it does not depend on the phase parameters while the NLS amplitude estimator essentially involves dephasing for amplitude estimation [11].

V. Robust Classifier with Two-Stage CFO Estimation

Fig. 2 depicts the proposed system. Consider the antenna array with L elements. To fully make use of the spatial diversity, MRC scheme is used to calculate likelihood functions for each possible modulation format. MRC is such a combiner that adds the incoming signals after phase correction and amplitude weighting for each branch. Thus, estimation of phase and amplitude for each branch is a must. At the same time, CFO has to be estimated and corrected as it causes accumulated phase shift. According to the investigation results presented in [9], NLS method is used for phase related parameters (CFO

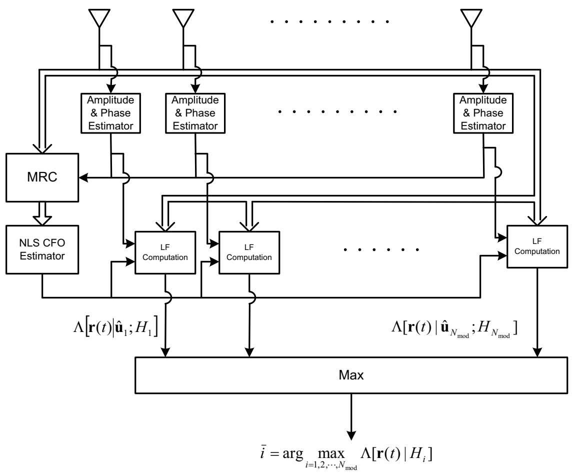


Fig. 2. Robust array-based qHLRT classifier with the two-stage CFO estimator.

and phase offset) estimation, whereas the MoM approach is applied to estimate amplitude.

It is well known that the imperfect estimate of CFO leads to the progressive rotation of constellation points around the origin. The problem becomes significant as the length of the observation interval NT increases. On the other hand, the calculation of likelihood function might need large number of symbols to achieve good classification performance. These contradicting requirements motivate the introduction of a two-stage CFO estimation scheme in the system.

In order to acquire the parameter estimates for each branch, the first stage of CFO estimation is carried out at each branch individually, together with amplitude and phase offset estimation. This paves the road for combining branches via MRC operation. The second stage of CFO estimation works with the output of MRC. It is known [8] that the mean square error of the nonlinear least square CFO estimator becomes smaller as SNR increases. Since MRC is an effective way to enhance SNR, the second NLS CFO estimate using the MRC output provides a better result, which is a critical factor for likelihood function computation and modulation classification. Furthermore, the spatial diversity of the antenna array offers the capability to combat deep fades. As a result, the performance of the array -based two-stage CFO estimator is improved in fading channel.

VI. SIMULATION AND DISCUSSION

In this section the proposed robust array -based qHLRT classifier is examined via computer simulations. The performance of the proposed classifier is compared not only to the classifier with the error-free estimate of unknown parameters, but also to the classifier with single CFO estimation on each branch only. The effect of spatial correlation among the antennas is also examined.

In the simulations, the pool of candidate modulations to be classified includes 4QAM, 16QAM, 64QAM, 2PSK, 8PSK, and 16PSK. Raised cosine pulse shaping filter is used with a roll-off factor of 0.35. Normalized constellations are generated to assure fair comparison, i.e., $E[|s_k^{(i)}|^2] = 1$, for each i. The received SNR per symbol per branch is defined as $\gamma = S_0T/N_0$, where S_0 is the power of the signal, which due to the normalization is independent of the modulation type. In all the experiments, we set $T = 1$ and $S_0 = 1$, and change SNR by varying N_0. The length of data symbol sequence is $N = 500$, and every simulation result is obtained via performing 1000 Monte Carlo trials. To evaluate the performance of classifiers, we define the probability of correct classification as $P_{cc} = N_{\text{mod}}^{-1} \sum_{i=1}^{N_{\text{mod}}} P_c^{(i|i)}$, where $P_c^{(i|i')}$ is the probability that the i-th modulation is received, where in fact the i' -th modulation has been originally transmitted.

A. Performance Comparison

The P_{cc} performance of the proposed array -based qHLRT classifier with double CFO estimates in Rayleigh fading is plotted in Fig. 3(a) using solid lines. All the CFO, phase offsets and amplitudes are estimated blindly. The curves plotted using dotted lines show the performance of the classifier with perfect estimates of all the unknown parameters. These curves can be considered as performance benchmarks.

Comparing with the curve with L=1, with all the parameters known, we note that the performance of our proposed classifier is better than any single antenna system. It is because in Rayleigh fading channels, some deep fades account for significant reception performance degradation and multiple receive antennas take care of them. For instance, for an acceptable performance ($P_{cc} \geq 0.9$), a two-antenna qHLRT system can have a 3dB gain over the ideal single antenna system, whereas with three antennas the gain increases to 5dB.

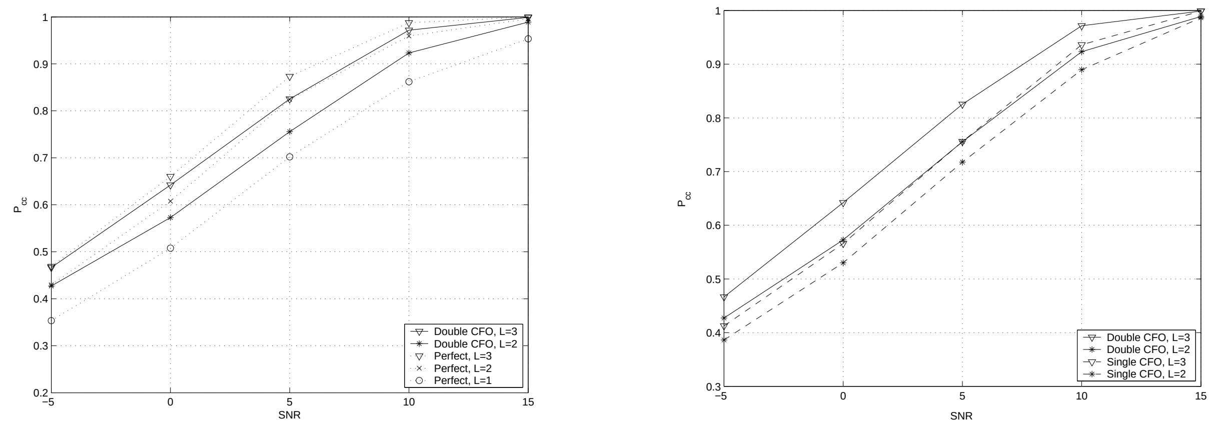
Fig. 3(b) illustrates the improvement of the classifier with two-stage CFO estimates, compared with single CFO estimates. The advantage is more significant in the low SNR regime. It is well known that unknown CFO impacts seriously the classification result. The proposed double CFO estimation scheme does increase the accuracy of CFO estimation, without extra hardware, and consequently, the overall system performance is boosted compared to the qHLRT approach with single-stage CFO estimation.

B. Effect of Correlation among the Antennas

In the derivation and simulation of the array classifier in previous Sections, the antenna elements were assumed to be far apart enough so that the branches were uncorrelated. To see the effect of correlation on the performance, in Fig. 4 we examined both ALRT with perfect estimates and the qHLRT with the double CFO setup, both with $L = 2$. The correlation between the two antennas is defined by

$$C = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \tag{8}$$

where $\rho_{11} = \rho_{22} = 1$, and $\rho_{12} = \rho_{21} = \rho$ is simulated with $0 \leq \rho \leq 1$. As expected, P_{cc} decrease when ρ increases. The performance degradation seems to be less at high SNRs.



(a) Classifier with double CFO estimates vs. classifier with perfect estimates

(b) Classifier with double CFO estimates vs. classifier with single CFO estimate

Fig. 3. Performance of the proposed classifier in Rayleigh fading, $f_e T = 0.05$.

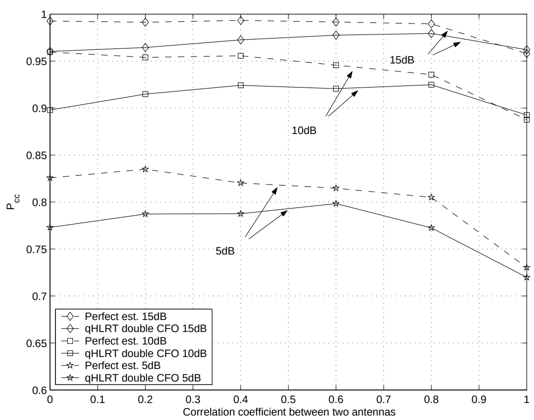


Fig. 4. Effect of the correlation among the antennas, modulation pool: 4-QAM,16-QAM and 64-QAM, L=2.

Nevertheless, our array classifier which assumes uncorrelated branches, appear to be reasonably robust to some possible correlations that may exist between the branches.

VII. Conclusion

A quasi-hybrid likelihood ratio test approach, combined with antenna arrays, is employed to classify signals with unknown carrier frequency offset, phases and amplitudes. A double CFO estimation scheme is proposed to increase the accuracy of CFO estimation. The maximal ratio combining technique, applied before the second CFO estimator as well as to the computation of likelihood functions, provides an efficient method to deal with channel fading. For estimation of phase-related parameters, the symbol-rate-sampling nonlinear least-squares estimator with a monomial nonlinearity which shows a good trade-off between complexity and performance is used, whereas for amplitude estimation the method-of-moment estimator performs better. As shown in the simulation results, our scheme offers an effective way to recognize linear modulation formats with unknown parameters in fading channels.

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